THE MECHANICS OF A STOCHASTIC CORPORATE FINANCIAL MODEL

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Abstract

Much has been written in recent years about the types of factors that should be considered in a dynamic financial analysis model. Much less has been written that actually provides a reader with an understanding of how the various pieces of a dynamic financial analysis model need to fit together. This paper is intended to provide a reader with a look "under the covers" at the structure of a model being used for dynamic financial analysis.

A second and equally important aspect of dynamic financial analysis is the determination of appropriate, or at least reasonable, parameters for different elements within the model's mechanical framework. Unfortunately, those organizations that have been the most active in the development of model parameters are in the uncomfortable position of having to choose between divulging the specifics of their parameterization studies, at the risk of losing a competitive advantage, or keeping the knowledge to themselves, to the long-term detriment of the actuarial profession's ability to effectively use models of this nature. The authors of this paper are no less constrained by our respective organizations. As such, we have largely excluded model parameterization from the subject matter of this paper.

1. INTRODUCTION

There are many facets to the problem of corporate financial model development. It is useful to begin with an analogy to the common actuarial problem of distinguishing between specification, parameter, and process risks. Specification risk relates to the question "Are the model structure and the selected probability distributions correct?" Parameter risk narrows the question to "Assuming the specification is correct, are the distributional parameters correct?" Lastly, process risk is concerned with randomness; i.e., answering the question "Assuming everything else is correct, what can happen in my universe of possible outcomes?"

One might quibble between modeling loss severity with a Weibull distribution instead of a lognormal distribution. Selecting from the universe of possible probability distributions in model design is coping with *specification* risk. In some situations, the specification risk may degenerate into subjective probability assessment—the knowledge set about a dynamic process may be so sparse that a rigorous description of the underlying probability distribution is not possible.[1] Even after this exercise is completed successfully, the analyst still must deal with describing the parameters of the chosen process model. This second stage investigation is the source of *parameter* risk. This risk involves the selection of incorrect parameters, even if the probability distribution is correctly chosen. This leaves only *process* risk to address. Ideally, process risk becomes insignificant under the weight of many, many recalculations of the model.

In financial modeling, there are many of these "risks," and the model designer should not be oblivious to them. The model designer must leap many hurdles while formulating a corporate financial model, particularly one for dynamic financial analysis (DFA). Examples of hurdles to be overcome or pitfalls to be avoided include:

1. The model can use the wrong equations when attempting to define causality or linkages among model constants and variables.

[1] The mathematics describing the fitting of distributions with only sparse knowledge of the underlying risk characteristics is described by Filshtein [4].

2. Important components of the operational or economic environment might be omitted.
3. Elements that should be rendered in a dynamic manner are kept static.
4. Model designers can be consumed by uncertainty regarding the dynamic behavior of those components deemed to be dynamic.
5. The model's accounting framework may be inaccurate.
6. The model could contain programming problems or other embedded divergent behavior.
7. It might not be possible to achieve a consensus among decision makers about the metrics (i.e., output results) of comparison.
8. Model results may not exhibit one clearly preferable alternative among different strategies under investigation.
9. Model results cannot be implemented or only can be implemented with constraints (e.g., the decision path that leads to the "best" long-term outcome is not feasible, either because it violates internal management operating constraints or regulatory boundaries).
10. The model can expand to consider such a wide array of possible situations, interrelationships, and outcomes that it becomes too time-consuming to use in a realistic and useful manner.

In summary, the risks include functional mis-specification of the model, errors in risk and process identification, and failure of the accounting framework to adequately divulge the metrics needed for decision making.

Let us begin with a disclaimer to all readers who hope to find an easy recipe for modeling. There is no panacea for model, functional, or dynamic variable mis-specification. Very often, there is not even a good place to start looking for a definition. With that in mind, we believe that (a) a definition that describes the event in question is better than no definition at all and (b) it is not worth quibbling over the finer points of parameterization—in the overall perspective of what we are trying to model, the error introduced by using a Weibull instead of a lognormal distribution to fit empirical claims severity data is not going to make or break our results.

We now turn to the three key concepts that form the basis for this paper:

- The model to be discussed is a *corporate financial model*; one that already has been deployed in the marketplace.
- The model is *stochastic*, with the capability of being made *dynamic.*
- There is often no clearly preferable solution among alternative decision paths.

2. KEY CONCEPTS

Corporate Financial Model

Day-to-day operations of a property/casualty insurance company include buying and selling assets, underwriting insurance policies, collecting premiums, administering claims, and running the insurance enterprise. A financial model of a property/casualty insurance enterprise needs to be able to model each of these operations separately and in conjunction with each other in order to produce realistic financial projections for the entity.

In order to perform a comprehensive dynamic financial analysis, a corporate financial model should have linkages and interrelationships between activity on the asset and liability sides of the business. For example, the model should:

- apply the same macroeconomic environmental conditions (e.g., interest rates, inflation rates, catastrophic events) across *all* operations of the company;

- allow investment decisions to be made after consideration of both operating needs and investment opportunities in the financial markets;
- look at the risk/return tradeoffs generated by both investment and operating decisions in the context of the entire company's risk/return spectrum rather than in isolation; and
- provide a universal set of metrics or decision criteria by which diverse company operations can be measured and managed.

These critical model components are couched in terms of one or more accounting frameworks (i.e., statutory, GAAP, or economic). The accounting mechanisms serve to organize the model's projected results into a readily understood and consistent set of outputs.

Stochastic vs. Static Corporate Financial Modeling

One of the purposes of a corporate financial model is to help company management understand how decisions made today can be expected to affect the company's financial well-being tomorrow. Traditionally, corporate financial modeling has relied on static evaluations of current and future events and predetermined cause and effect relationships. Static methods of analysis limit the ability to analyze the sensitivity of outputs to changes in input variables, especially if the number of input variables is large and the interrelationships among them are complex. Yet it is critical that strategic decisions be made with the understanding of how each decision impacts the following ones or how changes in the internal or external environment can alter the anticipated outcomes arising from each decision.

The essence of stochastic modeling is the ability to describe critical assumptions in terms of ranges of possible outcomes, rather than in terms of fixed values. Once each critical assumption is defined by a range of possible outcomes and the interrelationships among critical assumptions are mapped out, a series of model recalculations can be performed to obtain ranges of results that we can reasonably expect to see. The parameters used to model stochastic variables and the accounting interrelationships ultimately define the key criteria or metric variables that are of interest to management, regulators, and stockholders. Differences in financial results arising from alternative strategic decisions can be evaluated by replacing one set of strategic decisions with another, re-running the modeling exercise, and comparing the ranges of possible outcomes under each decision rule set.

A stochastic model should also be able to address *dynamic* modeling considerations. A dynamic modeling consideration is one that responds in a time-dependent manner to other events that are unfolding or have unfolded at an earlier point in the modeling environment. Dynamic modeling considerations might be as simple as adjusting the price adequacy of the premium in a line of business if previous years' loss ratios are higher than expected, or as complex as adjusting the mix of taxable and tax-exempt bonds in an investment portfolio in order to minimize tax payments. While dynamic modeling considerations of this nature are not discussed at great length in the remainder of the paper due to the individuality of their construction and application, the ability to implement such decision logic later is an important consideration in the construction of a dynamic financial analysis model.

Choosing Between Competing Strategic Decision Paths

Very often, a company is faced with deciding between two or more strategic options. Under some situations, one option may be clearly superior, while under other situations a different option is preferable. An evaluation of multiple alternative strategies understand their relative risk/reward tradeoffs provides the information needed to answer questions such as "What additional risks must I assume to achieve a higher long-run return on my investment?"

FIGURE 1

COMPARISON OF AFTER-TAX PORTFOLIO YIELD WITH CAPITAL GAINS

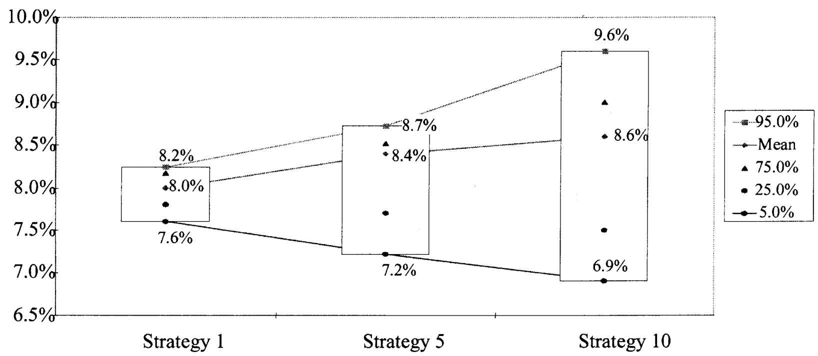

TABLE 1

ASSET COMPOSITION OF STRATEGIES IN FIGURE 1

Strategy 1	Strategy 5	Strategy 10
54% taxable bonds	80% taxable bonds	65% taxable bonds
31% tax-exempt bonds	0% tax-exempt bonds	0% tax-exempt bonds
0% stocks	15% stocks	30% stocks
4% cash	2% cash	3% cash
11% other	3% other	2% other

or "What is the probability of an important financial goal exceeding its expected value?" The answer to these questions should be the core DFA support for management decisions.

As an example, Figure 1 compares the three asset allocation strategies whose asset compositions are displayed in Table 1. The measurement criterion is the internal rate of return on the change in book value of all invested assets over the five-year projection horizon plus investment income, plus realized and unrealized capital gains, less the difference between the market value of assets maturing and sold and those purchased during the five years. Table 2 provides a numerical example of the metric displayed in Figure 1.

While there is no one alternative that is clearly superior, the picture illustrates that, in this case study, higher return is only achieved at the price of higher risk. The ultimate choice is a business decision; there is no alternative in this decision set that is superior to the others in all cases. This finding may seem to be a bane of dynamic financial analysis—there is no mechanically driven choice within a loosely defined utility framework. However, it points out the reality underlying strategic business decisions—it is not very often that one strategic direction is clearly superior to all others.

3. MODEL STRUCTURE OVERVIEW

The corporate financial model has been developed to include a minimum of one year of actual results and to produce pro-forma financial projections for the subsequent five years. For the purposes of simplification throughout the remainder of the article, it is assumed that the actual results are valued as of December 31, 1996 and that the projection period encompasses the years 1997 to 2001.

The model includes five separate and distinct components that must interact with each other in a structured and sequential manner. The components include

- an economic scenario generator,
- a projector of underwriting cash flows and accounting accruals,
- a projector of investment returns and asset valuations,
- a tax calculator, and
- a financial statement structure.

TABLE 2

NUMERICAL EXAMPLE OF A "PORTFOLIO YIELD WITH CAPITAL GAINS" CALCULATION

		1996	1997	1998	1999	2000	2001
(1)	**Investment Income**		40,000	38,000	44,000	43,000	45,000
(2)	**Realized capital gains**		2,000	−1,000	1,000	8,000	−3,000
(3)	**Unrealized capital gains**		10,000	−4,000	−10,000	26,000	−1,000
(4)	**Assets maturing or sold**		100,000	110,000	120,000	130,000	140,000
(5)	**Assets purchased**		120,000	135,000	150,000	165,000	180,000
(6) = (4) − (5)	**Net Sales**		−20,000	−25,000	−30,000	−35,000	−40,000
(7)	**Book Value**	−500,000					650,000
(8) = (1) + (2)+ (3) + (6) + (7)	**Cash Flows**	−500,000	32,000	8,000	5,000	42,000	651,000
	Internal rate of return (portfolio yield with capital gains)	8.66%					

FIGURE 2

CORPORATE MODEL STRUCTURE

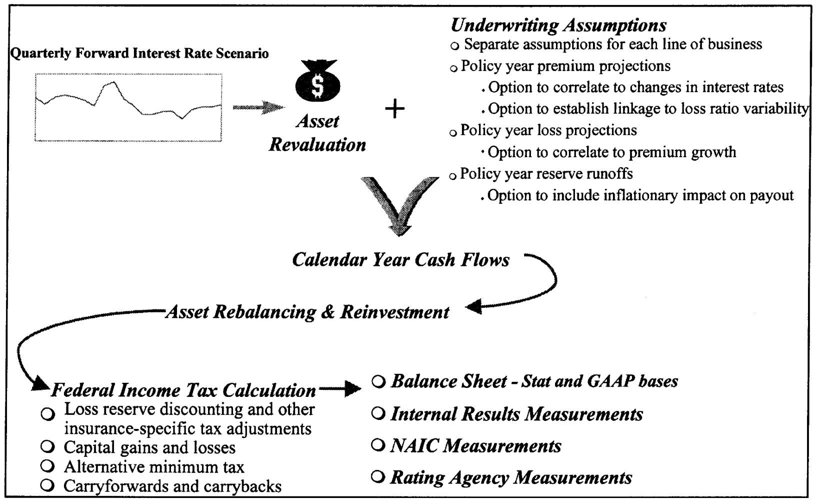

Figure 2 displays a flowchart of the period-by-period interactions of the five model sections.

The modeling starts with initial conditions—the beginning balance sheet, including accident year modeling of liabilities, knowledge of accruals, tax carry-backs and carry-forwards, costs and valuations of assets, and so forth. The following sequence of steps is replicated for each time period over which the model projects financial results:

1. Stochastically generate an economic scenario (interest rates, inflation, competitive conditions, etc.) for the next period.

2. Develop underwriting projections without consideration of the economic scenario (e.g., correlated, random effects on loss volume or severity that are independent of economic effects).

3. Overlay the economic scenario on top of the underwriting projections; quantify the effects of the economic scenario on the underwriting projections.
4. Apply the economic scenario to value existing assets.
5. Apply an asset rebalancing strategy based on current liability and asset conditions or on functions of previously observed or future expected ones.
6. Rebalance the portfolio of assets (and/or liabilities); i.e. buy and sell assets as needed.
7. Develop taxation effects and other fiscal period closing entries.
8. Tally assets and liabilities under the appropriate accounting scheme(s).
9. Create end-of-period financials, operating statistics, and metrics.

4. ECONOMIC SCENARIO

There is much literature describing models for the projection of economic scenarios. In fact, this may be the most well-documented of all DFA model parameters. The economic scenario model used in conjunction with the corporate financial model being discussed in this paper is of the family of "one-factor" interest rate projection models. It is closely based on the first of two interest rate generation algorithms described in a paper by James Tilley [8]. It is a one-factor lognormal model that reverts interest rates to short-term expectations. In other words, projected interest rates have a tendency to move from an initial seeding (e.g., the actual December 31, 1996 interest rate level) to an equilibrium that represents historic interest rate expectations in the short-term spectrum of the yield curve. Long-term interest rates, inflation rates, and projected movements in equity markets are produced by algorithms that relate each of these economic variables to short-term interest rates. One example of such a set of algorithms can be found in a paper written by Gary Venter in 1996 [9].

The economic scenario generation model is independent of the corporate financial model. Actions taken in the corporate financial model do not affect the future interest rate environments projected by the economic scenario generation model, but the future interest rate environments do impact the corporate financial model.

5. UNDERWRITING SECTION

The underwriting section performs seven basic tasks:

1. It converts held loss and allocated loss adjustment expense (ALAE) reserves into calendar year payouts.
2. It converts indicated redundancies or deficiencies in held loss reserves into calendar year payouts and captures the accounting impacts of reserve redundancy or deficiency emergence. Reserve redundancies or deficiencies can arise either from variability in the held reserves (i.e., the held reserves represent the best estimate of ultimate losses, but actual loss emergence might vary in some range around the best estimate) or from deliberately holding reserves at a level other than the best estimate.
3. It calculates the inflationary impact on loss payments arising from differences between a simulated future level of inflation and a level of inflation that was implicitly (or explicitly) assumed when the held reserve level was established.
4. It allows the emergence of reserve redundancies or deficiencies into the model's accounting results to be scheduled at the same rate or faster than the redundancies or deficiencies emerge into the model's cash flows.

5. It calculates any additional premium inflows that might be derived from policies already written (i.e., audit premium, premium from retrospectively rated policies) and earns premiums on in-force and new business according to a user-defined premium earning pattern.

6. It calculates discounted loss reserve levels for federal income tax calculations.

7. It provides the vehicle for entering a five year underwriting plan, including future premium inflows and associated loss and variable expense outflows at a line of business level of detail. (Only variable expenses are included in the line of business section. Fixed expenses are addressed in a different section of the model.)

Interrelationship of Held Reserves, Indicated Reserves, Payout Patterns, and Inflation on Income Statement and Cash Flow Projections

When the model was developed, it was decided that four factors needed to be considered in the loss reserve runoff structure

- the adequacy of held reserves,
- the speed with which reserves are paid out,
- the effects of unanticipated inflation on loss payments, and
- the way company management chooses to recognize adverse or favorable loss emergence through reserve additions or reserve takedowns.

The modeling structure allows each of the four pieces to be examined and modified separately for each line of business. This enables the model user to address each element with separate assumptions.

Conversion of Held Loss and ALAE Reserves Into Calendar Year Payouts

Predetermined loss and ALAE payout patterns are applied to *held* loss reserves and *held* ALAE reserves, respectively, to yield the calendar year payments that would be made, assuming the held reserves are correct. Each accident year's reserves are paid out over successive calendar years in accordance with the incremental calendar year payout percentages. The model should be flexible enough to allow for the application of different payout rates to different accident years, but in most situations only one underlying payout pattern will be needed.

An example of the conversion of held loss reserves at time T_0 (December 31, 1996) into payouts over the successive four years might look like the pattern shown in Table 3.

The calculation of the incremental calendar year payment is equal to:

$$\text{Held Reserve at Time } T_0 \\ * \frac{\text{Incremental Payout Percentage at Time } T_i}{\sum_{i=1}^{n} \text{Incremental Payout Percentage at Time } T_i}.$$

In this example, the remaining incremental payout percentages for accident year 1995 are 20% at time T_1, 15% at time T_2, and 10% at time T_3. The calendar year 1997 payment amount equals the product of the \$8,000 T_0 held reserve and the 20% T_1 incremental payment percentage divided by the sum of the T_1 to T_3 payment percentages, or \$8,000 $*$ 20% $\div$ (20% + 15% + 10%) = \$3,556.

Treatment of Reserve Adequacy in the Model

The model addresses the relative level of reserve adequacy at the accident year level of detail within a line of business. It captures held reserves and their payout in one payout triangle,

TABLE 3

PAYOUT TRIANGLE

Payout Pattern					
	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	30%	25%	20%	15%	10%
Cumulative %	30%	55%	75%	90%	100%

Calendar Year Reserve Payouts						
Accident Year	Held Reserve	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	2,000	2,000	0	0	0	0
1994	5,000	3,000	2,000	0	0	0
1995	8,000	3,556	2,667	1,778	0	0
1996	10,000	3,571	2,857	2,143	1,429	0
Calendar Year Total	25,000	12,127	7,524	3,921	1,429	0

as described in Table 3, and it captures any reserve redundancies or deficiencies and their payout in a second payout triangle, identical in format to the held reserve payout triangle.

The model assumes that the payout pattern being applied to held reserves is a correct representation of the rate at which exactly adequate reserves will become paid. However, held reserves are often not exactly adequate. If no adjustments for reserve inadequacies/redundancies are made, the resulting paid loss projections will understate/overstate actual future paid loss amounts.

The model addresses this by assuming that the sum of the held reserves and the indicated redundancy or deficiency amounts are equal to exactly adequate reserves. The model applies the same payout pattern to the indicated reserve deficiencies or redundancies (by accident year) as is used on the held reserves.[2] This ap-

[2]The assumption here is that the inadequacy or redundancy of held reserves is evenly spread across all outstanding claims. In reality, there are more likely to be differences in the relative adequacy levels of claims according to the length of time remaining until

proach projects incremental future paid loss amounts attributable to the difference between the level at which reserves should be held and what is being held. If held reserves are understated, the projections are positive (i.e., calendar year payments will be greater than held reserves would otherwise indicate), and, conversely, if held reserves are overstated, the projections are negative. The sum of the paid loss projections derived from the held reserve triangle and paid loss projections derived from the reserve redundancy/deficiency triangle equal the *correct* future payout amounts. Intuitively, this makes sense if one considers that, all other things being equal, there should be no impact on actual losses paid whether or not the held reserves at time zero are exactly equal to the future payment amounts.

Table 4 uses the payout pattern from Table 3 with modifications to the held reserves at time T_0. We can see how the assumption of equivalent payout patterns for held reserves and reserve redundancies/deficiencies results in the same calendar year payouts as if held reserves were equal to needed reserves.

Impact of Changes in Reserve Payout Speed

The model is structured so that changes in the speed with which reserves are paid out do not change the total amount to be paid out, only the timing with which the reserves are paid out. If a situation arises in which both the amount and timing of reserve payouts are impacted, the amount component would be captured through the reserve redundancy/deficiency triangle and the timing component would be captured through a shift in the reserve payout pattern.

The impact of a change in the payout pattern is therefore described as an "accordion effect" on calendar year underwriting cash flows. Any payout pattern change has the effect of stretching or compressing the cash flow pattern without changing the total

settlement, with those that are furthest from settlement being less adequately reserved than those that are closer to settlement.

amount to be paid out.[3] Table 5 uses an example to illustrate this phenomenon.

TABLE 4

CALENDAR YEAR RESERVE PAYOUTS

Accident Year	Held Reserve	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	2,000	2,000	0	0	0	0
1994	4,000	2,400	1,600	0	0	0
1995	6,000	2,667	2,000	1,333	0	0
1996	8,000	2,857	2,286	1,714	1,143	0
Calendar Year Total	20,000	9,924	5,886	3,047	1,143	0

Accident Year	Reserve Deficiency	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	0	0	0	0	0	0
1994	1,000	600	400	0	0	0
1995	2,000	889	667	444	0	0
1996	2,000	714	571	429	286	0
Calendar Year Total	5,000	2,203	1,638	873	286	0

Accident Year	Overall Total	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	2,000	2,000	0	0	0	0
1994	5,000	3,000	2,000	0	0	0
1995	8,000	3,556	2,667	1,778	0	0
1996	10,000	3,571	2,857	2,143	1,429	0
Calendar Year Total	25,000	12,127	7,524	3,921	1,429	0

[3]Because the model assumes payout pattern variability might be induced through the generation of incremental payout period adjustment amounts, the payout pattern after adjustment might not sum to 100%. This is accounted for in the model through formulas that rescale the adjusted payout pattern to total 100%.

TABLE 5

IMPACT OF CHANGES IN PAYOUT SPEED

Original Payout Pattern

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	30%	25%	20%	15%	10%
Cumulative %	30%	55%	75%	90%	100%

Payout Pattern Adjustment Amounts

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	+10%	+10%	no change	−5%	−5%

Revised Payout Pattern, Prior To Rescaling

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	40%	35%	20%	10%	5%
Cumulative %	40%	75%	95%	105%	110%

Revised Payout Pattern, After Rescaling

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	36.4%	31.8%	18.2%	9.1%	4.5%
Cumulative %	36.4%	68.2%	86.4%	95.5%	100%

Revised Calendar Year Reserve Payouts

Accident Year	Held Reserve	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	2,000	2,000	0	0	0	0
1994	5,000	3,333	1,667	0	0	0
1995	8,000	4,571	2,286	1,143	0	0
1996	10,000	5,000	2,857	1,429	714	0
Calendar Year Total	25,000	14,904	6,810	2,572	714	0

Table 5, based on the payout pattern used in Table 3, demonstrates the effects of changing the speed of the payout pattern.

Effects of Unanticipated Inflation on Loss Payments

By including the inflationary impacts on loss payouts, we incorporate a linkage between the macroeconomic environment affecting assets and the macroeconomic environment affecting losses. We are also in a position to examine the financial statement implications of unanticipated inflationary pressures on losses.

In addition to the speed of payment and inaccuracy in original reserve estimates, a third variable affecting losses in the model is inflation that affects claim costs through the claim payment date. It is assumed that the reserves quantified in the first two reserve triangles (the held reserve payout triangle and the reserve redundancy/deficiency payout triangle) implicitly or explicitly contemplate an *anticipated* level of inflation. Not contemplated in these two payment triangles are the differences between actual and anticipated payments arising from changes in inflation.[4]

As an example, suppose that if a claim were paid today, it would cost $1,000. But, with an annual inflation rate for claims of this nature at five percent per annum, if the claims are paid five years later, then it will cost $1,276 [$= 1{,}000 * (1.05)^5$]. The $1,276 reserve is included in the reserves quantified in the held reserve and the reserve redundancy/deficiency payout triangle. Suppose now that the actual inflation rate increases to ten percent per annum in the third, fourth and fifth years. Now, the amount paid will be $1,467, not $1,276. The additional $191 paid in year five is captured in the paid loss amounts for that year and also

[4]Robert Butsic, in his 1981 paper [2, pp. 58–102], describes two ways in which inflation can impact losses. One is through a claim's accident date and the second is through a claim's payment date. For the model described in this paper, the authors elected to contemplate and quantify the second way only; i.e., inflation impacting claims through the claim payment date.

shows up as adverse loss experience in the company's income statement in the fifth year.

Accounting for Reserve Additions or Takedowns

The first three components of the model's loss reserve runoff structure have been concerned with converting reserves into cash flows. The fourth component is concerned with the quantification of the accounting implications of reserve adjustments. It is not enough to know when a reserve redundancy or deficiency is converted into a cash event; the model must also know when a reserve redundancy or deficiency is recognized in the financial statements, either as an increase to held reserves or a decrease of held reserves. It should be noted that the model makes no provision for the recognition of unanticipated changes in loss payments arising from a change in the inflationary environment.

Returning to the numbers from Table 4, we examine a few different ways in which a reserve deficiency might manifest itself in a company's income statement. The reserve deficiency amounts and their calendar year payouts as shown in Table 4 were as follows:

Accident Year	Reserve Deficiency	Calendar Year 1997	Calendar Year 1998	Calendar Year 1999	Calendar Year 2000	Calendar Year 2001
1993	0	0	0	0	0	0
1994	1,000	600	400	0	0	0
1995	2,000	889	667	444	0	0
1996	2,000	714	571	429	286	0
Calendar Year Total	5,000	2,203	1,638	873	286	0

One approach to the income statement recognition of the reserve deficiency might be to recognize it as the deficiency emerges in loss payments. This approach would have no effect on the level of held reserves that appear on the company's balance sheet, but there would be an impact on the company's income statement in calendar years 1997 through 2000. The impact on the company's income statement in calendar year 1997 would be $2,203, the impact in calendar year 1998 would be $1,638, and so on.

A second approach might be to recognize the entire reserve deficiency in calendar year 1997. The 1997 financial statements would show a reserve increase of $2,797 over what would otherwise have been held at year end 1997, and the 1997 income statement would show incurred losses to be $5,000 higher than they would otherwise have been.

Suppose a company had one year of loss reserves on its books with the following additional information:

- Held reserves: $100,000, based on a reserve range of $80,000 to $105,000.
- Actual reserve need: $90,000.
- Reserve payout pattern: 25% over each of the next four years.
- Inflation level implicit in held reserves: 5%.
- Actual annual inflation rates: 5% in years 1 and 2, 8% in years 3 and 4.
- Company takes reserve redundancy into financial statements equally in years one and two by lowering reserves $5,000 in each year.

An example of the inter-relationship between the four elements (payout of held reserves, indicated reserve redundancy/deficiency emergence through payments, inflationary impacts on payments, and indicated reserve redundancy/deficiency emergence through the financial statements) is shown in Table 6 using the assumptions above.

TABLE 6

IMPACT OF RECOGNITION OF RESERVE REDUNDANCY/DEFICIENCY

	Year 0	Year 1	Year 2	Year 3	Year 4
1. Payout percentage		25%	25%	25%	25%
2. Expected inflation rate		5%	5%	5%	5%
3. Actual inflation rate		5%	5%	8%	8%
4. Held reserve cash flow	n/a	25,000	25,000	25,000	25,000
5. Redundancy cashflow	n/a	−2,500	−2,500	−2,500	−2,500
6. Inflationary impact	n/a	0	0	643	1,304
7. Reserve lowering	n/a	−5,000	−5,000	0	0
8. Held reserves	100,000	72,500	45,000	22,500	0
9. Net cash flow		22,500	22,500	23,143	23,804
10. Income statement impact (−gain/ + loss)		−5,000	−5,000	+643	+1,304

Line 6 formula:

$$\left[\left(\frac{\prod_{i=1}^{N}(1+\text{Line } 3_{\text{Year } i})}{\prod_{i=1}^{N}(1+\text{Line } 2_{\text{Year } i})}\right)-1\right]*(\text{Line } 4_{\text{Year } i}+\text{Line } 5_{\text{Year } i})$$

Example:

Year 3 inflationary impact

$$=\left[\frac{(1.05)(1.05)(1.08)}{(1.05)(1.05)(1.05)}-1\right]*(25{,}000-2{,}500)$$

Line 8 formula: $\text{Line } 8_{\text{Year } i-1}-\text{Line } 4_{\text{Year } i}+\text{Line } 7_{\text{Year } i}-\text{Line } 5_{\text{Year } i}$
Line 9 formula: $\text{Line } 4_{\text{Year } i}+\text{Line } 5_{\text{Year } i}+\text{Line } 6_{\text{Year } i}$
Line 10 formula: $\text{Line } 6_{\text{Year } i}+\text{Line } 7_{\text{Year } i}$

C. K. Khury develops this concept in a similar manner [6, p. 14]. Khury notes that, given a balance sheet loss reserve liability at time T, "the actual experience corresponding to this estimate can generate two effects on the financial results of an insurer: (a) the effect of the difference between expected and actual claim payments, i.e. actual development and (b) the effect of any restatement of the remaining unpaid claim liability arising from changes in the underlying assumptions, i.e. change in expected development." Khury's first effect is comparable to the combined impacts of the redundancy cashflow and inflationary impacts on Lines 5 and 6 in Table 6. His second effect is comparable to the reserve lowering on Line 7 in Table 6.

Premiums: the Premium Writing, Earning and Collection Processes

Premiums can be earned on two types of business: those already written and those that will be written during the model's time horizon. For both, the premium earning process is identical. Premiums are earned according to a predefined earnings pattern that can be as short as one year or as long as twenty years. This earnings pattern is applied to *initial* policy year written premium levels, not ultimate policy year written premium[5] levels. Similarly, premiums can be collected for business already written and business that will be written during the model's time horizon. A second premium pattern, this one for premium collections, is applied to initial policy year written premium levels to determine when premiums are collected.

The model makes three simplifying assumptions:

1. All policies are annual policies.
2. The amount of premium collected in the calendar year in which the premium is written equals the initial written premium.[6]

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[5]Differences between calendar year written premium and ultimate policy year written premium might arise because of premium audits and/or retrospective premium adjustments.

[6]The simplifying assumption that the amount of premium collected equals written premium in the calendar year in which the premiums were written ignores the existence of "cash-flow" premium collection arrangements. In reality, a premium collection arrangement could exist in which premiums are booked in one calendar year but are not fully collected until one or more years in the future. This would be most likely to occur on long-tail policies of large commercial accounts, such as workers compensation. The rules described in this paper do not work for these situations and would need to be modified to fit the actual premium collection/premium booking structure of the entity being modeled. One option is to leave the modeling of premiums unchanged at the line of business level,

3. If a line of business does not calculate a reserve for anticipated rate credits and retrospective adjustments in advance of actually collecting such adjustments, there is no change in the collected premium in the calendar year after the premium is written—any adjustments to collected premium occur in the third and subsequent calendar years.

The interrelationships between the writing, earning, and collection of premiums are explained by the following set of rules. Each of these rules are applied on a policy year by policy year basis:

- Written premium: In the first calendar year, written premium is set equal to the user-input initial written premium amount. If the line of business being modeled includes a provision for anticipated rate credits and retrospective reserve adjustments, then written premium is assumed to change by the calendar year change in collected premiums in the second and subsequent calendar years. If the line of business being modeled does not include a provision for anticipated rate credits and retrospective reserve adjustments, then written premium is assumed to change by the calendar year change in collected premiums in the third and subsequent calendar years.
- Earned premium: Premiums are earned by applying the user-input premium earning pattern to the initial written premium amount.
- Unearned premium: Unearned premium is calculated as:

$$\text{Written Premium} - \text{Earned Premium} \\ + \text{Prior Calendar Period Unearned Premium Reserve.}$$

but adjust the accounting portion of the model to reflect the impact of the delayed premium collection on the company as a whole. The accounting entries that would require "overrides" to accomplish this include the balance sheet entry or entries for uncollected premiums and the cash flow statement entry for total collected premium.

This develops an unearned premium reserve that is consistent with the one on the Underwriting and Expense Exhibit, Part 2A of the statutory Annual Statement (page 8 in the 1997 statutory Annual Statement), not one that is consistent with the unearned premium reserve displayed on the Liabilities, Surplus and Other Funds page (page 3) of the statutory Annual Statement. In the statutory Annual Statement, the difference between these two numbers is the reserve for rate credits and retrospective adjustments.

- Reserve for rate credits and retrospective adjustments, if applicable: In the first calendar year, this reserve is equal to:

  $$(\text{Ultimate earned premium} - \text{initial written premium}) * \left(\frac{\text{\% of initial written premium that is earned in CY 1}}{\text{ultimate earned premium/initial written premium}}\right).$$

  In the second and subsequent calendar years, this reserve is equal to the difference between the ultimate earned premium and the premium collected to date.

- Collected premium: The timing and amount of premium collections are calculated by applying a user-input premium collection pattern to the initial written premium amount.

- Uncollected premium: If the line of business being modeled does not include a provision for anticipated rate credits and retrospective reserve adjustments, then this is calculated as the difference between initial written premium and the premium collected to date. If the line of business being modeled includes a provision for anticipated rate credits and retrospective reserve adjustments, then the uncollected premium is set equal to the calculated rate credit or retrospective reserve adjustment.

Assume:

- Ultimate earned premium = initial written premium.

- Premium is collected as it is written.
- The line of business being modeled does not include a provision for anticipated rate credits and retrospective reserve adjustments.

The results of these assumptions are shown in Table 7.

Assume:

- Ultimate earned premium = 110% of initial written premium.
- No reserve for rate credits is used; i.e., the additional premium is written and earned when it is collected.

In the extended premium earning pattern, the ultimate policy year premium is greater than the initial written policy year premium, possibly due to the receipt of audit premium in the third calendar year after the start of the policy period. The extended premium earning and collection patterns account for this by totaling to 110% instead of 100%. The results are shown in Table 8.

Assume:

- Ultimate earned premium = 110% of initial written premium.
- A reserve for rate credits is used; i.e., the additional premium is earned at the same time the initial written premium is earned.

The results are shown in Table 9.

New Business Production

The model's new business production logic requires assumptions about future premium writing levels and associated loss and variable expense ratios. There is no one structure for this section that is suitable for every need, but certain capabilities and functional considerations can be generalized. These are discussed in the subsequent paragraphs.

TABLE 7

SIMPLE EARNING PATTERN

Premium Earning Pattern, Applied to Initial Policy Year Written Premium

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	50%	50%	0%	0%	0%
Cumulative %	50%	100%	100%	100%	100%

Calendar Year Earning of Policy Year Premium Writings

Policy Year	Initial Written Premium	CY 1996 Earned Premium	CY 1997 Earned Premium	CY 1998 Earned Premium	CY 1999 Earned Premium	CY 2000 Earned Premium
1996	20,000	10,000	10,000	0	0	0
1997	25,000	n/a	12,500	12,500	0	0
1998	30,000	n/a	n/a	15,000	15,000	0
Calendar Year Total	75,000	10,000	22,500	27,500	15,000	0

Calendar Year Premium Collection

Policy Year	Initial Written Premium	CY 1996 Collected Premium	CY 1997 Collected Premium	CY 1998 Collected Premium	CY 1999 Collected Premium	CY 2000 Collected Premium
1996	20,000	20,000	0	0	0	0
1997	25,000	n/a	25,000	0	0	0
1998	30,000	n/a	n/a	30,000	0	0
Calendar Year Total	75,000	20,000	25,000	30,000	0	0

Calendar Year Accounting Results

Cal. Year	Written Premium	Earned Premium	Unearned Premium (AS p. 8)	Reserve for Rate Credits	Unearned Premium (AS p. 3)	Collected Premium	Uncollected Premium
1996	20,000	10,000	10,000	0	10,000	20,000	0
1997	25,000	22,500	12,500	0	12,500	25,000	0
1998	30,000	27,500	15,000	0	15,000	30,000	0
1999	0	15,000	0	0	0	0	0
2000	0	0	0	0	0	0	0

TABLE 8

EXTENDED EARNING PATTERN

Premium Earning Pattern, Applied to Initial Policy Year Written Premium

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	50%	50%	10%	0%	0%
Cumulative %	50%	100%	110%	110%	110%

Premium Collection Pattern, Applied to Initial Policy Year Written Premium

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	100%	0%	10%	0%	0%
Cumulative %	100%	100%	110%	110%	110%

Calendar Year Earning of Policy Year Premium Writings

Policy Year	Initial Written Premium	CY 1996 Earned Premium	CY 1997 Earned Premium	CY 1998 Earned Premium	CY 1999 Earned Premium	CY 2000 Earned Premium
1996	20,000	10,000	10,000	2,000	0	0
1997	25,000	n/a	12,500	12,500	2,500	0
1998	30,000	n/a	n/a	15,000	15,000	3,000
Calendar Year Total	75,000	10,000	22,500	29,500	17,500	3,000

Calendar Year Premium Collection

Policy Year	Initial Written Premium	CY 1996 Collected Premium	CY 1997 Collected Premium	CY 1998 Collected Premium	CY 1999 Collected Premium	CY 2000 Collected Premium
1996	20,000	20,000	0	2,000	0	0
1997	25,000	n/a	25,000	0	2,500	0
1998	30,000	n/a	n/a	30,000	0	3,000
Calendar Year Total	75,000	20,000	25,000	32,000	2,500	3,000

Calendar Year Accounting Results

Cal. Year	Written Premium	Earned Premium	Unearned Premium (AS p. 8)	Reserve for Rate Credits	Unearned Premium (AS p. 3)	Collected Premium	Uncollected Premium
1996	20,000	10,000	10,000	0	10,000	20,000	0
1997	25,000	22,500	12,500	0	12,500	25,000	0
1998*	32,000	29,500	15,000	0	15,000	32,000	0
1999*	2,500	17,500	0	0	0	2,500	0
2000*	3,000	3,000	0	0	0	3,000	0

*Note the change in written and earned premiums in calendar years 1998 through 2000. Both the written and earned amounts are increased by the premium earned three years after the start of the 1996, 1997 and 1998 policy periods.

TABLE 9

EXTENDED EARNING PATTERN WITH RESERVES

Premium Earning Pattern, Applied to Initial Policy Year Written Premium

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	55%	55%	0%	0%	0%
Cumulative %	55%	110%	110%	110%	110%

Premium Collection Pattern, Applied to Initial Policy Year Written Premium

	0 to 12 months	12 to 24 months	24 to 36 months	36 to 48 months	48 to 60 months
Incremental %	100%	0%	10%	0%	0%
Cumulative %	100%	100%	110%	110%	110%

Calendar Year Earning of Policy Year Premium Writings

Policy Year	Initial Written Premium	CY 1996 Earned Premium	CY 1997 Earned Premium	CY 1998 Earned Premium	CY 1999 Earned Premium	CY 2000 Earned Premium
1996	20,000	11,000	11,000	0	0	0
1997	25,000	n/a	13,750	13,750	0	0
1998	30,000	n/a	n/a	16,500	16,500	0
Calendar Year Total	75,000	11,000	24,750	30,250	16,500	0

Calendar Year Premium Collection

Policy Year	Initial Written Premium	CY 1996 Collected Premium	CY 1997 Collected Premium	CY 1998 Collected Premium	CY 1999 Collected Premium	CY 2000 Collected Premium
1996	20,000	20,000	0	2,000	0	0
1997	25,000	n/a	25,000	0	2,500	0
1998	30,000	n/a	n/a	30,000	0	3,000
Calendar Year Total	75,000	20,000	25,000	32,000	2,500	3,000

Calendar Year Accounting Results

Cal. Year	Written Premium	Earned Premium	Unearned Premium (AS p. 8)	Reserve for Rate Credits	Unearned Premium (AS p. 3)	Collected Premium	Uncollected Premium
1996*	20,000	11,000	9,000	1,000	10,000	20,000	1,000
1997*	25,000	24,750	9,250	3,250	12,500	25,000	3,250
1998*	32,000	30,250	11,000	4,000	15,000	32,000	4,000
1999*	2,500	16,500	−3,000	3,000	0	2,500	3,000
2000*	3,000	0	0	0	0	3,000	0

*In this situation, the accounting results have been altered in all years as a result of earning the additional premium over the same calendar periods as the initial premium is earned. The reserve for rate credits captures the amount of additional premium that is anticipated as "earned but not received." The only accounting entry that is not impacted is the unearned premium reserve that would appear on page three of the statutory Annual Statement.

New Business—Premium Volumes

The projection of future premium volumes can be as simple as a fixed set of assumptions or can be as complex as a system of assumptions that interrelate the relative amount of business that is expected to be retained each year, a company's internal growth objectives, the overall insurance market conditions, and company reactions to prior-year underwriting results. In general, it would seem that the more linkages that are established between new business production and other events being played out in the model, the better the model will be. The model then should be more reactive; it should do what the company itself might do when faced with similar circumstances. However, in some cases, the inclusion of additional dynamic elements in these linkages could lead to greater confusion in what the model is doing than is warranted by the additional realism that is gained.

Future Loss Ratios

When the model was being developed, two alternative approaches to developing future loss ratios were contemplated. One was to assume that, all other things being held constant, the loss ratios at time T and $T+1$ could be described as independent values selected at random from one statistical distribution. We call this a "force of loss" approach to loss ratio generation. The second approach assumes that the loss ratio at time $T+1$ should be equal to the loss ratio at time T plus or minus a volatility parameter. The second approach assumes the loss ratio at time $T+1$ is more or less dependent upon the loss ratio at time T, depending upon the size and shape of the volatility parameter. We call this approach to loss generation the "incremental volatility" approach.

From a theoretical standpoint, it seems that the force of loss assumption would be more valid for lines whose loss experience can be characterized as more directly attributable to external factors than to internal management decisions, or whose exposure base is highly volatile, or whose loss profile is one of low-frequency, high-severity claims. The incremental volatility approach would seem to be most appropriate for those lines of business that display a stable exposure base with a high retention of insureds from year to year, a tendency towards high-frequency, low-severity losses, and a small exposure to catastrophic loss. Examples of "force of loss" lines might include homeowners (if catastrophes are not explicitly separated from non-catastrophic claims), commercial liability or umbrella. Examples of "incremental volatility" lines might include personal automobile, commercial automobile, or any non-catastrophic portion of property lines.

From a practical standpoint, however, the force of loss approach is much simpler to program. All that is required to successfully implement a force of loss approach is to have a random number generator return values from a distribution that reasonably replicates the desired shape, spread, and mean of the loss ratios being modeled. To successfully implement an incremental volatility approach, formulas must be established that (a) cap the overall upwards or downwards movement to reasonable floor and ceiling values and (b) have "mean-reverting" tendencies (i.e., the incremental volatility in time $T + 1$ will be more likely to move the overall loss ratio towards the long-term mean than away from it), while still returning mean values that are consistent with the expected value.

It should be noted that both the force of loss and the incremental volatility approaches to loss ratio selection describe the loss ratio that would arise if there were no other changes occurring that have an impact on the final loss ratio. Other changes might include premium rate changes, inflationary increases in the premium exposure base, or inflationary impacts on loss costs. When the model is run stochastically, the final projected loss ratio is developed by first randomly sampling from the probability distribution that describes this force of loss, then modifying the random sample to reflect the other changes.

Exhibit 1 provides an example of the interrelationships between premium development and loss ratio development, including inflationary and rate impact influences. A more realistic rate change formula would consider many more parameters than the relationship between one year's actual and expected loss ratios.[7]

6. ASSET MODELING

There are two basic components to modeling assets: valuing assets and rebalancing an asset portfolio through the sale of existing assets and the purchase of new assets. The first component is concerned with determining the book and market value at time $T+1$ of assets the company owned at time T. The second component is concerned with how the asset portfolio owned at time T should be adjusted at time $T+1$, including the manner in which net cash inflows between times T and $T+1$ should be invested.

In order to perform these tasks, a model must be able to:

- quantify at any valuation date the book and market values of assets held at that point in time and
- quantify the amount of cash generated by the insurance operation between the previous asset revaluation and rebalancing and the current asset revaluation and rebalancing.

A model must also contain one or more decision algorithms that tell it what assets to sell at time $T+1$, if asset sales are needed or desired and what assets to purchase at time $T+1$.

Asset Categorization

In developing the model, we elected to evaluate assets at an aggregate level of detail. We feel that this approach is in keeping with the strategic nature of the questions the model is expected

[7] The formula for determining whether or not a rate change occurs, and by how much, is there for example only and is not a realistic rate change formula.

to address. We recognize that there are many situations when the level of aggregation described herein is neither sufficient nor appropriate for the desired analysis. At these times, a more refined, maybe even seriatim, approach to asset analysis may be needed. We leave the task of describing a seriatim approach to asset modeling to other papers and instead turn to a discussion of the manner in which assets have been aggregated in this model.

Bonds

The model assumes that bonds mature in no more than thirty years and that bonds are either taxable or tax-exempt, resulting in sixty possible bond categories.

The starting bond portfolio is sorted by tax status and maturity date into the sixty available bond categorizations. Sixty "proxy" bonds are then created from the underlying bond portfolio. Each proxy bond's market value, statement value, and par value are calculated as the sum of the values of the underlying bonds. The maturity date of each proxy bond is assumed to be equal to the midpoint of the calendar year in which the underlying bonds were to mature. Each proxy bond's coupon rate is a weighted average of the underlying bonds' coupon rates, using the par values for weights. The model assumes that each proxy bond will pay coupons semi-annually.

Suppose at December 31, 1996, the XYZ Company has an asset portfolio with the five bonds shown in Table 10.

Three proxy bonds would be created to summarize this portfolio, as follows:

	Maturity Date	Years to Maturity	Statement Value	Market Value	Par Value	Coupon Rate
Proxy 1	7/15/2000	3–4 years	3,000,000	3,009,000	2,950,000	6.6%
Proxy 2	7/15/2003	6–7 years	5,000,000	5,331,000	5,000,000	7.5%
Proxy 3	7/15/2010	13–14 years	7,000,000	7,608,000	7,000,000	7.5%

TABLE 10

ASSET PORTFOLIO

	Maturity Date	Years to Maturity	Statement Value	Market Value*	Par Value	Coupon Rate
Bond 1	6/15/2000	3–4 years	1,000,000	965,000	950,000	6.5%
Bond 2	9/30/2000	3–4 years	1,500,000	1,540,000	1,500,000	6.8%
Bond 3	12/30/2000	3–4 years	500,000	504,000	500,000	6.2%
Bond 4	7/15/2003	6–7 years	5,000,000	5,331,000	5,000,000	7.5%
Bond 5	1/1/2010	13–14 years	7,000,000	7,608,000	7,000,000	7.5%

*The market values are approximations that assume the bonds have no embedded options, no default risk, and that the "current," or December 31, 1996 risk-free interest rate for new bonds maturing in the year 2000 is 6.0%, for new bonds maturing in the year 2003 is 6.25%, and for new bonds maturing in the year 2010 is 6.5%.

Suppose one year has elapsed. The XYZ company has decided not to purchase any new bonds. The proxy bond portfolio now might look as follows:

	Maturity Date	Years to Maturity	Statement Value*	Market Value†	Par Value	Coupon Rate
Proxy 1	7/15/2000	2–3 years	2,987,500	3,015,000	2,950,000	6.6%
Proxy 2	7/15/2003	5–6 years	5,000,000	5,410,000	5,000,000	7.5%
Proxy 3	7/15/2010	12–13 years	7,000,000	8,019,000	7,000,000	7.5%

*The change in proxy one's statement value reflects the amortization of one year's bond premium. As a simplification, this example assumes that the bond premium will be amortized evenly over remaining time to maturity; i.e., one-fourth of the difference between the December 31, 1996 statement and par values. A more accurate approach would be to calculate the change in the present value of the bond based on the initial interest rate.

†The market values are approximations that assume the bonds have no embedded options, no default risk, and that the now "current," or December 31, 1997 risk-free interest rate for new bonds maturing in the year 2000 is 5.65%, for new bonds maturing in the year 2003 is 5.75%, and for new bonds maturing in the year 2010 is 5.80%.

Now suppose that the same one year has elapsed, but the company decides to purchase a new risk-free bond with a $1,000,000 par value that will mature in 2003. The model assumes that this bond is purchased at cost, so the statement value and the market value are equal to the $1,000,000 par value. The coupon rate for this bond is 5.75%.

The model recalculates Proxy Bond 2 as a weighted average of the old and new bond characteristics, resulting in a revised bond with the following information:

	Maturity Date	Years to Maturity	Statement Value	Market Value	Par Value	Coupon Rate
Proxy 2	7/15/2000	2–3 years	3,987,500	4,015,000	3,950,000	6.38%

Preferred and Common Stocks

Preferred and common stocks are aggregated into two groups, with one proxy equity for each group. The proxy equity reflects the total market value, book value and actual cost of the underlying equities within that group.

Assumptions with regard to the projection of future market values can be varied, but not the basic framework. We believe such a simplification is acceptable in most situations and that only when a company has a large preferred stock portfolio would it be inappropriate.

Within each of the preferred and common stock groups, the model assumes there exists an average dividend rate that can be applied to the proxy equity for that group. While it is theoretically possible that the rates might be the same, it would be more likely that the rate applied to the preferred stock group would be higher than that applied to the common stock group. The model further assumes that any unrealized capital gains or losses within a stock grouping are spread evenly across all of the underlying securities within the grouping. These assumptions are maintained as equities are bought and sold during each asset rebalancing.

Suppose the XYZ company had common stock holdings at December 31, 1996 of:

Statement Value of Proxy Equity	Market Value of Proxy Equity	Dividend Rate for Proxy Equity
1,800,000	2,500,000	2.0%

During 1997, the stock portfolio's market value increased by fifteen percent, to $2,875,000. During the year-end 1997 asset rebalancing, the XYZ Company decides to sell ten percent of its equity portfolio. The result is:

Statement Value of Retained Stocks in Proxy Equity	Market Value of Retained Stocks in Proxy Equity	Dividend Rate on Retained Stocks in Proxy Equity	Realized Capital Gains on Sold Portion of Proxy Equity
1,620,000	2,578,500	2.0%	$107,500

Alternatively, the XYZ Company might have decided to purchase additional equities at year-end 1997. Suppose that instead of selling ten percent of the 12/31/96 proxy equity, the XYZ Company decides to purchase an additional $1,000,000 of stocks. The model assumes the stocks purchased will have the same average dividend rate of the previously existing stock portfolio. The proxy equity is restated to reflect the newly purchased stocks as follows:

	Statement Value of Proxy Equity	Market Value of Proxy Equity	Dividend Rate for Proxy Equity
Prior to new stock purchases	1,800,000	2,875,000	2.0%
New purchases	1,000,000	1,000,000	2.0%
After inclusion of new purchases	2,800,000	3,875,000	2.0%

Real Estate

The model tracks real estate in two categories that are consistent with the NAIC Annual Statement: "properties occupied by the company" and "other properties." Besides the desire to match the model's asset categories as closely as possible to the Annual Statement, real estate is maintained as its own asset category in order to better address the accounting impacts of depreciation and capital improvements. The model's accounting and asset valuation structures are designed to allow annual depreciation of real estate assets to flow through the modeled company's balance sheet and income statement without affecting either calendar year cash flows or projected market values for the real estate. Capital improvements flow through the financial statements as a direct outflow from "cash" into the real estate's statement and market values.

Suppose the XYZ company owned real estate with a statement and market value of $10,000,000 at December 31, 1996. The property has annual depreciation of $500,000. Assume that:

(a) no real estate is bought or sold during 1997,

(b) no capital improvements are made to the property, and

(c) the market value remains unchanged from year-end 1996 to year-end 1997.

The December 31, 1997 balance sheet would show a statement value of $9,500,000 and a market value of $10,000,000. The income statement would reflect $500,000 of real estate expenses incurred during the year, and the cash flow statement would not be impacted at all.

If, instead, $1,000,000 of capital improvements were performed during 1997, the balance sheet would show a statement value of $10,500,000 and a market value of $11,000,000. The income statement would again reflect $500,000 of real estate expenses incurred during the year. The cash flow statement would reflect the conversion of $1,000,000 from "cash" to "real estate" through the accounting entry "cost of real estate acquired."

Short-Term Investments

Short-term investments are aggregated into one group and treated in a manner similar to cash. They are assumed to generate an investment income yield that is commensurate with the three-month risk-free interest rate projected by the economic scenario. The model assumes that the statement and market values of this asset group are identical.

All Other Asset Groups (Mortgage Loans, Collateral Loans, Other Invested Assets)

The model makes no special provisions for any other invested asset group. All assets invested in mortgage loans, collateral loans, and other invested assets are consolidated into one proxy asset for each group. A statement and a market value are entered and the model user can specify the annual investment income return anticipated from each of these proxy groups.

Asset Rebalancing

The model rebalances assets at the end of each calendar year. The amount of money that can be rebalanced at year-end equals the sum of:

- cash flow from operations during the year (premium collected less losses and underwriting expenses paid);
- investment income collected during the year, net of investment expenses paid during the year, including investment income derived from the insurance operation's average cash balances, which are deemed to be invested at the yield for "Cash" until the end of the year;
- the cash value of any bonds maturing during the calendar year; and
- the market value of all other invested assets at the end of the calendar year.

The asset rebalancing strategy is ad hoc; it is a user-defined strategy that defines how much money should be invested in any asset class at year-end. Examples of different asset rebalancing strategies might be: "invest 50% in taxable bonds, 30% in equities, and 20% in cash," or "invest 40% in taxable bonds, 40% in tax-exempt bonds, 15% in equities and 5% in cash." These ad hoc strategies are consistent with active investment management portfolio rebalancing where allocations among asset categories are important.

Depending on the rebalancing strategy, some existing assets may be sold and the proceeds reinvested to produce approximately the asset distribution dictated by the chosen asset strategy. The determination of whether to sell or buy assets in an asset class is based on a comparison of the market value of assets held in that class prior to rebalancing and the desired market value of that class. If the amount being held prior to rebalancing is greater than the desired amount, then some portion of assets in that class are sold. If the amount being held prior to rebalancing is less than the desired amount, then additional assets in that class are purchased.

An example might be as follows:

- Suppose we have $1,000 available for reinvestment.
- We want to invest the $1,000 in 3 asset classes; $500 in Asset Class 1, $300 in Asset Class 2 and $200 in Asset Class 3.
- Prior to rebalancing, we have $500 in Asset Class 1, $500 in Asset Class 2 and $0 in Asset Class 3.

The rebalancing algorithm compares the amount held in each asset class prior to rebalancing to the desired amount and causes the following asset redistributions to occur:

- Asset Class 1: no change. Held prior to rebalancing equals desired amount.
- Asset Class 2: Sell $200. New held amount equals $300.
- Asset Class 3: Buy $200. New held amount equals $200.

The final allocations are subject to modifications attributable to year-end closing transactions, primarily tax payments and the payment of investment expenses. The rebalancing can result in capital gains or losses, which are combined with operating results to determine the federal income tax liability for the year.

Calculation of Future Asset Market Values

One of the most important aspects of the model's asset calculations is determining new market values at time $T + 1$ for the assets held at time T. The new market values must be developed in concert with the projected interest rate environment. This calculation is essential because, as noted in the previous section, the model uses market values in the asset rebalancing algorithm as the basis for determining whether to sell some or all of the existing assets in any asset group or to buy additional assets for any asset group.

Different techniques are employed for valuing different categories of assets.

Calculating the Market Value at Time $T + 1$ of Bonds Owned at Time T

Traditional bond valuation methods are used to calculate the market value at time $T + 1$ of bonds owned at time T. As described earlier, the model retains information about the pertinent characteristics of each proxy bond, namely amount and timing of coupon payments and principal repayment.[8] From this infor-

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[8]The future cash flows of bonds held at December 31, 1996 are known because the bonds themselves are known quantities. We know their coupon rate and timing, their maturity date, and their par, book, and market values. This is sufficient information to project future cash flows arising from the December 31, 1996 bond portfolio.

The future cash flows of bonds purchased during 1997–2001 are known because we know (a) the risk-free interest rate environment at the time the bonds are purchased, (b) the risk factor that is added to the risk-free interest rate for each bond category, (c) the time to maturity of the bonds that are purchased, and (d) the total dollar amount of new investments in each bond category. With this information, we can calculate an appropriate coupon rate for each dollar of investment in each bond category. We make a mation, the future cash flows from each bond category can be developed. This future cash flow stream, in conjunction with the model-generated interest rate environment at time $T + 1$, is sufficient information to allow the creation of a market value for the bond category at time $T + 1$. It should be noted that this procedure does not contemplate the calculation of option-dependent market values, which are influenced not only by the interest rate at time $T + 1$, but the likelihood of exercising the bond option(s) at time $T + 2$, $T + 3$, etc.

Calculating Future Market Values for Equities

Future market values for equities are derived by projecting values for calendar year equity rates of return, and multiplying the market value of equities at time T by the projected rate of return during the $T + 1$ calendar period. For example, let us assume the XYZ company has a stock portfolio with a market value of $1,000,000 at time T. By some manner, we project this portfolio will generate a fifteen percent return during the upcoming calendar year. The model will calculate the market value of the stock portfolio at time $T + 1$ to be $1,150,000.

The more interesting aspect of this calculation is the way in which the portfolio's rate of return is developed. In some economic scenario generators, this process is embedded within the generator itself, so that the projected economic scenario "automatically" contains projected equity index returns that are correlated with interest and inflation rates. In our experience, the mathematics underlying this type of equity projection methodology tends to be proprietary to the entities that have developed the economic models.

In the absence of such an economic scenario generator, a few alternatives exist for projecting future equity returns. One is to base the rate of return on equities on a normally distributed ran-

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simplifying assumption that new bonds are purchased at par, so the new bonds' market values at the time of purchase equal their book, par and statement values. We now have sufficient information to project future cash flows arising from new bond purchases.

dom variable with a mean market return and standard deviation based on investor expectations. This alternative uncouples equity pricing from changes in interest rates and inflation and is a conventional random walk model. A second alternative is to postulate a relationship between equity returns and interest and inflation rates so that future equity returns can be related to future projections of interest rates and inflation rates. As noted earlier, an example of a postulated relationship between interest rates and equity projections that also attempts to incorporate a time-dependant element is described in Gary Venter's paper [9].

Another method of relating equity returns to the projected interest rate environment is through the use of the Capital Asset Pricing Model (CAPM). Recall that the CAPM formula is $R = R_f + \beta(R_m - R_f)$, where

R = the expected return on a given stock,

R_f = the risk-free interest rate, such as the rate on Treasury bills,

R_m = the overall expected market return, and

β quantifies the undiversifiable or systematic risk associated with the stock (or stock portfolio) in question.

$(R_m - R_f)$ can also be thought of as the market risk premium, or the amount by which the return on stocks is expected to exceed the risk-free rate.

Under this approach, changes in the risk-free rate of return lead directly to changes in the projected equity return. The magnitude of the change felt by the company is driven by the volatility of the company's stock portfolio (β) in relation to the movement in the index portfolio, R_m. Unlike the Venter algorithm, no attempt is being made here to include a time-dependency.

An example of the relative differences in projected returns is shown in Table 11. The example assumes the risk-free rate of return at time T is 6% and that the expected return of the stock market as a whole, R_m, is 15%.

TABLE 11

PROJECTED EQUITY RETURN

	Risk Free Rates		
Betas	6%	8%	4%
1.00	15.00%	15.00%	15.00%
1.50	19.50%	18.50%	20.50%
0.50	10.50%	11.50%	9.50%

When $\beta = 1$, the projected equity return is identical to the projected return that would be achieved by basing the rate of return on a normally distributed random variable with a mean market return and standard deviation based on investor expectations.

It is worth noting that when using the CAPM equation, and assuming a value greater than zero for β, an inverse relationship is developed between changes in interest rates and equity returns. Other authors have postulated the appropriateness of such a relationship.[9]

Revaluing All Other Assets

As was noted earlier, the model makes no special provisions for any other invested asset group other than bonds and stocks. Short-term investments owned at time T are assumed to mature before time $T + 1$, and as such are valued as cash at time $T + 1$. The market value of real estate at time $T + 1$ is changed only if it is specified that capital improvements were made to the property during the $T + 1$st time period. For all other invested assets, it is up to the model user to specify when and how the market value of each asset class will change from time T to time $T + 1$.

Amortization of Bond Original Issue Discount

Because new bond purchases are made at par, it is assumed that only the starting bond portfolio can have a difference be-

[9] See Becker [1], Feldblum [3], or Hodes et al. [5].

tween par value and amortized cost. The reader might recall that the starting bond portfolio is summarized into a set of proxy bonds, one for each maturity grouping. The difference between par value and amortized cost is calculated separately for each proxy bond. This difference is assumed to be "original issue discount," deriving from bond purchases at either a premium or a discount. The original issue discount is amortized over the proxy bond's remaining time to maturity.

7. TAX ALGORITHMS

Some financial models, rather than addressing the complexities of tax algorithms, will stop short of developing after-tax financial statements. We believe this presents a misleading view of the world. Consequently, we believe it is an important and worthwhile endeavor to have a model include tax calculations that are in keeping with the financial statements being developed. If the model develops only statutory financial statements, then it is sufficient that the model address only current federal income tax considerations. If the model develops GAAP financial statements as well as statutory ones, the model should also address deferred federal income tax considerations.

Current Income Taxes

Current income taxes are calculated in accordance with insurance company tax procedures [7, Chapter 13]. Current taxes are calculated by adjusting current year statutory net income as follows:

1. Increase or (decrease) current year net income by 20% of the change in the unearned premium reserve.
2. Increase or (decrease) current year net income by the difference in the amount of tax discount in held reserves.[10]

[10]The model is seeded with historical tax discount factors, either industry, company-specific or a combination of the two, depending upon what tax discount factor elections were made in 1987 and 1992. Projected future discount rates are developed using either

3. Decrease current year net income by 85% of the amount of tax-exempt investment income earned during the year.
4. Reduce current year net income by 59.5% of the amount of dividends received from common and preferred stock (the dividends received deduction is 70%, but 15% of the deduction must be added back into net income for tax purposes).
5. Apply a 35% tax rate to the resulting taxable net income amount.

Alternative minimum taxes also are calculated for the current year by increasing taxable net income by 75% of the amount of tax-exempt investment income and dividends received deduction excluded from regular taxable net income and multiplying the resulting alternative minimum taxable net income by the 20% AMT tax rate.

These calculations develop the preliminary current year tax position. If a projection year develops an operating loss, that loss is compared against the three prior calendar years to see if it can be used to offset prior years' operating gains. If not, it is retained for possible use as an operating loss carryforward, to be applied against operating gains in a later projection year.

Deferred Income Taxes for GAAP Accounting

The major components of the deferred income tax calculation are the tax discount in held loss reserves, deferred taxes on deferred acquisition expenses, and deferred taxes on unrealized gains or losses on equities and bonds available for sale or trade. The GAAP income statement includes the calendar year change in the portion of the deferred tax asset arising from the tax discount in held loss reserves, the deferred taxes on deferred

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pre-seeded industry payout patterns or company-specific payout patterns derived from the line of business underwriting structure and a rolling sixty-month average interest rate that is linked to the model's projected risk-free interest rate projections.

acquisition expenses, and the deferred tax asset or liability arising from unrealized capital gains or losses on that portion of the bond portfolio available for trade.

8. THE FINAL STEP: FINANCIAL STATEMENT PRESENTATION

It is our belief that a model must begin by quantifying cash flows—if cash flows cannot be projected in a reasonably accurate manner, it does not matter how accurately the accounting accruals are developed. In keeping with this belief, we have tried to present mechanics that allow a model to establish with some amount of realism the details of insurance company asset and liability cash flows. It now remains to build up the balance sheet and income statement structure around the cash flows. We proceed by creating a series of general-ledger type accrual accounting entries that extend the underlying cash-basis modeling. We populate these accounting entries by relating them to elements of the insurance company operations that have already been modeled by the underwriting or asset valuation components. Some examples of the types of ratios and the underwriting or asset valuation components to which they might reasonably be related are as follows:

Item	Relationship to:
Agents' balances in course of collection	Written premium, possibly by line of business
Reinsurance recoverable on paid loss	Calendar year paid loss
Interest income due and accrued*	Interest income earned during the year
Expenses due and unpaid	Calendar year expenses incurred
Taxes due (federal or state)	Ratio to calendar year taxes incurred
Provision for reinsurance (Schedule F penalty)-unpaid loss and LAE portion	Year-ending ceded reinsurance balances due or net loss reserves
Provision for reinsurance (Schedule F penalty)-paid loss and LAE portion	Calendar year paid loss

*Depending on the level of detail and sophistication with which assets are modeled, this accrual item may be calculated as part of the asset valuation process. However, if assets are analyzed at even a moderate level of aggregation, this accrual item will need to be estimated instead of calculated directly.

After establishing the asset and liability accrual accounts, we begin the process of creating a full-fledged income statement and cash flow statement. Each of these must contain formulas that respond correctly to changes in the asset and liability accrual values. Much of the information needed to perform this task is described in various chapters of the *Property-Casualty Insurance Accounting* textbook [7]. This is not a step in the model development process to be taken lightly or to be treated superficially. Ultimately, a financial model's results will be shared with many non-actuaries. For them, the test of whether or not the model is (a) believable and (b) worthy of relying on for decision-making will rest in the model's ability to communicate valuable information through the medium of standardized financial statements. A model with sound underlying fundamentals can be undone by such seemingly trivial issues as balance sheets and income statements with minor discrepancies in surplus reconciliation amounts or an incorrect treatment of accounting entries. Accounting rigor also provides model developers with a way of verifying and validating the correctness of the underlying logic so that model users can be comfortable that the model has sound fundamentals.

For those readers already engaged in model development, we hope that this paper provides some ideas on alternative ways of addressing specific modeling issues. For those readers not yet engaged in the model development process, we hope that this paper provides some useful concepts to keep in mind when thinking about the ways in which a financial model might be structured for different organizations. Just as catastrophe models have come to be viewed as a necessity for companies writing property insurance, we believe that DFA-type models will soon be viewed as a necessary tool for examining the overall strategic direction of insurance enterprises. As computer capabilities expand the toolkit available to the actuarial profession, it becomes reasonable to contemplate and actually develop ever more sophisticated and realistic models that will be useful in guiding insurance company decisions.

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EXHIBIT 1

PART 1

INSURANCE INDUSTRY COMPOSITE WORKERS COMPENSATION LINE

	Actuals 1996	Projected 1997	Projected 1998	Projected 1999	Projected 2000	Projected 2001
INCOME STATEMENT RELATED ASSUMPTIONS						
Premiums						
1. Net written premiums prior to rate, competitive impacts:		18,656	18,656	17,441	17,842	19,648
2. Rate impact on net written premiums:		n/a	−10.0%	−2.5%	6.7%	−10.0%
3. Competitive impact on net written premiums:		n/a	n/a	n/a	n/a	n/a
4. Other impact on net written premiums:		n/a	3.9%	4.9%	3.2%	2.7%
5. Final expected net written premiums = (1) * [1 + (2)] * [1 + (4)]	18,656	18,656	17,441	17,842	19,648	18,161
Net Loss & ALAE Ratio, net of subrogation/salvage						
6. Net loss & ALAE ratio, prior to rate, inflation impacts		71.5%	71.6%	82.4%	75.5%	79.0%
7. Impact due to premium rate changes		0.0%	9.5%	3.6%	−5.2%	8.4%
8. Inflationary impact		n/a	2.3%	2.3%	1.2%	1.2%
9. Other impact		n/a	n/a	n/a	n/a	n/a
10. Final expected loss & ALAE ratio = (6) * [1 + (7)] * [1 + (8)]		71.5%	80.2%	87.1%	72.6%	86.7%
Inflation and interest rate information						
11. Inflation rate that is implicitly embedded in premium growth levels		3.2%	3.2%	3.2%	3.2%	3.2%
12. Inflation rate that is implicitly embedded in loss payout pattern		4.7%	4.7%	4.7%	4.7%	4.7%
13. Risk free interest rate underlying asset valuations in current scenario		7.3%	9.2%	6.0%	5.1%	3.9%
Premium earning information						
14. Percent of written premium that is earned during first twelve months		86.5%	86.5%	86.5%	86.5%	86.5%
15. Percent of written premium that is earned during second twelve months		13.5%	13.5%	13.5%	13.5%	13.5%

EXHIBIT 1

PART 2

FORMULA EXPLANATIONS

2. **Rate impact on net written premiums:** Model is assuming an expected loss ratio of 81.9%. If the prior year's loss ratio is less than 81.9%, a rate decrease is implemented. The rate decrease is the lesser of a 10% decrease and the complement of the ratio of the prior year's loss ratio and 81.9%. For example, the projected 1997 loss ratio equals 71.5%, so the rate change in 1998 is the smaller of −10% and (71.5%/81.9%), or −12.7%. A similar formula exists for rate increases if the prior year's loss ratio exceeds 81.9%. The 81.9% expected loss ratio was derived from the ten year average industry loss and ALAE ratio. The 10% cap was implemented based on judgment. *This formula is not meant to be representative of a realistic rate change formula. It is used for example only. A realistic rate change formula would consider many more factors than just the relationship of one year's actual loss and expected loss ratios.*

3. **Competitive impact on net written premiums:** This premium adjustment element is not being used in this example. It could be used to incorporate an underwriting cycle element in pricing.

4. **Other impact on net written premiums:** This factor is used to quantify the impact of wage inflation on premium levels. Based on Bureau of Labor Statistics data, wage inflation has averaged 3.2% over the past ten years. For example purposes, it was assumed that the risk-free interest rate over the past ten years has averaged 6.0%. The projected wage inflation impact is equal to [(3.2%/6.0%) ∗ prior year projected risk free interest rates in (13)].
Example: 1999 premium inflation impact = (3.2%/6.0%) ∗ 9.2% = 4.9%.

6. **Net loss and ALAE ratio, prior to rate, inflation impacts:** This is a stochastically generated loss ratio. The distributional parameters were developed from historical industry loss ratios. This is the underlying "force of loss" that is associated with the policies being earned during the year. This is the loss ratio that would develop, absent any other influences on the loss ratio, such as premium rate changes, premium inflation, and loss inflation.

7. **Impact on loss ratio from premium rate changes:**

$$\frac{1}{\{[(1+\text{current year rate change \% from }(2))*(14)]+[(1+\text{prior year rate change \% from }(2))*(15)]\}}-1$$

Example: 1999 impact = 1/{[(1 − 2.5%) ∗ 86.5%] + [(1 − 10.0%) ∗ 13.5%]} − 1 = 3.6%.

EXHIBIT 1

PART 2

(Continued)

8.	**Inflationary impact on loss ratio:** This is the result of inflationary pressure on loss costs, partly offset by inflationary increases in premium volume. Based on Bureau of Labor Statistics data, loss inflation has averaged 4.7% over the past ten years. Again, for example purposes, it was assumed that the historical risk-free interest rate over the past ten years has averaged 6.0%. Formula: $\frac{\{[(4.7\%/6.0\%) * \text{prior year risk-free interest rate in (13)}] + 1\}}{\{[(1 + \text{current year prem inflation \% from (4)}) * (14)] + [(1 + \text{prior year prem inflation \% from (4)}) * (15)]\}} - 1$ Example: $1999 \text{ impact} = \frac{\{[(4.7\%/6.0\%) * 9.2\%] + 1\}}{\{[(1 + 4.9\%) * (86.5\%)] + [(1 + 3.9\%) * (13.5\%)]\}} - 1 = 2.3\%$
9.	**Other impact on net written premiums:** This element is not being used in this example. It could be used as the counterpart to item (3) in the premium development calculation.
11.	**Inflation rate that is implicitly embedded in the premium growth levels:** This is the ten-year average wage inflation statistic from the Bureau of Labor Statistics Consumer Price Index.
12.	**Inflation rate that is implicitly embedded in the loss payout pattern:** This is an average of the ten-year average wage inflation statistic from the Bureau of Labor Statistics Producer Price Index and the medical care inflation index from the Bureau of Labor Statistics Consumer Price Index.
13.	**Risk-free interest rate underlying asset valuations in current scenario:** This is a stochastically generated future interest rate path, based on a one factor mean-reverting interest rate model.
14.	**Percent of written premium that is earned in first twelve months:** This was calculated from industry statistics [10].
15.	**Percent of written premium that is earned in second twelve months:** The complement of (14).