Compendium of credit risk resources

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March 13, 2017

Abstract

This compendium summarizes the various aspects of credit risk that are important to insurance companies in general, namely corporate credit risk (single and multi-name), typical credit-sensitive securities, credit risk for individuals (including mortgage insurance), municipal credit risk, sovereign credit risk, counterparty risk, and regulatory and enterprise risk management. The document also includes considerations for property and casualty insurers and about their practices. Finally, we also list and link to important resources for practitioners and graduate students.

Keywords

- Actuarial Applications & Methodologies
    - Capital management: Capital allocation, Capital requirements
    - Dynamic risk modeling: ALM, solvency analysis
    - Enterprise risk management: Analyzing/Quantifying risks, Financial Risks
    - Regulation and law: Rating agencies, Risk-based capital, Solvency
- Business Areas
    - Credit
    - Surety
- Financial and Statistical Methods
    - Asset and econometric modeling
        - Asset classes (ABS, Corporate bonds, Equities, MBS, Municipal bonds)
        - Credit spreads
- Practice Areas
    - Consulting
    - Risk management

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Chapter 1

Fundamentals of Credit Risk

1.1 Credit risk

An organization (company, government, etc.) that has issued debt (or any other security) or has been extended credit and is unable to meet its obligations, partially or fully, is deemed to be insolvent. Thus, credit risk arises from the potential loss a lender may suffer due to the borrower's insolvency. Credit risk has two components: the uncertainty related to the timing of default (which may never occur) and the amount of loss at default (loss given default, which is the inverse of the recovery rate given default). A financial instrument (asset, derivative, etc.) with a price that depends directly or indirectly on the solvency of the underlying company is known as a credit-sensitive instrument (asset, security, etc.).

There are several types of credit risk, depending on the type of issuer of credit-sensitive securities:

- Corporate credit risk, which involves private and public companies, primarily through corporate bonds but also stocks, credit default swaps, etc.
- Consumer credit risk, which involves ordinary people through credit cards, lines of credit, loans, mortgages, etc.
- Sovereign, state/provincial/county, municipal credit risk, which arises primarily from bonds issued by countries and their governments or their entities (such as utilities);

Losses resulting from credit risk can be extremely large and can easily spread to impact the solvency of several companies and even an entire economy. Life insurance companies and pension plans are heavily invested in long-term bonds to match long-term cash flows and are thus particularly exposed to credit risk.

There are many of examples of insolvency in the recent past. For example, Lehman Brothers went bankrupt in September 2008, and General Motors filed for bankruptcy protection in June 2009 and reorganized the company thereafter. Russia defaulted on its debt in 1998, whereas Greece restructured its debt in 2012. Finally, the city of Detroit filed for bankruptcy in 2011.

1.2 Legal aspects of credit risk

In the credit risk literature, various terms are often employed to define a similar event: insolvency, default, bankruptcy, credit event, etc. However, there are subtle differences among these terms, and thus, this section will clarify some legal terms used to define credit risk.

A default results from the failure of a debtor to make a payment on a debt, whereas insolvency is the legal term equivalent to default. Bankruptcy is tied to an order from the court supervising an insolvent firm. Moreover, a credit event is a term often used when a credit derivative is established. Usually, the contract needs to specify all the events that trigger a payment. For participants belonging to the International Swaps and Derivatives Association (ISDA), credit events must be defined in the ISDA Master Agreement. Finally, technical default is a term used to designate a quasi-default or a default that has been avoided by government intervention, such as a distressed sale or exchange.

When a company nears insolvency or has defaulted on a payment, the company may seek protection from creditors through the courts. In the US, the Bankruptcy Code (which is technically known as *Title 11 of the*

United States Code) establishes the various types of bankruptcies:

- Chapter VII: bankruptcy for individuals and corporations, involving liquidation of assets
- Chapter IX: bankruptcy for cities
- Chapter XI: bankruptcy for individuals and corporations, involving restructuring of debt

In Canada, the *Bankruptcy and Insolvency Act* oversees the bankruptcy process for individuals and corporations.

Corporate defaults thus generally fit into chapters VII and XI, but most corporations first seek to renegotiate the terms of their debt while protected by the courts (Chapter XI bankruptcy). That was the case for General Motors in 2009, which reorganized its debt and eliminated several automotive brands. Lehman Brothers also initially filed for Chapter XI bankruptcy before Barclays acquired the investment bank the following day. Few companies go directly to a Chapter VII bankruptcy, although that was indeed the case for consumer electronics retailer Circuit City (acquired much later) and video game developer and publisher Acclaim. Many financial institutions and investment banks technically defaulted in 2008 after requesting help from the US government through the Troubled Asset Relief Program (TARP) to find a buyer for their toxic assets.

There are various types of creditors in a company who are entitled to the cash flows of the debt when a company goes into restructuring or liquidation. The most senior creditors are always repaid first when the company is insolvent, whereas junior creditors and stockholders usually receive what remains. This is known as the seniority of a debt, whereas the latter money distribution mechanism is known as the absolute priority rule. For example, when a Canadian individual goes bankrupt, any amount owed to the Canada Revenue Agency (the Canadian equivalent of the IRS) has to be paid first, followed by mortgages, credit cards and other types of loans. Senior debt is thus less risky and trades at a lower interest rate.

When countries, states or cities default, they can restructure their debt with their creditors or seek help from other countries. For example, in the period from 2010 to 2012, Greece received help from the International Monetary Fund (IMF) and other European countries in exchange for implementing drastic austerity measures. Privatization of national services and utilities can also be used to quickly raise money in the event of default.

1.3 Credit risk assessment

Given the extent of the risk involved, it is very important for an investor to assess the credit risk on a security. However, making such an assessment is far from easy. Although it is possible for an investor to use models and data to infer the quality of an asset, there are firms that specialize in evaluating the credit risk of any entity (corporation, municipality, country, etc.).

1.3.1 Credit Rating Agencies

A credit rating agency is a private corporation with the core business of assessing the quality of a debt contract. Investors demand information on the quality of the issuer's credit, and typically, the issuer pays the rating agency to evaluate the credit risk linked to a specific debt issue by considering the likelihood of the firm's insolvency, the loss at default and the seniority of the issue.

A credit rating is a grade or score associated to a specific debt issue. Rating agencies usually use letters, numbers, and plus and minus signs to differentiate the various types of ratings. Long- and short-term debt are usually evaluated differently. In the US, the three major credit rating agencies are Moody's, Standard & Poor's, and Fitch. The highest rating for long-term debt is AAA (S&P and Fitch) or Aaa (Moody's), which is considered a prime investment. Ratings between BBB- and AA+ (S&P and Fitch) or Baa3 and Aa1 (Moody's) are investment-grade investments, whereas lower ratings are designated as high-yield or speculative investments. DBRS (formerly known as Dominion Bond Rating Service) is a rating agency founded and headquartered in Canada; the company uses a scale resembling those of its American counterparts.

In the aftermath of the 2008 financial crisis, the rating agencies were highly criticized, notably by the Financial Crisis Inquiry Commission, as well as by countless journalists and economists.

1.3.2 Modeling, valuation and risk management

Credit risk assessment focuses on the likelihood of default (in a broad sense) and the distribution of the loss in the event of default. Such an assessment can be used for risk management and/or to price credit-sensitive instruments. Credit risk is complicated to model, and unfortunately (from the modeler's point of view), defaults occur very rarely, thereby further complicating the evaluation of credit risk.

Statistical approaches seek to uncover factors that can explain a firm's survival or default. They rely, for example, on financial statements, industry data, default counts, transitions (from one credit rating to another) and similar information. A probit regression is an example of a statistical model that can be used to approximate a company's default likelihood (probit models are discussed in Chapter 4). One of the earliest attempts to evaluate credit risk is Altman's Z-Score, which is based on financial ratios.

Actuarial approaches usually represent the total loss in a portfolio in a manner similar to aggregate loss models in actuarial mathematics. Refinements are usually necessary to account for the possible dependence relationship between the number of defaults in a portfolio and the individual loss on each debt issue. CreditRisk+ by CreditSuisse is a famous example of an actuarial credit risk model (details are provided in Chapter 3).

Actuarial and statistical approaches usually perform better when the portfolio (the number of different firms) is large and sufficient data are available. They are designed primarily for risk management purposes because they cannot be used to consistently price credit-sensitive financial instruments.

Credit risk models can also be designed with the intent of pricing credit-sensitive securities such as stocks, corporate bonds, and credit default swaps. It is important to insist that the term "pricing" is defined consistently with that in financial engineering, i.e., a price that prevents arbitrage opportunities. For example, in the earliest credit risk model, Merton (1974) views stockholder equity as a call option on the firm's assets. Further details on credit risk models can be found in Chapters 2-3.

1.4 References

- Altman, E.I. (2000). Predicting financial distress of companies: revisiting the Z-score and ZETA models. Stern School of Business, New York University. http://pages.stern.nyu.edu/~ealtman/Zscores.pdf
- Merton, R.C. (1974). On the pricing of corporate debt: The risk structure of interest rates. *Journal of Finance* 29, 449–470.

1.5 List of resources

See also Chapter 12.

1.5.1 Books

Presented in alphabetical order.

- Bielecki, T.R., M. Rutkowski (2002). Credit Risk: Modeling, Valuation and Hedging, Springer Finance.
  - As with other Springer Finance books, the treatment of credit risk models is highly technical, but this is one of the few that rigorously addresses the mathematics behind these models. A solid background in mathematics is required.
  - Chapter 1 takes a broad view of credit risk and is perhaps the easiest chapter to understand.
  - Technical/Mathematical level: Accessible/technical
- Bluhm, C., L. Overbeck, C. Wagner (2010). An Introduction to Credit Risk Modeling, Second Edition, CRC Press.
  - Well-rounded book that covers many areas of corporate credit risk.
  - Targeted to both professionals and academics.
  - Some basics of credit risk management are discussed in Chapter 1, while Chapter 6 examines default probabilities.
- Crouhy, M., D. Galai, R. Mark (2000). Risk management, McGraw-Hill.
  - Crouhy et al. (2000)'s book is a well-rounded book that discusses the regulatory system and capital requirements and exhaustively covers credit risk.
  - The same authors published a book in 2014 (Essentials of risk management) that does not seem to go into as much detail on credit risk.
  - Credit ratings and ratings agencies are discussed in Chapter 7.
  - Technical/Mathematical level: Very accessible
- De Servigny, A., O. Renault (2004). Measuring and Managing Credit Risk, McGraw-Hill.
  - An excellent book written for practitioners, devoted entirely to the topic of (corporate) credit risk.
  - Chapter 1 covers the fundamentals of credit risk in a broader context, whereas Chapter 2 examines credit ratings.
  - Technical/Mathematical level: Very accessible
- Duffe, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton Series in Finance.
  - Book mostly targeted to academics and professionals with a solid background in mathematics.
  - Basics of credit risk are covered in Chapter 2.
  - Technical/Mathematical level: Accessible/technical
- Fabozzi, F.J., S. Mann (2012). The Handbook of Fixed Income Securities, 8th edition, McGraw-Hill.
  - A definite must-have book for those interested in various aspects of credit risk (especially corporate and municipal). At over 1800 pages, the book covers a very wide range of topics.
  - The fundamentals of credit risk are briefly discussed in Chapters 1 and 12.
  - Technical/Mathematical level: Very accessible
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
  - Hull's book is always a good starting point for learning about credit risk, a classic finance text for MBA students.
  - Chapter 24 covers (corporate) credit risk in general with a few subsections on credit

ratings, historical default probabilities and recovery rates.
- Technical/Mathematical level: Very accessible

- Hull, J.C. (2015). Risk Management and Financial Institutions, Wiley Finance.
  - Book mostly aimed at practitioners.
  - Chapter 19 is an excellent introduction to the various aspects of corporate credit assessment and valuation.
  - Technical/Mathematical level: Very accessible.

1.5.2 Websites and online reports

- US Bankruptcy Code for individuals and corporations (Official government website): http://www.uscourts.gov/services-forms/bankruptcy. Links to the various bankruptcy Chapters are clearly indicated.
  - US Code: Title 11 - Bankruptcy, Chapter 7 - Liquidation (through Cornell University Law School): https://www.law.cornell.edu/uscode/text/11/chapter-7
  - US Code: Title 11 - Bankruptcy, Chapter 11 - Reorganization (through Cornell University Law School): https://www.law.cornell.edu/uscode/text/11/chapter-11
  - US Code: Title 11 - Bankruptcy, Chapter 13 - Adjustment of Debts of an Individual with Regular Income (bankruptcy for individuals) through Cornell University Law School): https://www.law.cornell.edu/uscode/text/11/chapter-13

- Canadian bankruptcy laws:
  Summary from Industry Canada: https://www.ic.gc.ca/eic/site/cilp-pdci.nsf/eng/h_cl00021.html
  - *Bankruptcy and Insolvency Act* (Liquidation, corporations and individuals): http://laws-lois.justice.gc.ca/eng/acts/b-3/ (Official government website)
  - *Companies' Creditors Arrangement Act* (Reorganization): http://laws-lois.justice.gc.ca/eng/acts/C-36/ (Official government website)
  - US Department of Treasury, Troubled Asset Relief Program (TARP) (Official government website): http://www.treasury.gov/initiatives/financial-stability/TARP-Programs/Pages/default.aspx
- Ratings agencies' official websites:
  - Moody's: https://www.moodys.com/
    - Moody's Analytics – Moody's KMV: http://www.moodysanalytics.com/About-Us/History/KMV-History
  - Standard & Poor's: http://www.standardandpoors.com/en_US/web/guest/home
  - Fitch: https://www.fitchratings.com/
  - Dominion Bond Ratings Services (DBRS): http://www.dbrs.com/
- Ratings definitions:
  - Moody's (March 2015): https://www.moodys.com/researchdocumentcontentpage.aspx?docid=PBC_79004
  - Standard & Poor's (August 2016): https://www.standardandpoors.com/en_US/web/guest/article/-/view/sourceId/504352
  - Fitch (December 2014): https://www.fitchratings.com/web_content/ratings/fitch_ratings_definitions_and_scales.pdf
- Annual corporate default and recovery rate studies:
  - Moody's (2008 study, latest public study): https://www.moodys.com/sites/products/DefaultResearch/2007400000578875.pdf
  - Moody's (up to 2016, not public): https://www.moodys.com/Pages/GuideToDefaultResearch.aspx
  - Standard & Poor's (2014 study, in PDF format, unofficial source):

http://www.nact.org/resources/2014_SP_Global_Corporate_Default_Study.pdf (from the National Association of Corporate Treasurers (NACT).
- Standard & Poor's (2014 study, in HTML, official source): can be found through their Global Credit Portal https://www.globalcreditportal.com/ or through the Search tool from S&P's official website. Title of the report: Annual Global Corporate Default Study And Rating Transitions.
- Fitch (2014 study, through NRSRO Annual Certification): https://www.fitchratings.com/web_content/nrsro/nav/NRSRO_Exhibit-1.pdf
- DBRS (2014 study, official): http://www.dbrs.com/research/278497/2014-dbrs-corporate-rating-transition-and-default-study.pdf

1.5.3 Data

In addition to the major databases mentioned in Chapter 12, two additional databases focus on bankruptcy filings.

- UCLA (University of California, Los Angeles) LoPucki Bankruptcy Research Database (BRD): Chapter 7 and 11 filings for companies with over $100 million in assets: http://lopucki.law.ucla.edu/ Sample data available.
- New Generation Research Bankruptcy Data: http://www.bankruptcydata.com/

Historical credit ratings by companies, sorted by credit ratings agency (see Chapter 12).

1.5.4 Computer programs

Matlab programming language:

- Credit risk modeling with Matlab

Chapter 2

Single-name corporate credit risk models

A wealth of scientific and professional literature has been published on this subject, and this chapter summarizes the most significant approaches proposed to assess the credit risk of debt issuers.

2.1 Structural models

Structural credit risk models rely on an explicit definition of the company's assets, liabilities and equity. Default is triggered when assets are insufficient to meet the company's obligations, either at the exact moment the payment is due or before.

2.1.1 Merton (1974)

Merton's (1974) model is the cornerstone of modern credit risk assessment. The idea is that a firm is composed of risky assets (A_t) and has committed to paying a specific sum of money (F) on a known future date (T), which is the maturity of this debt. Before the debt matures, the company pursues normal business, regardless of the valuation of the assets. At debt maturity T, the firm is dissolved and the assets redistributed between debtholders and equityholders according to the absolute priority rule, whereby the creditors are paid in full first and the equityholders receive the remainder.

Using mathematical notation, we have

payment to debtholders at $T = D_T = F - max(F - A_T, 0)$
payment to equityholders at $T = E_T = max(A_T - F, 0)$.

Thus, one can view equity as a call option on the firm's assets, whereas the payment to debtholders corresponds to the face value of the debt (the promised payment) minus a put option on the assets, which represents the debtholders' loss given default. Therefore, using put-call parity, we easily recover the fundamental accounting equation, which is

$$A_T = L_T + E_T$$
$$A_T = F - max(F - A_T, 0) + max(A_T - F, 0).$$

That is, the total value of assets is equal to the sum of liabilities and equity.

When the firm survives, the debtholders recover F (the put option is out-of-the-money), whereas the equityholders have a right to the excess of A_T over F. When the firm defaults, the debtholders have a right to the liquidated value of the firm (A_T), and debtholders suffer a loss of $F - A_T$. However, equityholders receive nothing (the call option is out-of-the-money).

Assuming that the market value of the assets of the company evolves as a geometric Brownian motion, which is the standard Black-Scholes assumption, one can use the Black-Scholes equations to value the company's equity and liabilities.

In computing the value of the equity (or of the liabilities), one important quantity often arises, which is

$$d_1^{\mu} = \frac{\ln\left(\frac{A_T}{F}\right) + \mu(T-t)}{\sigma\sqrt{T-t}}.$$

d_1^{μ} is known as a normalized "distance to default" metric[1]. Indeed, the ratio of the assets over the face value is some form of distance that has to be adjusted by the potential growth of the assets ($\mu(T-t)$) and by the volatility of the assets ($\sigma\sqrt{T-t}$). The original founders of the company KMV, Kealhofer, McQuown and Vasicek, designed an expected default frequency (EDF) based on the distance to default.

Merton's model is very intuitive and has done much to foster our understanding of the dynamics between the capital structure and the firm's credit risk. However, the default triggering mechanism is unrealistic, as most creditors would not wait until maturity to claim a fraction of the assets. Moreover, at least from a scientific point of view, the asset and liability dynamics are overly simplistic.

2.1.2 First-passage models

In first-passage credit risk models, default is triggered as soon as the value of the assets crosses a given barrier, known as the default barrier. This barrier is likely to be different from the value of the promised payment(s), as debtholders are risk-averse and would intervene to maximize the likelihood of being paid. First-passage models are similar in spirit to ruin theory, according to which the claim and premium arrival processes are used to represent the insurer's ruin.

To contrast the first-passage and Merton models, we will rely on Figure 2.1. Suppose that the debt matures in 20 years. In Merton's model, the firm easily survives because the assets (700) exceed the liabilities (200). However, we see that at approximately 10 years, the assets have fallen below the value of liabilities, and thus, in a first-passage model, that company would have defaulted.

First-passage models are also intended to quantify value for debtholders and equityholders. The idea is similar to Merton's, namely that equityholders and debtholders have a right to assets whenever there is a default. Thus, whenever the value of assets crosses the default barrier, the absolute priority rule applies, and debtholders are paid in full before equityholders. Instead of having plain vanilla call and put options to represent the equity value and the loss given default, in a first-passage model, we instead have barrier call and put options. The authors who pioneered the first-passage model and equity pricing include Black & Cox (1976) and Brockman & Turtle (2003). At present, most structural models are based on the first passage of assets across a default barrier.

A notable professional model was also inspired by the spirit of first-passage models, namely CreditMetrics by RiskMetrics (the model was formerly owned by JP Morgan). In CreditMetrics, the creditworthiness of a firm evolves according to a Markov chain, the role of which is to mimic changes

[1] Note that when d_1 is used to price equity and liabilities, μ has to be replaced with r to be consistent with absence of arbitrage arguments.

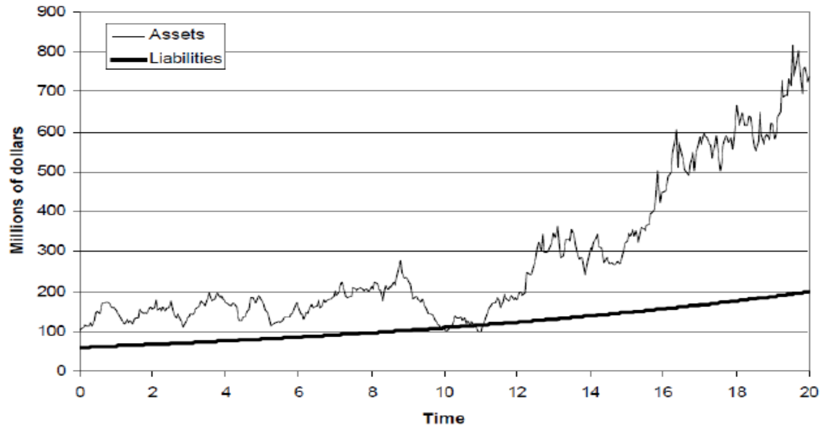

Figure 2.1: Evolution of assets and liabilities of a fake company

in the credit ratings of a company over time. For example, a firm can evolve from A to B, AAA or D (default). Further details can be found in Chapter 3.

2.1.3 Other structural models

Numerous extensions have been proposed to extend Black & Cox (1976) with the similar goal of evaluating the equityholder's and debtholder's value. For example, stochastic interest rates were introduced in Longstaff & Schwartz (1995) and Briys & de Varenne (1997). Collin-Dufresne & Goldstein (2001) introduced a structural model with additional debt issuance, thus allowing for mean reversion in leverage ratios under a stochastic interest rate environment. Finally, Leland (1994) and Leland & Toft (1996) introduced models in which debt is a (perpetual) coupon bond and the default trigger is endogenously determined by external frictions such as bankruptcy costs, and taxes.

2.1.4 Conclusion

Structural models are extremely useful for better understanding the interplay of assets and debt in valuing a firm and its default risk. However, most structural models are unable to replicate the level of credit spreads observed for short-term securities.

The main reason for this deficiency is that in a structural model, default or survival is predictable in the very short term. When the distance to default is large, which means that the firm's assets are currently far from the default barrier, multiple declines in asset value need to be generated in the short term to generate a default. This event is generally nearly impossible to include in a model, and hence, these models cannot replicate the observed level of credit spreads for short-term securities (see the discussion by Jones, Mason & Rosenfeld (1984) and more recently the books mentioned below). It is clear that default holds an element of surprise that increases short-term credit spreads.

When structural models are used to value debtholders' and equityholders' shares of the assets, the absolute priority rule is a crucial assumption. However, according to Eberhart et al. (1990) and more recently Bris et al. (2006), deviations from the absolute priority rule are common and are even expected and priced in the financial markets.

2.2 Reduced-form models

Instead of modeling the assets and the debt of a company to obtain the value of the debtholders and equityholders, a reduced-form credit risk model takes the completely opposite view. The idea is that we directly model the instantaneous default probability (also known as default intensity, hence intensity models) as a stochastic process. This process is completely unrelated to the firm's capital structure. The stochastic process is chosen and estimated to replicate the observed prices of credit-sensitive assets as closely as possible.

Default intensity or forces of mortality are similar concepts in which we model the instantaneous probability of the event: default or death. Even if death probabilities can evolve over time with the policyholder's behavior, it is clear that default probabilities will vary considerably more over time. Therefore, reduced-form models have grown to be increasingly sophisticated to better fit prices of credit-sensitive securities.

The idea of a reduced-form model was pioneered by Jarrow & Turnbull (1995) and further extended in Jarrow et al. (1997). In the former article, a model similar to an exponential distribution was used for the moment of default, whereas in the latter, the rate of arrival of default was modeled as a continuous-time Markov chain, thus mimicking ratings transitions from credit rating agencies. Other important and early contributions to reduced-form models are Lando (1998) and Duffie & Singleton (1999).

From the latter articles, the number of reduced-form models has exploded over the years as authors have often changed the default intensity process. Because the T-year default probability (as opposed to default intensity) is computationally equivalent to the price of a zero-coupon bond in a stochastic interest rate model, the former literature has borrowed much from the latter. The paper by Duffie, Pan & Singleton (2000) helped many authors to find closed-form formulas for default probabilities. Because the speed of calibration and estimation is determined primarily by the speed at which these quantities are computed, analytical expressions can become substantial.

Whereas structural models use information from the firm's capital structure to deduce default probabilities and the price of credit-sensitive securities, reduced-form models use the observed prices of these securities to deduce the market's view of the firm's solvency. Therefore, parameters obtained from such a calibration are only valid to price other credit-sensitive instruments.

2.3 Hybrid models

Although the fit of reduced-form models to security prices is known to be much better than that of structural models, reduced-form models lack the financial intuition and justification of structural models. However, structural and reduced-form models can be viewed as a single, consistent type of model, depending on the amount of information observed by an investor. The information paradigm of credit risk models was introduced by Duffie & Lando (2001) and further justified by Jarrow & Protter (2004).

To be usable in practice, structural models rely on full and continuous knowledge of the market value of the company's assets and debt structure. However, only firm managers have that much information, whereas investors only receive partial and periodic information from the company, often in the form of accounting statements. Duffie & Lando (2001) demonstrate that for a given capital structure, when investors receive only partial information, default may prove to be partially predictable and thus better replicate short-term credit spreads. Contrarily, when investors have absolutely no information, the perceived credit risk may behave as in a reduced-form model. The result is what is known as a hybrid credit risk model, i.e., a model that features components of both structural and reduced-form credit risk models.

2.4 Loss given default

The vast majority of the previous models focus primarily on determining the likelihood of default, whereas another component of credit risk has an important impact on the prices of credit-sensitive securities: the loss given default (LGD) (or, conversely, the recovery rate). Altman et al. (2005) and Acharya et al. (2007) report an inverse relationship between default probabilities and recovery rates. In other words, solvent companies that default (say, for liquidity reasons) have higher recovery rates (and lower LGDs), whereas insolvent companies have lower recovery rates[2]. Thus, the factors that explain the likelihood of default also seem to explain the LGD. Altman et al. (2005) report that losses can be highly underestimated when the relationship between the two is ignored.

Earlier attempts to integrate stochastic recovery rates possibly related to company solvency are Frye (2000), Jarrow & Yu (2001) and Jokivuolle & Peura (2003). For example, Frye (2000) and Jokivuolle & Peura (2003) focus on the value of the collateral on a loan, with recovery provided by the value of collateral on default. The negative relationship between recovery rates and default intensity/probability is integrated in the models of Gaspar & Slinko (2008), Das & Hanouna (2009), Bruche & Gonzalez-Aguado (2010), Bade et al. (2011) and Boudreault et al. (2013).

When recovery rates are stochastic and depend on the same factors as default, a term structure of recovery rates appears. In other words, the timing of default impacts the distribution of recovery rates in a credit-sensitive instrument. Boudreault et al. (2013) find that the term structure of recovery rates decreases for high-rated firms but increases for low-rated firms. For example, a AAA-rated firm can only maintain or downgrade its rating and hence, its recovery rate can only be stable or decrease in the future. For a CCC-rated firm, if it survives, its solvency will improve (or it will default and be excluded), and thus its recovery rate will increase.

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[2] This was observed as early as in Altman & Kishore (1996) and in Table III of Elton et al. (2001).

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2.6 List of resources

See also Chapter 12.

2.6.1 Books

Presented in alphabetical order.

- Ammann, M. (2002). Credit Risk Valuation: Methods, Models, and Applications, Springer
  - Mostly focused on single-name credit risk models, aimed at academics.
  - Chapters 1–5 examine traditional single-name models.
- Bielecki, T.R., M. Rutkowski (2002). Credit Risk: Modeling, Valuation and Hedging, Springer Finance.
  - The mathematics behind structural models (Chapters 2 and 3) and reduced-form models (Chapter 8) are rigorously described.
  - Technical/Mathematical level: Very technical.
- Bluhm, C., L. Overbeck, C. Wagner (2010). Introduction to Credit Risk Modelling, Second Edition, CRC Press.
  - Merton's model is covered in Chapter 3.
- Crouhy, M., D. Galai, R. Mark (2000). Risk management, McGraw Hill
  - Chapter 9 is dedicated to structural models, also known as the contingent claim approach. It is mostly focused on Merton's model and its commercial equivalent, Moody's-KMV.
  - Chapter 10 discusses reduced-form models.

  - Technical/Mathematical level: Very accessible
- De Servigny, A., O. Renault (2004). Measuring and Managing Credit Risk, McGraw-Hill.
  - Merton's model and Moody's-KMV models are described in Chapter 3.
  - It is interesting to note that this book is one of the few that examines credit scoring techniques, i.e., using regression models (statistics) to evaluate solvency (from Altman's Z-score to generalized linear models).
  - There is also an entire chapter devoted to loss given default (recovery rate) and its impact on credit risk management.
  - Technical/Mathematical level: Accessible
- Duffie, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton Series in Finance.
  - Structural, reduced-form models and some statistical models are covered in Chapter 3. The authors contrast pricing and risk-management applications (physical and risk-neutral probabilities) in Chapter 5.
  - Technical/Mathematical level: Accessible/technical
- Fabozzi, F.J., S. Mann (2012). The Handbook of Fixed Income Securities, 8th edition, McGraw-Hill.
  - Single-name models are very well summarized in Chapter 45. The content is scientifically accurate but citations are centered on the chapter's co-authors.
  - Technical/Mathematical level: Very accessible
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
  - Chapter 24 has some subsections on credit risk modeling, but coverage is very light. There is not much on the two families of credit risk models.
  - Technical/Mathematical level: Very accessible
- Lando, D. (2004). Credit risk modeling, Theory and Applications, Princeton University Press.
  - Lando's book reviews a wide range of credit risk models and methods, especially from an academic standpoint.
  - Structural models are exhaustively covered in Chapters 2 and 3, whereas reduced-form models are discussed in Chapter 5.
  - It is worth mentioning that there is an entire chapter devoted to credit scoring techniques using regressions (Chapter 4).
  - Technical/Mathematical level: Technical
- McNeil, A.J., R. Frey, P. Embrechts (2005). Quantitative Risk Management, Princeton University Press.
  - Textbook aimed at advanced undergraduates, graduates or professionals with an applied mathematics background. Covers a wide range of topics in quantitative risk management. Very well done if the reader has the technical knowledge.
  - Chapters 8 and 9 discuss some single-name and portfolio credit risk models in detail.
  - Technical/Mathematical level: Technical
- O'Kane, D. (2008). Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance.
  - Book focusing specifically on pricing credit derivatives. The Wiley Finance series is usually accessible to practitioners.
  - Brief summary of single-name credit risk models in Chapter 3.
  - Technical/Mathematical level: Accessible

2.6.2 Computer programs

Matlab programming language

- Merton structural credit risk model

- Moody's KMV Credit Risk Model Probability of default

Chapter 3

Portfolio corporate credit risk models

In Chapter 1, we laid down the foundations of credit risk, whereas in Chapter 2, we addressed how to assess credit risk for a single corporation. We now extend some of the principles found in Chapter 2 to portfolio credit risk management. To adequately transition from Chapter 2 to this chapter, we must understand that a key aspect of portfolio credit risk is assessing the dependence between assets. Therefore, Section 1 summarizes some of the key concepts of dependence, whereas Sections 2 and 3 briefly review some of the most important classes of portfolio models.

3.1 Notions of dependence

3.1.1 Sources of dependence

From the probabilistic or statistical perspective, dependence between two random variables X and Y is defined as the influence that a specific realization of X exerts on the distribution of outcomes of Y. In finance and actuarial science, dependence can occur in several ways: as a direct influence of one asset on the other or as a result of a common source of shock or influence affecting both.

An example of dependence resulting from direct influence occurs in life insurance. The broken heart syndrome is a well-known phenomenon whereby the death of a spouse alters the lifetime distribution of the surviving spouse. Natural catastrophes in P&C (property and casualty) insurance provide many situations of common shocks that simultaneously affect the claims distribution of a portfolio of policyholders. For example, the occurrence of an earthquake can affect thousands of policyholders living close to the epicenter.

In credit risk modeling, dependence also occurs as either a direct influence or a result of exposure to common variables. The most common source of dependence in credit risk is exposure to common macroeconomic (state of the economy, interest rates, etc.) or industry variables (those that affect one sector as a whole but not another). An example of how defaults can directly influence the solvency of other companies occurs when an important insolvency on the asset side of an investor's balance sheet can also provoke its own bankruptcy. This phenomenon is known as a type of contamination process and can occur when large financial institutions default (systemic risk).

3.1.2 Dependence measures

To assess the impact of dependence on portfolio losses, we need to determine the level of dependence contained in the asset portfolio. To do so, we use what is known as a dependence measure. Such a measure evaluates the degree to which X influences Y and vice versa.

The attentive reader might have noticed that thus far we have discussed dependence without mentioning the word "correlation". Correlation, as defined by Pearson, is

$$Corr(X,Y) = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where μ_X and σ_X are the mean and standard deviation of the random variable X, respectively. Correlation describes whether one variable is above its mean when the other variable is either above or below its mean. It is situated between -1 and 1.

Correlation is the unique dependence parameter in the joint normal distribution and plays a fundamental role in linear regressions. This is also why it is known as a linear dependence measure, as it can only capture linear types of dependence. Therefore, it can fail to measure many types of dependence relationships, as shown in Figure 3.1.

There are alternative dependence measures known as rank correlation. The idea of rank correlation is to measure the capability of generating small/large realizations when the other variable has generated a small/large outcome. However, instead of determining the relative size based on the mean, as is the case with Pearson's correlation, the idea is to compare ranks. The most well known rank correlation measures are Kendall's tau and Spearman's rho, two measures included in most statistical software such as R and Matlab.

Finally, a dependence measure that is extremely useful in finance is the tail dependence measure. Tail dependence is the ability of one variable to influence the other when either has generated an extreme outcome. For example, in times of crisis, stocks and other assets traded on financial markets tend to show more dependence than in more "normal" times. Because the tails of a normal distribution are very light (it is incapable of consistently generating large extremes), the joint normal distribution does not have tail dependence.

3.1.3 Creating dependence

To create dependence in a portfolio of credit-sensitive assets, the most popular but not necessarily most appropriate choice is to choose a multivariate normal distribution. In the latter

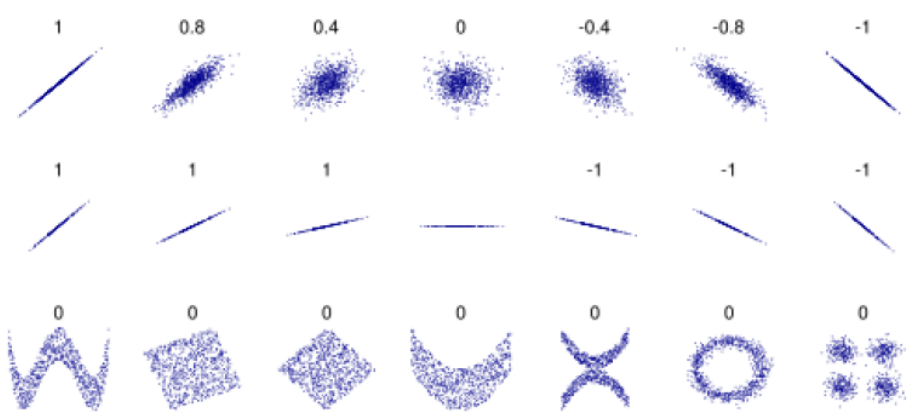

Figure 3.1: Illustration of how Pearson's correlation can fail in detecting several types of dependence (Source: Wikipedia.)

case, dependence between risks is summarized by each pairwise correlation.

Another popular approach used in finance to create dependence between random variables is to use what is known as copulas. A copula is a mechanism that creates dependence using the marginal cumulative distribution functions (c.d.f.s). Mathematically, suppose we have two risks X and Y with known c.d.f.s $F_X(x)$ and $F_Y(y)$. Instead of writing X as a function of Y or linking X and Y with another variable Z, the copula works by assuming that the joint c.d.f. $F_{X,Y}(x,y)$ is some function of $F_X(x)$ and $F_Y(y)$. Therefore,

$$F_{X,Y}(x,y) = C(F_X(x), F_Y(y))$$

where the bivariate function C is known as the copula. The copula creates dependence between X

and *Y* by linking each c.d.f. $F_X(x)$ and $F_Y(y)$.

Using copulas is very popular because it allows the modeler to separate the problem of fitting the distributions of X and Y and assessing the dependence in the variables. An excellent and accessible reference on the topic is Frees & Valdez (1998).

A very popular copula in finance and credit risk modeling is the Gaussian or normal copula[3]. The Gaussian copula links two or more random variables that are not necessarily normally distributed using the dependence structure that stems from a multivariate normal distribution. For example, X can be gamma distributed, and Y can be distributed as a Weibull, but we can create a dependence structure between *X* and *Y* based on their correlation.

The Gaussian copula is tractable; correlation between variables is relatively easy to estimate, but as mentioned earlier, it lacks appropriate tail dependence. The problem remains that for large portfolios, there are many correlations to estimate (large correlation matrix).

There are alternatives to reduce the number of correlations to estimate. One can assume that correlation is constant for subgroups of firms (block constant correlation matrix). Another popular approach is to use factor models. The idea is that risk X can be regarded as the sum of random common factors (common to all firms) and an idiosyncratic element, which is firm specific. The one-factor model is very popular, and when risk factors are Gaussian, we retrieve the Gaussian copula.

The widespread use of Gaussian copulas (and some factor models) in the pricing of collateralized debt obligations (CDOs) (see Chapter 7) was highly criticized in the aftermath of the 2008 financial crisis. However, realistic alternatives to Gaussian copulas for large portfolios (of more than 5–10 assets!) are unfortunately not readily available either. The Student-t copula is one of the few realistic alternatives that are also based on correlations, with a feature that allows tail dependence to be gauged.

3.2 Professional models

Professional portfolio credit risk management models are designed primarily to compute the credit value-at-risk (VaR), i.e., the VaR related to credit losses. Two popular approaches used in the industry are CreditMetrics (by RiskMetrics, MSCI) and CreditRisk+ (by Crédit Suisse). The two methods are briefly described below, with further details provided in their respective technical documents, also below.

3.2.1 CreditMetrics

CreditMetrics is mostly based on ratings provided by agencies such as Standard & Poor’s and Moody’s. The idea is (1) to evaluate each bond (or any credit-sensitive asset) held in the investment portfolio for each possible ratings transition (upgrade, downgrade and default) and (2) to do so using joint transition probabilities to calculate the credit VaR.

The first step is to compute the hypothetical price of each bond for every possible rating. This is usually done with the 1-period forward yield curves specific to each possible rating.

The second step is to compute the joint transition probabilities, e.g., the probability that issuer no. 1 is upgraded to AA while issuer no. 2 is downgraded to B. While the ratings agencies provide transition matrices, this only applies to one company, and thus yield marginal transition probabilities. To compute joint transition probabilities, CreditMetrics maps ratings to a normal distribution such that the Gaussian copula can be used to compute the joint probabilities.

For large portfolios, CreditMetrics is best handled with a simulation. The multivariate

[3] The Archimedean family of copulas is also well known in statistics (including Frank, Clayton & Gumbel), but we will not discuss these further. This is because when attempting to link more than two random variables in an Archimedean copula, it is difficult to ensure that dependence between all pairs differs from one pair to the next.

Gaussian copula is very easy to simulate, even on large scales. With this simulation, retrieving the transitions for each bond and thus the gain or loss is straightforward.

Correlation is a critical input in CreditMetrics, and theoretically, we require a correlation for each specific pair of firms in the portfolio. For 100 firms, there are 5,000 correlations to estimate. This is a daunting task that can be simplified by using constant correlations across sectors. For example, two companies from the same sector have the same correlation, and the correlation between any two companies in each of the two sectors is also constant (block constant correlation matrix). CreditMetrics suggests using equity correlation, i.e., correlations of stock returns, although this is only an approximation, as stock return dynamics do not necessarily reflect the credit risk dynamics (see. e.g., Boudreault et al. (2015)).

3.2.2 CreditRisk+

The core of the CreditRisk+ model is actuarial in nature. The simplest version assumes a very large portfolio with small individual default probabilities. In this case, the number of defaults in the portfolio can be reasonably approximated by a Poisson distribution. Typical severity distributions can be used to approximate the loss on each default such that the aggregate loss can be evaluated. This simplistic framework does not account for dependence in defaults or individual bond characteristics.

This framework can be extended to account for (a) heterogeneity in the bond loss distribution and (b) dependence in default occurrences. However, doing so means addressing individual default occurrences and every possible loss through a Monte Carlo simulation. Models similar to CreditRisk+ are presented in Chapter 9 of McNeil et al. (2005).

3.3 Academic models

Although they can also be used for risk-management purposes, most academic portfolio credit risk models are also designed for pricing. Pricing in this context is taken in the financial engineering sense, meaning that we are required to find the price that eliminates arbitrage opportunities. In this pricing framework, models are able to price basket credit default swaps (such as k-th to default), collateralized debt obligations, and so forth (further details on these products are provided in Chapter 7).

A range of multivariate extensions of structural and reduced-form credit risk models exist. In a multi-name context, the idea is that each company's assets and liabilities are bound using dependence models such as copulas. This will create clusters of defaults in times of crisis. Similarly, in multi-name reduced-form models, the default intensity of each company has some form of dependence, which increases the likelihood of default clusters. Often-cited papers are Li (2000), Duffie & Garleanu (2001), Andersen & Sidenius (2004), and Hull, Predescu & White (2010). Special models of contagion and contamination have appeared in the literature in which the default of one company provokes a sequence of defaults (see, for example, Davis & Lo (2001) and Jarrow & Yu (2001)).

The financial crisis also prompted additional research on the topic of systemic risk, the risk that one very large institution provokes the insolvency of other financial institutions. When investigating banks and insurance companies, a notable finding is that the insolvency of major banks may cause the insolvency of insurance companies but not the opposite. Key papers on that topic are Billio et al. (2012) and Chen et al. (2014). There is also an entire scientific journal devoted to the issue: Journal of Financial Stability (launched in 2004).

Finally, it is well known that recovery rates are inversely proportional to firms' solvency (see Chapter 2, Altman et al. (2006) and Acharya et al. (2007)). During recessions, the number of defaults rises, and we should expect recovery rates to decrease as solvency deteriorates. While this effect on aggregate losses has seldom been investigated, this double-whammy effect is examined in Boudreault et al. (2014).

3.4 References

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- McNeil, A.J., R. Frey, P. Embrechts (2005). Quantitative Risk Management, Princeton University Press.

3.5 List of resources

3.5.1 Books

Presented in alphabetical order.

- Bielecki, T.R., M. Rutkowski (2002). Credit Risk: Modelling, Valuation and Hedging, Springer Finance.
  - Dependence in portfolio structural models is briefly described toward the end of Chapter 3, while this topic is also discussed in Chapters 9 and 10.
  - Technical/Mathematical level: Very technical.
- Bluhm, C., L. Overbeck and C. Wagner (2010). Introduction to Credit Risk Modelling, Second Edition, CRC Press.
  - Correlated defaults are discussed in Chapter 2, and CreditRisk+ is covered in Chapter 4.
- Crouhy, M., D. Galai, R. Mark (2000). Risk management, McGraw Hill

  - Chapters 7 and 8 examine the building blocks of CreditMetrics and the evaluation of the credit VaR in this model.
  - Moody's-KMV, which is often used in portfolio models, is discussed in Chapter 9.
  - CreditRisk+ is presented as an actuarial model in Chapter 10.
  - Technical/Mathematical level: Very accessible
- De Servigny, A., O. Renault (2004). Measuring and Managing Credit Risk, McGraw-Hill.
  - Two complete chapters are devoted to the issue of portfolio credit risk modeling. Chapter 5 examines the sources and measures of dependence, while portfolio models are discussed in Chapter 6.
  - Technical/Mathematical level: Accessible
- Duffie, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton Series in Finance.
  - Correlated defaults are covered in Chapter 10.
  - Technical/Mathematical level: Accessible/technical
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson
  - Chapter 22 discusses default correlation and credit VaR, which is well explained.
  - Technical/Mathematical level: Very accessible
- Hull, J.C. (2015). Risk Management and Financial Institutions, Wiley Finance.
  - Book mostly targeted at practitioners.
  - Credit VaR is covered in Chapter 21.
- Lando, D. (2004). Credit risk modelling, Theory and Applications, Princeton University Press.
  - Entire chapter devoted to dependence modeling, from correlation and copulas to network dependence.
  - Technical/Mathematical level: Technical
- McNeil, A.J., R. Frey, P. Embrechts (2005). Quantitative Risk Management, Princeton University Press.
  - Textbook targeted at advanced undergraduates, graduates or professionals with an applied mathematics background. Covers a wide range of topics in quantitative risk management. Very well done if the reader has the technical skills.
  - Chapter 5 addresses copulas and dependence.
  - Chapters 8 and 9 discuss in detail some single-name and portfolio credit risk models.
  - Technical/Mathematical level: Technical
- O'Kane, D. (2008). Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance.
  - Chapters 13–16 discuss portfolio models and how they can be used to price multi-name derivatives.

3.5.2 Websites and online reports

- CreditMetrics' (official) technical document (2007), RiskMetrics, MSCI: https://www.msci.com/resources/technical_documentation/CMTD1.pdf
- CreditRisk+ (official) document (1997), Credit Suisse | First Boston: http://www.csfb.com/institutional/research/assets/creditrisk.pdf

3.5.3 Computer programs

R Programming Language

- crp.CSFP package: According to CRAN "Modelling credit risks based on the concept of "CreditRisk+", First Boston Financial Products, 1997 and "CreditRisk+ in the Banking Industry", Gundlach & Lehrbass, Springer, 2003."
- CreditMetrics package: According to CRAN "A set of functions for computing the CreditMetrics risk model"

Chapter 4

Credit risk for individuals

This chapter discusses credit risks that are sold by insurance companies. Instead of proposing how to model credit risk on the asset side, we discuss credit risk on the liability side.

In the case of short-term credit insurance products, for example insurance on credit cards, mortgages[1], surety insurance, and similar products of this type, actuaries can use the traditional approach of P&C insurance. Insurance companies can then operate under a micro-level approach, where each insured is observed over several years. The risk characteristics, the presence or absence of default risk, as well as the date and the amount paid for those defaults are recorded in the insurance company database.

In the analysis of such insurance products, P&C actuaries are on familiar ground, and the usual pricing techniques can be used. This chapter will summarize the statistical approaches used in pricing in general insurance. As opposed to the other chapters of this compendium, a more technical and theoretical approach is needed in this chapter, as the knowledge of the P& C actuaries is more advanced.

4.1 Credit Scoring

The basis for the credit risk modeling for individuals is the notion of risk scoring. Scoring is a classic segmentation technique used in finance, marketing or actuarial sciences. This technique is to assign an overall rating to an individual based on some of his personal characteristics, such as his age, his sex, his marital status or his income but can also be based on past activities or past defaults. These characteristic variables can be categorical or continuous and may also be used independently or in interaction. As we shall see, the construction of a score uses predictive modeling techniques.

The main idea of credit score modeling is to use a historical database, in which we have the characteristics of each individual, a variable indicating the number of credit defaults, and ideally, the cost involved in each of his bankruptcies. Table 4.1 shows a small sample of a database illustrating the type of observations with which a scoring analyst could work[2]:

# Observation	**Sex**	**Age**	**Civil Status**	**…**	**Number of defaults**	**Loss given default**
1	F	51	Married	…	0	.
2	M	45	Single	…	1	15,733
3	M	27	Single	…	0	.
…	.	.	…	…	…	…
155,000	F	71	Divorced	…	2	7,845

[1] Mortgage insurance has average durations of 4-7 years, with some policies extending beyond 7 years.

[2] In the U.S., many of those rating variables represent a protected class and therefore cannot be used for pricing in practice.

Table 4 1: Sample of a scoring database

The general challenge in scoring techniques is therefore to determine a way to estimate the probability of default, or the number of future defaults, and the loss given default for some specific individuals having various characteristics.

4.2 Basic Modeling and Actuarial Techniques

Table 1 shows a database for credit risk, but could also represent a database for conventional insurance products, such as automobile or property insurance. Indeed, the risk characteristics used to price an insurance product, such as gender, the age, or the marital status, are used in ratemaking to predict the realization of a random variable. In the pricing of auto insurance or home insurance, an actuary attempts to predict the number of claims and the cost of each of these claims. In credit risk analysis, the analyst will instead attempt to predict the number of defaults and amount of loss given default. In essence, the situations are almost identical. Thus, P&C actuaries already have the basic knowledge to perform modeling for this type of product.

4.2.1 Minimum Bias

Ratemaking, pricing and using segmentation variables is an old problem in actuarial science, for which many solutions and techniques have been proposed in the literature. Historically, the minimum bias technique that was introduced by Bailey and Simon (1960) and Bailey (1963) were used to find the parameters of some classification rating systems. These techniques are intended to find parameters that minimize the bias of the premium using iterative algorithms.

With the development of regression models and statistical theories, theoretical models have been used instead of the minimum bias. As we will see in the following section, advanced predictive models based on generalized linear models (often simply called GLMs) are used to estimate the parameters of a rating system. Interestingly, note that it has been shown that the results obtained from GLM theory are very similar to those obtained by the minimum bias technique (see, for example, Brown (1988)).

4.2.2 Statistical Approaches

The statistical approaches to ratemaking, as well as those that can be used for scoring risk ratings, are based on the following pure premium expression:

$$PP_i = \sum_{j=1}^{N_i} S_{i,j}.$$

By convention, we suppose that PP=0 when N=0, meaning that there is no related cost when there are no claims. This models the total amount of claims paid by the insurer; specifically, for an insured i, for a given level of insurance coverage (such as insurance protection against defaults) is the sum paid S on all N_i claims. It can be shown that the expected value of PP can be expressed as the product of the expected value of N times the expected value of S. In other words, the pure premium is equal to the frequency times the severity. Having defined the total potential cost to be assumed by the insurer, it is now necessary to specify how actuaries must model and segment the random variables N, S and PP.

4.2.3 Number of defaults, N

The starting point for modeling the number of events, claims or defaults, a random variable denoted Y, is the Poisson distribution. The Poisson distribution is the basis for almost all analysis of count data. The Poisson distribution has some interesting properties, such as an equidispersion property, which means that the expected value of the distribution is equal to its variance. By examining a database, it is then quite simple to verify whether the Poisson distribution is appropriate by comparing the empirical mean and the empirical variance.

The characteristics of the insured that should influence the premium, such as age, sex or marital status, can be included as regressors affecting the parameter representing the count distribution. As in classic regression models, the exogenous information can be coded with binary variables. For example, we can model sex with a variable *x* that takes value 1 if the insured is a man and 0 otherwise. In statistical modeling, we use the link function $h(x_i'\beta)$, where $\beta' = (\beta_0, \beta_1, \dots, \beta_k)$ is a vector of regression parameters for the binary explanatory variables $x_i' = (x_{i,0}, x_{i,1}, \dots, x_{i,k})$. Usually in insurance, the link function *h()* is an exponential function. Consequently, we have a mean parameter $\lambda_i = t_i\, exp\,(x_i'\beta) = exp\,(x_i'\beta + \ln(t_i))$, where t_i represents the risk exposure of the insured *i*. Because the mean parameter can be seen as the premium to be charged to an insured, there is a substantial advantage from using an exponential function. Indeed, it allows the insurer to construct a premium based on multiplicative relativities:

$$\begin{aligned}
\lambda_i &= t_i\, exp(\beta_0 + \beta_1\, x_{i,1} + \cdots + \beta_1 x_{i,k}) \\
&= t_i\, exp(\beta_0)\, exp(\beta_1 x_{i,1}) \dots exp(\beta_1 x_{i,k}) \\
&= t_i p_0 \times r_{i,1} \times \dots \times r_{i,k},
\end{aligned}$$

where p_0 can be viewed as the base premium and $r_{i,j}, j = 1, \dots, k$ as the relativities applied to insureds having the property *j* (i.e., $x_{i,j} = 1$).

Note that because the Poisson distribution is a member of the exponential family, it has some useful statistical properties (see the list of resources at the end of this chapter for further details). One such property is the fact that GLM theory can be used to estimate the parameters. Thus, instead of manually maximizing the sum of the log-probability function, one can use packages in R or SAS procedures to estimate the parameters. The properties of the MLE are well known and allow us to compute the variance of the estimators.

Because the Poisson distribution has some severe drawbacks, such as its equidispersion property, that limit its use, some generalizations are often needed. Indeed, we can still use the estimated parameters obtained by the Poisson distribution assumption and simply *correct* the overdispersion of the Poisson distribution by adding a multiplicative factor ϕ to the variance of the estimators. We can estimate the ϕ parameter using several techniques that are often included in R packages and SAS procedures, for example. The statistical significance of the estimators can be tested using classic Wald or likelihood ratio tests. Deviance can also be used to verify the fit of the Poisson because this distribution is a member of the linear exponential family.

Other properties of the Poisson distribution

The scientific literature proposes many other models that can correct problems related to the use of the Poisson distribution (see the list of resources at the end of this chapter for further details). An interesting property of the Poisson distribution refers to the time between two defaults. If we define the time between the $i-1^{th}$ and the i^{th} default as a random variable, we can define the arrival time of the j^{th} default by summing all the j^{th} times between defaults. By examining the relationship between the count process of the time between defaults, we can show the if we suppose that the waiting time between two defaults is exponentially distributed with mean $1/\lambda$, the number of defaults before time t can be expressed as a Poisson distribution of mean λt. Note that because the hazard function does not depend on *t* (the memory less property of the exponential distribution), the Poisson does not imply duration dependence, meaning that a default does not modify the expected waiting time until the next default. Another important property of the Poisson distribution that follows the waiting time interpretation is that the mean parameter of the model is proportional to the observed time length. Normally, t is regarded as the number of years of coverage. For example, an insured covered for 6 months will have a premium half as high as if he were insured for 1 year because his mean parameter of the Poisson distribution will be 0.5λ, compared to λ. Some papers consider waiting times that are not exponentially distributed.

An interesting generalization of the Poisson distribution can be constructed by adding a heterogeneity term to the mean parameter of the Poisson distribution. Intuitively, we suppose that the overdispersion of the insurance data is caused by the omission of some important classification variables. Consequently, by supposing that the insurance portfolio is heterogeneous, we can generalize the Poisson distribution by adding a random heterogeneity term:

$$Pr(Y = y) = \int_0^\infty Pr[Y = y|\theta] g(\theta) d\theta,$$

where $Pr[Y = y|\theta]$ is the conditional distribution of Y, and $g(\theta)$ is the density of Θ. The introduction of a heterogeneity term means that the mean parameter is also a random variable. When the random variable Θ follows a gamma distribution of mean 1 (to ensure that the mean heterogeneity is equal to 1), the resulting mixed model is then a negative binomial distribution. This count distribution is also well known in actuarial science. It can be proved that the negative binomial distribution accounts for overdispersion. As for the Poisson distribution, we can estimate the parameters of the distribution by maximum likelihood by maximizing the log-probability function, or by using R and SAS, as they include preprogrammed packages and procedures for this distribution. Obviously, other heterogeneity distributions can be used with the Poisson distribution, such as the Inverse Gaussian distribution, the lognormal distribution, or mixtures of continuous distributions. However, in those cases, parameter inference is more difficult.

Other count distributions

The actuarial literature, as well as the statistical literature, and studies on the number of defaults propose many other types of count distributions. We refer the interested reader to the book by Cameron and Trivedi (2013) or that by Denuit et al. (2007) for an overview of all possible count distributions. It could be interesting for a credit risk modeller to fit many count distributions to his datasets and compare the fit of the model, as well as its predictive ability.

4.2.4 Loss given defaults, S

To model the loss given defaults, i.e., the amount of claims, denoted S, we restrict ourselves to strictly positive distributions. As for the count distributions, many parametric distributions can be used to model the amount, but we will restrict ourselves to a few of them. The distributions were chosen because of their importance in the actuarial literature (see the list of resource for further details).

1. The gamma distribution is usually the first distribution used to model the cost of claims in P&C insurance and should also be the first distribution to use to model the loss given default. The gamma distribution is a member of the linear exponential family, and as done with the Poisson distribution, GLM theory can be used. Consequently, regressors can be added quite easily in the mean parameter of the distribution.
2. The inverse-Gaussian distribution is an interesting alternative to the gamma distribution. As it is also a member of the linear exponential family, covariates and regressors can be added easily in the mean parameter of the distribution, and GLM theory can be used. The inverse-Gaussian distribution has a heavier tail that the gamma distribution.
3. The beta-prime distribution, also called the inverse-beta distribution or second-type beta distribution, is also very popular in actuarial science. The distribution is quite flexible, but it is not a member of the linear exponential family. Consequently, covariates and regressors

can still be used in the model, but maximum likelihood estimators are difficult to identify, as the log-likelihood function should be maximized numerically. Like the inverse-Gaussian distribution, the beta-prime distribution can be used for heavy-tail data.

4. The Pareto distribution is another interesting distribution to consider. This distribution is widely used in the literature (in actuarial sciences and statistics) to model extreme values.
5. The Champernowne distribution is less known by actuaries but possesses many advantages. For example, the tail of the Champernowne distribution converges toward a Pareto distribution.

Parametric modeling allows the actuary to summarize data into a distribution with a small number of parameters. However, the analysis of the loss given default can be performed by first exploring the data. Histograms and kernel estimators, for example, should be used to understand the data. Finally, goodness-of-fit tests should be used to verify the fit, and information criteria can be used to compare distributions. Classic statistical books can provide further details.

4.3 Typical Credit Risk Products Sold by Insurers

As indicated in the introduction of this chapter, the statistical approach developed by actuaries for traditional P&C products is perfectly suitable for credit products sold by P&C insurers. To complete the overview, we present some typical insurance products and the approaches employed by actuaries and risk modelers.

Mortgage Credit Risk

As mentioned in Mrotek and Schmitz (2010), before the financial crisis of 2008, many financial analysts based their risk evaluations of mortgage credit risk on ratings agencies and securities brokers. Since the crisis, more techniques and approaches have been proposed for actuaries to analyze such risks.

There are numerous scientific papers discussing the financial crisis of 2008. These papers often attempt to explain how such a crisis occurred and frequently discuss the techniques used to quantify and manage mortgage credit risk. Regarding actuarial science, let us mention the overview of the subprime mortgage crisis by Donnely and Embrechts (2010). The authors explain how actuaries quantified risk at that time. The paper discusses some of the actuarial models that were used in the pricing of credit derivatives. The authors explain how important it is to correctly model the dependence between risks. Indeed, since mortgages all depend on the same economic and social conditions, it is necessary to rule out the independence assumption between risks. However, as the authors note, adding dependency between risks is not a simple task, and it is important to understand how to model dependent random variables.

A data source, and significant expertise in the modeling of mortgage credit risk in the United States, is offered by the Federal Housing Finance Agency (FHFA) (Dunsky et al. (2014)). This agency provides an overview of the mortgage market in the United States, and the FHFA notes the following:

"The motivation to build the FHFA Mortgage Analytics Platform derived from the Agency's need for an independent empirical view on multiple policy initiatives. Academic empirical studies may suffer from a lack of high quality data, while empirical work from inside the industry typically represents a specific view. The FHFA maintains several vendor platforms from which an independent view is possible, yet these platforms tend to be inflexible and opaque. The unique role of the FHFA as regulator and conservator necessitated platform flexibility and transparency to carry out its responsibilities."

Surety Insurance

Jiang and Dunn (2013) offer a survey of techniques that were historically used to model credit card risks, as well as mortgage default risk. As previously mentioned, statistical approaches based on GLM or panel data modeling are used. The paper lists several databases that can be used to adjust models. Dunn and Kim (1999) also use regression techniques to explore the important covariates that can be used to explain credit card default.

Surety insurance modeling is less popular, and few papers can be found in the literature. However, a paper published by actuaries, Alwis and Steinbach (2003), explains the product:

Surety is unique in the insurance industry in that it is the only three-party insurance instrument. It is a performance obligation, meaning it is a joint undertaking between the principal and the surety to fulfill the performance of a contractual obligation.

Alwis and Steinbach (2003) also describe the historical context and propose intuitive techniques to model risk. Advanced techniques using statistical approaches should be developed by the actuarial community. However, these products do not seem to have attracted interest from the actuarial community or academics.

Credit Scoring and Credit Cards

A credit score corresponds to a numerical value to represent the credit risk associated with an individual or company. This score is used to better express the creditworthiness or the probability of default of an individual or a company. Usually, as mentioned at the beginning of this chapter, the credit score is based on individual characteristics and the past financial operations carried out by the person or company concerned. Such information is often held by credit bureaus. Banks and credit companies use the credit score to assess the risk of default posed during a financial loan and when issuing credit cards. There is vast literature on credit scoring. For example, two books, Siddiqi (2012) and Thomas (2009), explore this domain. More recently, Koh et al. (2015) propose data mining techniques to construct credit scoring models.

In actuarial science, credit scoring is directly used as a covariate in many insurance products, such as automobile or homeowner insurance. Actuaries should not have to model credit scores directly, but they should understand how such scoring works. Credit scoring is highly related to bonus-malus systems in automobile insurance. As mentioned in Denuit et al. (2007), bonus-malus systems are class systems in which the insured's level increases or decreases depending on the number of reported accidents. A specific entry level is determined for new insured individuals. Each year, the level of each insured is adjusted depending on that person's claim experience. The number of claims is usually considered in determining the level of the BMS, but the amount of the claim could also be considered.

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4.5 List of resources

See also Chapter 12.

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4.5.3 Database

- Canadian Statistics for Credit Cards Defaults
- Canadian Statistics for Mortgage and Credit Cards Defaults
- Statistics for Credit Cards Defaults
- Canadian consumer credit trends, Equifax Analytical Services
- Consumer Finance Monthly

4.5.4 Computer programs

- ActuR package
- Arthur Charpentier's website: http://freakonometrics.hypotheses.org/
- Credit scoring demo

Chapter 5

Single-name credit-sensitive assets

In the first part of this chapter, we discuss three credit-sensitive assets traded on a company, i.e., stocks, corporate bonds and credit default swaps, and in the second part, we address how to use these securities to evaluate a company's creditworthiness[3].

5.1 Securities

A company that needs to raise money generally has two choices: issue stocks or issue bonds. In the first case, it cedes part of the ownership in the company in exchange for money. In the second case, it acquires a loan that needs to be repaid with interest. For the lender, the loan bears credit risk tied to the uncertainty over whether the borrower will fully repay the loan.

5.1.1 Stocks

A stock is a title of ownership in a company. Its value evolves randomly over time and may or may not pay dividends. A dividend is an income stream paid for each share owned. Standard stock valuation methods rely on discounting cash flows such as dividends and earnings during some holding period. Fundamental and technical analyses are methods used by investors to assess the relative value of a stock, whether it is overpriced or underpriced. Fundamental analysis relies on financial ratios such as earnings per share and price to earnings, whereas technical analysis relies mostly on patterns in historical stock prices. For further details, see Bodie et al. (2013) and Brealey et al. (2013).

The standard stock valuation approaches rarely explicitly discuss the issue of credit risk and how stocks can reflect the financial health of a firm. In fact, changes in earnings due, for example, to an economic downturn will affect a firm's solvency and credit risk. However, the structural models described in Chapter 2 (which are inspired by Merton (1974)) often view stocks as a contingent claim or a derivative on the assets of the firm, incorporating the capital structure as a whole. The two approaches thus fundamentally differ.

5.1.2 Corporate bonds

A corporate bond is essentially a loan contracted between the borrower and thousands of investors on the financial markets and is part of a larger family known as fixed-income securities. The price of a bond is expressed on a common basis, which is known as the par or face value. This value is often set at 100 by convention. Corporate bonds entail credit risk in the sense that cash flows are not guaranteed, i.e., the borrower may or may not fully repay the bond's principal and interest. When one or more payments are missed, we say that the borrower is in default.

The price of a corporate bond is also often expressed in terms of yields or spreads. The yield, or yield-to-maturity, is the unique interest rate that reprices a corporate bond. In other words, given the future cash flow structure and assuming that the bonds are paid with certainty, the yield-to-maturity is the unique interest rate used to discount cash flows such that it matches the current

[3] This is in contrast with statistical tools that are based on default frequencies, ratings transitions or financial ratios.

observed price. The yield is unique to a specific corporate bond, as it depends not only on the general level of interest rates or the credit risk of the issuer but also on the cash flow structure. Conversely, credit spreads are firm-specific measures that do not necessarily depend on the bond's characteristics.

The relationship between the duration of a loan and its annualized interest rate is known as the term structure of interest rates. Such a term structure can be obtained on default-free securities (such as those issued by a highly rated country) or on (defaultable) corporate bonds. The credit-spread curve is the difference between the term structures of interest rates on a defaultable bond and the equivalent on a default-free bond. Thus, the credit spread is the compensation required by bondholders to invest in a risky bond for a given maturity. In theory, credit risk should be the main driver (along with interest rates) of corporate bond prices (and spreads).

However, this is not entirely the case empirically. Various authors have investigated the question of what proportion of corporate bond spreads is truly attributable to credit risk. The first authors examining this issue are Elton et al. (2001), who find, quite surprisingly, that the proportion of corporate spreads related to default is at most 25%. Using a different methodology and dataset, Dionne et al. (2010) find that this proportion can vary between 30% and 75%, depending on the credit rating and period. Other notable contributions in this area are Collin-Dufresne et al. (2001), Eom et al. (2004) and Huang & Huang (2012), who obtain similar results while considering different datasets, periods and methodologies. The consensus is that (1) credit risk alone cannot fully explain corporate bond spreads, (2) liquidity can also be an important factor, especially for long-term bonds that are held by insurance companies and rarely traded, and (3) the proportion of credit spreads explained by credit risk decreases as a firm's credit quality increases.

A difficulty that may arise with these studies is the presence of embedded options (also known as optionalities or covenants) in corporate bonds. These options are included in corporate bonds to give bondholders further protections, making the bonds more attractive to the market. However, these covenants also affect the price of the bond and the corporate spread. Most bonds have numerous embedded options such as callable (redeemable), putable, exchangeable, extendible, and so forth.

For a more thorough treatment of corporate bonds and other fixed-income securities, the reader should consult Fabozzi & Mann (2012).

5.1.3 Credit default swaps

A credit default swap (CDS) is a credit derivative that offers protection against the default (or any credit event) of the underlying company in exchange for a premium. A company may take on the role of the protection seller (which receives a premium but assumes default risk) or of the protection buyer (which pays a premium in exchange for protection). CDSs are generally available with maturities of 1 to 10 years, with 5 being the most traded maturity and 1 being the least traded.

Credit default swaps are quoted with a reference bond that is employed in calculating the amount of loss paid on default. When the CDS is settled in cash, a sum of money equivalent to the loss on the reference bond is paid on default. When physical settlement is chosen, on default, the insurance buyer cedes the defaulted bond to the seller, and the latter also pays the notional amount on the bond.

A CDS is essentially an insurance contract against default and is used chiefly by investors who face default risk on their assets. CDSs are therefore used mostly as a hedging instrument. However, following the financial crisis of 2008, CDSs have been in the spotlight due to lack of CDS market regulation. A first reason is that the protection buyer is not required to have an equivalent insurable interest in the company for which it seeks protection, thus allowing for major speculation. For example, an investor holding $100M in bonds in company ABC is allowed to buy CDS protection of more than $100M in notional value. Second, to avoid its own bankruptcy (also known as counterparty risk), the issuer of a CDS (protection seller) needs to set aside sufficient funds in case it has to pay for a default.

Although much less common since the 2008 financial crisis, CDSs can also be issued on structured financial products (such as asset-backed securities). In this case, default is interpreted as the inability of the investment to deliver the totality of the promised payments. American Insurance Group (AIG) suffered very significant losses in 2008 from CDSs issued on collateralized debt obligations on securitized subprime mortgages.

The valuation of CDSs is similar in spirit to the valuation of T-year term insurance with premiums payable 4 times a year until the end of the contract or on default, whichever comes first. The main difference, which is essential, is that pricing is subject to the absence of arbitrage arguments. A CDS combined with a corporate bond should yield the risk-free rate, as the combination is supposed to be risk free. Market makers will exploit arbitrage opportunities, and thus, expectations should be taken under the pricing (risk-neutral) probability measure. The quarterly premium is determined based on the equivalency principle, meaning that the expected benefits should be equivalent to the expected premiums. In theory, the spread or premium paid by the protection seller should be determined by the firm's default risk and interest rates, given that payments can be as far as 10 years in the future.

However, in practice, when we study the determinants of CDS premiums, it is found that default risk and interest rates are indeed major components of the spreads (see Ericsson et al. (2009)). However, other components—liquidity and counterparty risk (see Arora et al. (2012))—are also significant, especially since the financial crisis, although their overall impact on the spreads can be small.

5.2 Credit risk assessment using security prices

To assess the credit risk of a firm using a credit risk model, one needs to estimate the model parameters. An increasingly popular approach is to use security prices such as stocks, corporate bonds and/or CDSs[4]. The idea is that when the parameters are known, most credit risk models can be used to calculate the price of securities. Thus, the opposite can also hold, that is, to take observed security prices as given and use them to estimate a model.

An intuitive way to do so is to minimize the squared difference between the theoretical (model) and observed prices over time and/or a cross-section of securities. This method is better known as calibration, which is popular among practitioners and often performed daily on multiple securities at once, yielding a different set of parameters every day. The quality of the fit is assessed on the basis of the pricing errors. Despite being very intuitive, it is difficult, albeit not impossible, to assess the uncertainty associated with a given parameter.

The scientific community has turned instead to statistically based estimation approaches such as maximum likelihood or the generalized method of moments (GMM), which are based on the frequentist paradigm (as opposed to the Bayesian paradigm). In the maximum likelihood approach, one seeks to identify the parameters that maximize the probability of observing a given sample. The GMM, as its name indicates, generalizes the method of moments by using more moments than the number of parameters or by placing different weights on the moments (such as assigning greater weight to tail measures).

The estimation of credit risk models using maximum likelihood with security prices was pioneered by Duan (1994, 2000). The idea is that the main driver of a company's creditworthiness, that is, the market value of assets, the default intensity, etc., is not directly observed but indirectly observed through the value of a security (such as a stock, corporate bond or CDS). Since the latter is a transformation of the former, the likelihood function has to account for this link between the two, yielding a slightly modified likelihood function. One recovers all the usual properties of the maximum likelihood by following Duan (1994).

The disadvantage of this approach is that it assumes that all changes in the price of the security

[4] Other estimation approaches will be discussed throughout the compendium. Portfolio models are also further discussed in Chapter 3.

are attributable to changes in creditworthiness, which is not necessarily the case. For example, Duan & Fulop (2009) report that asset volatility in Merton's model can be overstated if model error is not considered. Therefore, we need to disentangle the changes in the security price that are due to changes in the fundamentals (the true credit risk of the firm) and those due to other factors. This is the role of filtering techniques such as non-linear Kalman filters or particle filters. For a review of these methods, see Boudreault et al. (2015).

More recently, academics and practitioners have focused on models that include a random recovery rate. The estimation of such models presents a challenge because of the identification issue. The price of a credit-sensitive security is affected by two variables: the likelihood of default and the loss given default. Depending on the model and the data used, it is possible to vary the two variables in opposite directions to obtain the same price. This identification issue can be solved by two methods: (1) modify how recovery payments occur in the model (see Pan & Singleton (2008)) and/or (2) use various types of securities (or datasets) that are exposed differently to recovery rate risk in the estimation process.

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- Pan, J., K.J. Singleton (2008). Default and recovery implicit in the term structure ofsovereign CDS spreads. Journal of Finance 63, 2345–2384.

5.4 List of resources

See also Chapter 12.

5.4.1 Books

Presented in alphabetical order.

- Ammann, M. (2002). Credit Risk Valuation: Methods, Models, and Applications, Springer
  - Chapter 6 discusses the pricing of single-name credit derivatives.
- Bluhm, C., L. Overbeck, C. Wagner (2010). Introduction to Credit Risk Modelling, Second Edition, CRC Press.
  - Single-name and multi-name credit derivatives (excluding CDOs) are introduced in Chapter 7.
- Crouhy, M., D. Galai, R. Mark (2000). Risk management, McGraw Hill.
  - Credit (single-name and portfolio) derivatives are discussed in the context of hedging credit risk in Chapter 12.
  - Technical/Mathematical level: Very accessible
- De Servigny, A., O. Renault (2004). Measuring and Managing Credit Risk, McGraw-Hill.
  - There is a brief discussion on credit default swaps and other single-name credit derivatives in Chapter 9.
  - Technical/Mathematical level: Accessible
- Duffie, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton Series in Finance.
  - The pricing of corporate bonds and credit default swaps is covered in Chapters 6 and 8, respectively.
  - Technical/Mathematical level: Accessible/technical
- Fabozzi, F.J., S. Mann (2012). The Handbook of Fixed Income Securities, 8th edition, McGraw-Hill.
  - Chapter 66 primarily examines at the characteristics of CDSs, whereas Chapter 67 discusses their valuation.
  - Technical/Mathematical level: Very accessible
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
  - Chapters 22 and 23 discuss single-name and portfolio credit derivatives. The method proposed to estimate Merton's model is outdated and provides biased estimates of the parameters[5]. Very good introduction to CDSs.
  - Technical/Mathematical level: Very accessible
- Lando, D. (2004). Credit Risk Modeling, Theory and Applications, Princeton University Press.
  - CDSs are discussed in Chapter 8.
  - Technical/Mathematical level: Technical
- O'Kane, D. (2008). Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance.
  - Book specifically focusing on the pricing of credit derivatives. The Wiley Finance series is usually accessible to practitioners.
  - Chapters 4 and 5 describe corporate bonds and CDSs in detail. Chapter 6 addresses pricing CDSs, while risk management of CDSs is discussed in Chapter 8.
  - Technical/Mathematical level: Accessible

5.4.2 Computer programs

R programming language

- creditr package: According to CRAN, it "Provides tools for pricing credit default swaps

[5] See Ericsson, J. and J. Reneby (2005), "Estimating Structural Bond Pricing Models," Journal of Business 78, 707-735.

using C code for the International Swaps and Derivatives Association (ISDA) CDS Standard Model"

- credrule package: According to CRAN, "It provides functions to bootstrap Credit Curves from market quotes (Credit Default Swap - CDS - spreads) and price Credit Default Swaps - CDS."

Matlab programming language

- CDS Pricer

Chapter 6

Municipal securities

6.1 Introduction

Municipal securities are bonds issued by government entities, such as a city, province/state or public utility. These products are better known as short-term municipal securities, or munis. The fact that munis are generally tax-free makes them well suited to investors taxed in the highest brackets. Government entities issue munis to finance expansion projects (airports, waterworks, parks, schools, hospitals, etc.) or simply to fund daily operations (infrastructure maintenance, landscaping, etc.). The main buyers of municipal securities are individuals, mutual funds, commercial banks and P&C insurance companies.

6.2 Type and characteristics

Municipal bonds come in many forms. They include zero-coupon bonds and bonds with fixed or variable rates. Munis also fall into two categories: general obligation bonds and revenue bonds. General obligation bonds are bonds secured by the taxation power of the issuer (often considered infinite). By comparison, revenue bonds repay bondholders directly from revenues generated by the infrastructure being funded (airport, schools, hospitals, power stations, maritime ports, etc.).

6.3 Ratings by recognized credit rating agencies

Until the 1980s and 1990s, investors considered ratings issued by rating agencies (Moody's, Standard & Poor's, etc.) to be infallible. In 1994, the case of Orange County created upheaval when the county was forced to seek protection under the bankruptcy act; its munis had previously been very highly rated. This case also led investors to ask more questions about fund management by municipalities and quashed the popular belief that muni fund managers invest prudently.

More recently, on July 18, 2013, the city of Detroit declared bankruptcy, and investors subsequently lost a total of $7 billion. In mid-2015, the island of Puerto Rico defaulted on a payment of nearly $58 million.

In the wake of all these improbable yet actual events, financial institutions have become more suspicious of the credit ratings of munis. They now use them as a starting point and continue their analysis using internal models. Note that rating agencies use the same rating system for munis as for corporate bonds. In addition, agencies use more than one rating style to represent several types of municipal bonds (see Moody's, Standard & Poor's).

6.4 Tax issues

Although munis are generally considered nontaxable, it is important to understand that this status applies only at the federal level and not necessarily at the state, provincial and municipal levels.

Evaluating munis is complex; options and many other factors need to be considered. In addition, the rate of return (nontaxable) of munis must be compared with that of corporate bonds (taxable) to determine how profitable they are. Because the muni rate is below the rate of return of equity, it is important to ensure that the investment is worthwhile (given the investor's taxation rate). Therefore,

$$\text{Taxable bond rate} \approx \frac{Muni\ rate}{1-T}$$

where T is the investor's marginal taxation rate. Note that this is a very rudimentary way to evaluate the tax impact on the total bond yield.

6.5 Insurance on municipal bonds

One method to reduce a muni's borrowing rate is by using insurance. This insurance guarantees that the investor will be paid the coupon and principal in full if the issuer defaults on the payment. There are two groups of municipal bond insurers: monoline and multiline. Monoline insurers provide only financial guarantees, whereas multiline insurers are P&C insurance companies. Although this insurance may appeal to issuers of lower-quality bonds, the financial crisis of 2008 sent insurance companies into turmoil. As a result, the addition of this insurance does not necessarily improve the quality of the bond due to the counterparty risk of the insurance company.

6.6 Factors determining credit risk

Before investing in munis, investors must analyze the clauses of the official document to determine their risk exposure. According to the SEC's Office of Investor Education and Advocacy, there are at least four factors to consider before investing in munis (SEC (2012)):

1. Type of municipal bond: as mentioned above, there are two main types of munis and, therefore, two different credit risks. For general obligation bonds, even if the taxation power is considered unlimited, it is important to verify the real taxation power of the entity issuing the bond. For revenue bonds, revenue may be more at risk if the future project is risky, and thus, this type of muni has a higher probability of default than general obligation bonds do.
2. Non-recourse financing: Some revenue bonds have a non-recourse financing clause, which means that the investor obtains nothing if the product does not generate revenue. This greatly increases the credit risk.
3. Financial condition of the issuer: The credit rating is a good starting point to anticipate the outlook for a municipal bond issuer. Evidently, the worse the rating is, the higher the default risk. As mentioned above, secondary analysis of the issuer is crucial to anticipate possible future issues.
4. Other sources to pay the principal and interest on the bond: Sometimes, the funds to repay the municipal bond come from an uncertain source, which makes the issuer's capacity to derive constant revenue from that source uncertain. One example is funds coming from taxes on the sale of a product in declining demand due to a social or economic cause, such as tobacco.

To begin investigating a municipal bond, one can consult the official release on the Electronic Municipal Market Access (EMMA) website at www.emma.msrb.org.

6.7 Credit risk model

As we have seen, and as Fama (1977) and Miller (1977) confirmed, municipal bonds are not risk-free, which is why their yields are higher than typical government bonds adjusted for tax considerations. An adjustment is made for credit risk. In practice, explaining the part of the spread attributed to credit risk is called the muni puzzle: this calculation is not easy because the taxation rate is not fixed for each individual. Few models of credit risk calculation exist. A recent approach proposes the use of a neural network to classify munis (Hjek 2011). Starting with the rating given to munis by an agency and through the use of several qualitative and quantitative variables, the algorithm attempts to find links between variables and performs a classification. With this type of model, we can see whether the chosen variables are representative of the evolution of credit risk and then predict the future rating of a muni. Chalmers (1998) demonstrates that default risk does not suffice to solve the muni puzzle. This means that another source of risk must be incorporated into the calculation.

6.8 Liquidity risk

One of the risks that should not be overlooked in evaluating municipal bonds is liquidity risk. According to Wan et al. (2005), the "Liquidity premium explains about 7 to 13 percent of the observed municipal yields for AAA bonds, 7 to 16 percent for AA/A bonds and 8 to 20 percent for BBB bonds with different maturities." Based on muni price data, Wang et al. (2005) use a model to find the percentage of return attributable to liquidity, default, taxes and the risk-free rate of a muni. Ang et al. (2014) offer a more recent study on this topic.

6.9 List of resources

6.9.1 Books

Presented in alphabetical order.

- Fabozzi, F.J. (2012). The Handbook of Fixed Income Securities, McGraw-Hill
  - Chapters 11 and 44 cover several aspects of munis.
  - This is one the most important references used to write this chapter.
  - Technical/Mathematical level: Very Accessible
- Fabozzi, F.J., S.G. Feldstein (2008). The Handbook Of Municipal Bonds, Wiley
  - Qualitative book addressing several aspects of municipal bonds.
  - Technical/Mathematical level: Very Accessible
- Mysak, J., M.R. Bloomberg (2010). Handbook for Muni-Bond Issuers, Wiley
  - Although this book is not intended for investors, we found it useful to include a reference on muni issuers because this subject is not very well known.
  - Technical/Mathematical level: Very Accessible
- SIFMA (2011). The Fundamentals of Municipal Bonds, 6th Edition, Wiley Finance
  - SIFMA is a group of private companies (banks, hedge funds, etc.) interested in munis.
  - Chapters 6 and 7 cover risk and credit analysis of munis, respectively.
  - Technical/Mathematical level: Very Accessible
- Handbook of Finance (2008). Financial Markets and Instruments, Wiley.
  - This three-volume work addresses a number of diverse topics in finance.
  - Chapter 22 covers municipal bonds.
  - Technical/Mathematical level: Very Accessible

6.9.2 Data

- The Municipal Securities Rule Making Board (MSR) is a company that protects municipal bond investors. Although the MSRB has compiled many databases on munis, they are not free to use.
- J.J. Kenny Drake, Inc.: "J.J. Kenny Drake, Inc. operates as a municipal bonds inter-dealer. It offers market data that provides regional and national coverage with approximately 500 screen pages of market activity; offerings and bid-wanted; coverage by region, state, and

bond type; and descriptions on various advertised issues through Web and digital feeds." (Bloomberg Finance website).

6.9.3 Bibliography

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- SEC (2012). Municipal Bonds: Understanding Credit Risk, Office of Investor Education and Advocacy, Access. Investor Bulletin.
- Standard & Poor's (2012). Standard & Poor's Ratings Definitions.
- Wang, J., C. Wu, F. Zhang (2005). Liquidity, Default, Taxes and Yields on Municipal Bonds, Finance and Economics Discussion Series Divisions of Research & Statistics and Monetary Affairs Federal Reserve Board.

Chapter 7

Portfolio credit risk derivatives and other structured assets

Multi-name credit-sensitive products are, in general, investments with values that depend on the credit risk of an entire portfolio of corporations or individuals. In this chapter, we will summarize some of the most popular multi-name credit-sensitive products, namely the basket CDS, collateralized debt obligations and other asset-backed securities.

7.1 Basket Credit Default Swaps

A basket credit default swap (CDS) is similar to a standard single-name CDS, but the credit event is triggered with the *k*-th default in a portfolio of reference assets. For example, the first-to-default CDS on a 100-name portfolio implies that a payment is expected whenever one reference in the portfolio defaults.

The behavior of the basket CDS depends on the dependence relationship in the portfolio. A first-to-default CDS will be more costly when assets in the portfolio are independent, whereas a last-to-default CDS will be the cheapest under independence.

Pricing, in the financial engineering sense (absence of arbitrage), requires a portfolio credit risk model, as described in Chapter 3. Because the distribution of the 1st, 2nd or *k*-th to default is rarely tractable in large portfolios, prices are determined by simulation.

7.2 Asset-Backed Securities

Asset-backed securities (ABS) are financial assets with values that are derived from the cash flows of a portfolio of obligations. Most of these instruments are created using what is known as securitization, which is the process of pooling non-tradable obligations, such as mortgages or credit cards, into a tradable security. An intermediary handles the legal aspect of the creation of the ABS by collecting the premium from the investors on one side and the cash flows from the obligations on the other. The obligations most commonly used in ABS are mortgages, auto loans, credit cards and student loans. We will discuss mortgage-backed securities (MBSs) and collateralized debt Obligations (CDOs) in further detail. MBSs are covered very extensively in Fabozzi & Mann (2012).

7.2.1 Mortgage-Backed Securities

We define as MBSs the subset of ABS based on a pool of residential or commercial mortgages. MBSs issued by either Fannie Mae (FNMA), Freddie Mac (FHLMC) or Ginnie Mae (GNMA) are known as agency MBSs, whereas non-agency MBSs are mostly issued by private investment banks. As of late 2014 and according to the Securities Industry and Financial Markets Association (SIFMA), US agencies have approximately $6 trillion in outstanding MBSs, whereas non-agencies have approximately $1.5 trillion in outstanding MBSs.

The most significant types of MBSs are pass-through securities, collateralized mortgage obligations (CMOs) and stripped MBSs. A pass-through security backed by mortgages simply repays a share of the cash flows of a pool of mortgages (residential or commercial). CMOs represent all structured securities (backed by mortgages) with cash flow designs that are meant to distribute prepayment risk differently among investors (see below). They are legally established by a special purpose entity. Finally, stripped MBSs are securities that redistribute part of the interest or principal of a pool of mortgages.

MBSs are affected by changes in interest rates, the borrower's prepayment behavior and credit risk. Prepayment means that a borrower repays its mortgage faster than initially planned. This occurs primarily when the mortgage is refinanced at a lower rate or whenever the home is sold. Prepayment is a risk to the investor since it changes the timing of cash flows and, in the case of refinancing, lowers the rate at which the cash flows are reinvested.

CMOs offer investors various ways to cope with prepayment risk. Principal can be paid sequentially to investors (sequential pay) or based on a planned amortization schedule with bounded repayment rates. For example, in the first design, an investor entitled to the second 25% of the principal will start receiving payments when the first 25% has been paid. Therefore, when prepayment is faster (slower) than expected, the investor will also receive its share of the cash flows faster (slower). The second design is known as planned amortization class (PAC). Whenever prepayments are within some given range, the investor assumes prepayment risk, and thus, the cash flows will be very stable. Otherwise, an excess or shortage of prepayments will be compensated by companion tranches. CMOs are popular assets for insurance companies to match the duration of their liabilities. More CMO structures are discussed in Chapters 26–29 of Fabozzi & Mann (2012).

The borrower's credit risk is tied to the borrower's ability to fully repay its mortgage. In many instances, it is considered negligible because lenders investigate the credit history of their borrowers, agencies provide guarantees against homeowner default, and the homes serve as collateral. However, the subprime mortgage crisis of 2008 proved that many financial institutions relied too heavily on the home equity to provide mortgages to insolvent individuals. When the subprime bubble burst, many borrowers defaulted, the value of the houses plunged, and Fannie Mae and Freddie Mac were placed in conservatorship by the US Treasury.

7.2.2 Collateralized Debt Obligations

A CDO is a type of ABS that was highly popular in the 2000s. It was used to securitize and repackage loans and tranches of other MBSs. CDOs have been blamed for fuelling the subprime mortgage crisis in 2008-2009. At its 2007 peak, the CDO market size was as large as $1.4 trillion but collapsed following the recession. As of the end of 2014, the market size of the CDO market was approximately $800B (total CDOs outstanding, SIFMA).

A CDO is established through a special purpose vehicle, and its cash flows are distributed in tranches using a waterfall structure, i.e., a set of priority rules. To illustrate how tranches are structured in a CDO, let us assume a simple two-tranche CDO, i.e., senior and junior tranches. According to the waterfall structure, the senior-tranche investors are first in line to receive the cash flows from the collateralized debts. Once the senior-tranche investors have been paid in full, the junior-tranche investors receive the surplus until they are also paid in full. The latter structure provides senior-tranche investors with better credit risk protection than junior-tranche investors, and therefore, the senior tranche trades at a lower premium.

The value of CDOs depends on the creditworthiness of their internal obligations, but it is highly dependent on the degree of diversification within the portfolio. From a statistical standpoint, the strength of the dependence between the constituents is a major driver of CDO value, as illustrated in Table 7.1.

In Table 7.1, a “1” implies a default, whereas “0” is survival. It is important to note that the sum over columns leads to the same default probabilities in both scenarios of high or low dependence. Thus, there are 9 defaults in each part of the table.

High dependence generates clusters of defaults, i.e., scenarios in which there are either 0 or 4 defaults, whereas under low dependence, in all scenarios, there are at most 1 or 2 defaults. Therefore, under low dependence, it is highly unlikely that one would observe multiple defaults.

The impact on a CDO is direct. Under low dependence, senior tranche holders are highly protected whereas junior tranche holders will most likely assume all defaults. While not numerous, these defaults are regular. Under high dependence, all tranche holders are paid in full under certain scenarios, but under others, there are multiple defaults that also affect senior tranche holders. Therefore, dependence affects the variability of outcomes for all tranche holders in non-trivial manner. Tranches were often designed to obtain a very safe rating such as AAA, whereas as shown in the

High dependence						
Scenario/Company	1	2	3	4	5	Total
ω_1	0	0	0	0	1	1
ω_2	1	1	1	0	1	4
ω_3	0	0	0	0	0	0
ω_4	0	1	1	1	1	4
ω_5	0	0	0	0	0	0
Default prob.	0.2	0.4	0.4	0.2	0.6	9
Low dependence						
Scenario/Company	1	2	3	4	5	Total
ω_1	1	0	0	1	0	2
ω_2	0	1	0	0	1	2
ω_3	0	0	1	0	0	1
ω_4	0	1	0	0	1	2
ω_5	0	0	1	0	1	2
Default prob.	0.2	0.4	0.4	0.2	0.6	9

Table 7.1: Scenarios of defaults in a portfolio under low or high dependence

previous illustration, in a high-dependence economy, even the safest tranches are affected.

In the CDO-related enthusiasm that preceded the financial crisis, CDOs squared (CDO^2) and CDOs cubed were also issued. A CDO squared is a CDO issued on a CDO, i.e., tranches of a CDO are further broken down into tranches to form the cash flows of the CDO^2. Similarly, a CDO cubed is a CDO issued on a CDO squared. In both cases, valuation was highly complex due to the tranching mechanism applied over other tranches.

Pricing, again in the financial engineering sense, requires a portfolio credit risk model, as described in Chapter 3. The Gaussian copula was very popular in pricing CDOs, in large part due to David X. Li’s paper in 2000. Li provided the first tractable approach to pricing CDOs and therefore linked hundreds or thousands of loans using correlations. In the aftermath of the financial crisis, the Gaussian copula and its misuse in CDO pricing was highly criticized.

7.3 References

- Fabozzi, F.J., S.V. Mann (2012). The Handbook of Fixed Income Securities. McGraw Hill

Professional.
- Li, D.X. (2000). On Default Correlation: A Copula Function Approach. Journal of Fixed Income 9, 4354.

7.4 List of resources

See also Chapter 12.

7.4.1 Books

Presented in alphabetical order.

- Bielecki, T.R., M. Rutkowski (2002). Credit Risk: Modelling, Valuation and Hedging, Springer Finance.
  - Chapter 9 examines the mathematical mechanics behind basket credit derivatives.
  - Technical/Mathematical level: Very technical.
- Bluhm, C., L. Overbeck, C. Wagner (2010). Introduction to Credit Risk Modeling, Second Edition, CRC Press.
  - CDOs are thoroughly covered in Chapter 8.
- Crouhy, M., D. Galai, R. Mark (2000). Risk management, McGraw Hill.
  - Credit (single-name and portfolio) derivatives are discussed in the context of hedging credit risk in Chapter 12.
  - Technical/Mathematical level: Very accessible
- De Servigny, A., O. Renault (2004). Measuring and Managing Credit Risk, McGraw-Hill.
  - CDOs are introduced in Chapter 9.
  - Technical/Mathematical level: Accessible
- Duffie, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton Series in Finance.
  - CDOs are covered in Chapter 11.
  - Technical/Mathematical level: Accessible/technical
- Fabozzi, F.J., S. Mann (2012). The Handbook of Fixed Income Securities, 8th edition, McGraw-Hill.
  - The coverage of the various MBSs is very thorough, with 10 chapters dedicated to several specific topics (Chapters 24–32, 41).
  - Moreover, ABSs on credit cards, auto and student loans are discussed in Chapters 33 and 34.
  - Technical/Mathematical level: Very accessible
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
  - Chapter 23 briefly discusses basket credit derivatives such as basket CDSs, CDOs and their valuation.
  - Technical/Mathematical level: Very accessible
- O'Kane, D. (2008). Modelling Single-name and Multi-name Credit Derivatives, Wiley Finance.
  - Chapters 12 and 22 address portfolio credit derivatives, while risk management of these products is discussed in Chapter 17.
  - Technical/Mathematical level: Accessible

7.4.2 Computer programs

Matlab programming language
- Fast Computation of the Expected Tranche Loss of CDO Credit Portfolio
- Li's Copula model for CDS and CDO default intensities and loss function

Chapter 8

Counterparty risk

8.1 Introduction

According to the Bank for International Settlements, counterparty risk is (BIS (2005))
[...] the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows. An economic loss would occur if the transactions or portfolio of transactions with the counterparty has a positive economic value at the time of default [...]

Counterparty risk is almost inevitable for an entity that wants to take a position in the financial market. As the definition states, counterparty risk is the risk that the entity holding the opposite position from that of the investor fails to meet its obligations. It can result from the default of the issuer of an option, the inability of the protection seller of a CDS to deliver the promised compensation, or the failure of the other side of the swap to make payments, and so forth.

For an insurance company, counterparty risk is not limited to the derivatives that it acquires to manage its risks. Reinsurers may also file for bankruptcy, which represents a major counterparty risk for insurers. For further details, see section 8.4.

To avoid default risk and reinforce the credibility of financial markets, governments and financial institutions have introduced counterparty policies. As one example, clearinghouses have been introduced; acting as intermediaries, they ensure that a financial security's cash flow streams are available when one of the two parties involved in a transaction demands settlement. Although clearinghouses are intermediaries that reduce the risk of payment default, the possibility that the clearinghouses themselves default is quite present; this risk must therefore also be considered.

Because all financial transactions involve a random future value, the credit risk of the counterparty must be assessed by means of calculation techniques stipulated by Basel, Solvency, etc. to determine the value of exposure to this risk.

8.2 Credit Risk

In a simple loan between a bank and an individual, counterparty risk is assumed by the bank. The bank loses money if the borrower does not repay the loan. It is important to understand that credit risk is quite different for a derivative. Hull (2012) (Chapter 23) notes three possible situations:

- "The contract is always a liability to the financial institution
- The contract is always an asset to the financial institution
- The contract can become either an asset or a liability to the financial institution."

In the first case, the financial institution bears no counterparty risk because the bank owes money to another entity (such as when a bank sells options). In the second case, the financial institution assumes counterparty risk because the product it trades has positive value (such as when a bank buys options). The last case refers to a situation in which the financial institution would have positive or negative counterparty risk depending on the product's characteristics (such as forwards).

It is therefore important for a financial institution to incorporate a premium in all the financial products that it sells, to cover this risk. By denoting the value of the financial product at time t_i by f_i, the (risk-neutral) probability of default at time t_i by q_i and the recovery rate by R, we can define the following equation:

$$\sum_{i=1}^{T} q_i(1-R)PV(E[max(f_i,0)])$$

where PV(.) denotes the present value.

We therefore obtain the value of the CVA (credit value adjustment), an adjustment made to all financial products to account for counterparty risk. This calculation is very rudimentary but represents the broad outlines very well. Several factors come into play in CVA valuation, including the addition of some clauses to the contracts that financial institutions issue. Three typical clauses help mitigate counterparty risk (see, for example, Hull (2012) (Chapter 23), Caouette et al. (1998)):

1. **Netting:** Netting is a clause that stipulates that if entity A has several contracts with entity B and if one of the two entities defaults on one of its contracts, the entity in question will have to close all its positions with the other party. Let us assume, for example, that A has two contracts with B. The first contract has a value of -$10 and the second a value of $15 for A. If A defaults on the first contract, it will receive $15 from the second contract if there is no netting clause. In the opposite case, A will receive only $5 because the value of the two products will be combined.
2. **Collateralization:** This clause refers to what we mentioned earlier in Section 8.1. The value of a security changes over time, and to ensure that funds are indeed available at maturity when the security becomes payable, the entity that owes money must set a particular amount aside. This amount is generally administered by clearinghouses. This clause is very similar to the margins that clearinghouses require on futures.
3. **Downgrade Triggers:** This clause stipulates that if the credit rating of one of the parties is downgraded, then the contract can be closed immediately at its market value to avoid a future default.

Counterparty risk can be modeled using classic credit risk models such as those presented in a previous chapter. Because the counterparty may default at any time, a first-passage structural model is a good starting point to model counterparty risk (see Chapter 2). Several other models can also be used to model this risk (see Brigo et al. (2013) for a more exhaustive list). According to Brigo et al. (2013), counterparty risk in a derivative product can be divided into three subcategories:

- Credit exposure (CE): the value of the product today, if the counterparty defaults (market value)
- Expected exposure (EE): the value of expected future losses
- Potential future exposure (PFE): the possible future risk with a given confidence interval; this risk can be assessed using VaR (value-at-risk)

8.3 Credit Default Swap (CDS)

Counterparty risk can be mitigated with CDSs (see also Chapter 5), which are similar to a conventional swap, where a periodic premium is paid by the protection buyer in exchange for a payment by the protection seller in the event that the reference entity defaults. The use of these products has expanded considerably since 2008 following the bankruptcy of Lehman Brothers.

Note that even if these products largely cover credit risk, CDSs are also exposed to the counterparty risk between the buyer and issuer of this product. The counterparty risk linked to CDSs has been increasingly studied since AIG sustained major losses with its CDSs, valued at over $ 182.5 billion (Longstaff et al. (2010)). It is therefore important to consider this risk when pricing CDSs. Classic finance models can serve as the basis for CDS valuation.

8.4 Reinsurer credit risk

Counterparty risk may exist between an insurer and its reinsurer. The role of the reinsurer is to cover the losses of an insurance company that exceed a pre-defined limit. We present two different viewpoints on the link between the insurer and reinsurer (Britt & Krvavych (2009) and Cummins & Weiss (2010)).

Britt & Krvavych (2009) argue that several factors increase counterparty risk between insurers and reinsurers:

- Given that the number of reinsurers on the market is small, counterparty risk has become very concentrated.
- The correlation between the insurer and reinsurer is very high because they both provide insurance (they trade the same type of financial products). In addition, the strong link between the insurer, reinsurer and catastrophic events raises the counterparty risk. When a disaster occurs, the insureds of several insurance companies will file insurance claims, and their insurers will in turn file a claim with the reinsurer. This increases the reinsurer's probability of default.

Therefore, it is difficult to evaluate the mitigation of counterparty risk between an insurer and its reinsurer. A Monte Carlo simulation should be used to assess the impact of this risk.

It is important to mention that counterparty risk is a precursor to credit risk contagion. In our insurer/reinsurer relationship, default contagion exists if a reinsurer is unable to cover the losses of several insurers following a natural disaster, for example. This point is raised by Cummins & Weiss (2010) who analyze insurance data to analyze their correlations. They determined that a reinsurer's insolvency crisis would trigger an intra-sector crash. However, their study found only a weak correlation between insurers and the financial market, which implies that a crisis affecting insurers would not necessarily lead to a financial crisis. The authors conclude that insurers can nonetheless create systemic risk on financial markets when they use options (such as CDSs) in their non-core activities (as in the case of AIG, see Section 8.3).

8.5 References

- Brigo, D., M. Morini, A. Pallavicini (2013). Counterparty Credit Risk, Collateral and Funding – With Pricing Cases for All Asset Classes, Wiley Finance.
- Britt, S., Y. Kravych (2009). Reinsurance Credit Risk Modelling – DFA Approach, in ASTIN Colloquium 2009.
- Caouette, J.B., E.I. Altman, P. Narayanan (1998). "Managing Credit Risk: The Next Great Financial Challenge," Wiley.
- Cummins, J.D., M.A. Weiss (2010). Systemic Risk and the US Insurance Sector, Journal of Risk and Insurance 81, 489–528.
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
- Longstaff, F.A., P. Gandhi, N. Arora (2010). Counterparty Credit Risk and the Credit Default Swap Market, Journal of Financial Economics 103, 280–293.

8.6 List of resources

8.6.1 Books

Presented in alphabetical order.

- Brigo, D., M. Morini, A. Pallavicini (2013). Counterparty Credit Risk, Collateral and Funding – With Pricing Cases for All Asset Classes, Wiley Finance.
  - Covers a wide range of financial models
  - Technical/Mathematical level: Technical

- Caouette, J.B., E.I. Altman, P. Narayanan (1998). “Managing Credit Risk: The Next Great Financial Challenge,” Wiley.
    - Chapter 5 discusses clearinghouses at greater length;
- Cesari, G., J. Aquilina, N. Charpillon, Z. Filipovic, G. Lee, I. Manda (2009). Modelling, Pricing, and Hedging Counterparty Credit Exposure, Springer.
    - Concentrates on modeling counterparty risk for plain vanilla and exotic derivatives.
    - Technical/Mathematical level: Technical
- Gregory, J. (2011). Counterparty Credit Risk: The new challenge for global financial markets, Wiley finance.
    - Discusses several topics in counterparty risk, from qualitative and quantitative perspectives.
    - Technical/Mathematical level: Technical
- Gregory, J. (2015). The xVA Challenge: Counterparty Credit Risk, Funding, Collateral, and Capital, 3rd Edition, Wiley.
    - Extensively discusses counterparty risk
    - Presents many financial products
    - Technical/Mathematical level: Accessible
- Hull, J.C. (2014). Options, futures and other derivatives, 9th edition, Pearson.
    - Chapter 24 discusses counterparty risk as part of credit risk.
    - Technical/Mathematical level: Very accessible
- Norman, P. (2011). The Risk Controllers: Central Counterparty Clearing in Globalised Financial Markets, Wiley.
    - Discusses clearinghouses.
    - Refers to many historical facts.
    - Technical/Mathematical level: Accessible

Chapter 9

Sovereign credit risk

9.1 Introduction

After the restriction on cross-border cash flows was lifted in the late 1960s, Argentina and Mexico nearly defaulted on their loans due to oil shocks. Since these events, sovereign credit risk has been an important element to consider when one country decides to lend money to another. Sovereign credit risk is the risk that a (sovereign) country that borrowed money internationally will not repay its creditors (interest and/or principal). Whereas corporate bond default must be covered with the company's assets, international loans carry a higher risk because if the issuer defaults, the borrowing country is not necessarily obliged to repay its debt.

Although it may seem advantageous for an issuing country to default on its debt, this choice has serious economic repercussions. Take, for example, a country (which we will call A) that exports wheat. Country A may experience years of abundance and shortages. During the lean years, A has no choice but to borrow internationally to meet the needs of its population. Conversely, during good years, the country has a surplus and can repay its debt. The fact that A repays this debt cements good relations with its trading partner nations. If A refused to repay the loan, relations between the countries would deteriorate, and the underwriting countries would stop lending money to A. Over the long term, this may create enormous problems for A if it were to experience a shortage lasting several consecutive years.

9.2 Sovereign credit risk factors

Frenkel et al. (2004) identified five factors to explain sovereign credit risk:

1. Debt service ratio: a country's ratio of debt service payments (principal + interest) to export earnings. A country with a high ratio is considered risky.
2. Import ratio: A country's ratio of total imports to its total foreign exchange reserves. Foreign exchange reserves are assets that central banks hold to cover their debt. The higher this ratio is, the higher the country's risk.
3. Investment ratio: A set of accounting ratios related to a country's financial health. These ratios must be examined carefully to determine their impact on credit risk.
4. Variability of export revenue: Export revenue varies with the quantity and price of goods and services. Risk therefore comes from the law of supply and demand for this good/service. The more variable the revenue is, the higher the credit risk.
5. Domestic money supply growth: Domestic money supply is the total monetary value of the assets of an economy. If growth is good, the credit risk is lower because the country's economy is improving.

In addition to the factors identified by Frenkel et al. (2004), we can add power of taxation. The power of taxation encompasses income tax, goods/services sold by the government, sales tax, and more. A country that supplies a very expensive service to its population for free always has the option of privatizing this service to release itself from these financial obligations. In addition, a government may increase the income tax to reduce its international debt.

9.3 Modeling and pricing

One of the principal references for sovereign credit risk modeling is Eaton & Gersovitz (1981). They introduced a model for the amount of debt observed and determined that this amount is the minimum between the amount of debt sought by the borrowing country and the maximum quantity of loans permitted by the other countries. Their model allows for the possibility of default, but this must be to the borrower's advantage; moreover, after a single default, the country may no longer borrow abroad. Research on payment incentives has also been conducted by Grossman et al. (1988)), Bulow & Rogoff (1989a, b), Atkeson (1991), Dooley & Svenson (1994), Cole & Kehoe (1996, 2000), Dooley (2000), and many others.

Edwards (1984) offers another viewpoint on sovereign credit risk. He sought to identify the factors that influence the difference (spread) between a country's borrowing interest rate and the London Interbank Borrowing Rate (LIBOR). Because this difference should naturally be tied to default risk, default probability factors were determined empirically. Edwards found that the spread is positively correlated with the debt/GDP ratio and debt service. In addition, the spread is negatively correlated with the international reserves/GDP ratio and the propensity to invest. Empirical studies on the determinants of the lending rate spread over LIBOR have been performed by Edwards (1986, 2002), Berg & Sachs (1988), Boehmer & Megginson (1990), Duffie et al. (2003), and Zhang (2003).

Pan & Singleton (2007) demonstrate that the spread of the borrowing rate over LIBOR may be caused by common global factors. In line with these findings, Longstaff et al. (2007) perform a principal component analysis (PCA) on several CDS spreads and find that 50% of those spreads are "[…] more related to the US stock and high-yield bond markets, global risk premia, and capital flows than they are to their own local economic measures." More recently, Longstaff & Ang (2013) study the sensitivity to systemic risk of American and European issuers and find it to be quite heterogeneous.

9.4 Default history or sovereign insolvencies

9.4.1 Russia (1998)

The Russian financial crisis occurred on August 17, 1998, triggering the devaluation of the Russian ruble and the government's default on its obligations. This crisis may be explained by a decline in productivity, recurrent budget deficits and the high exchange rate between the ruble and other currencies. Productivity plunged following the Asian financial crisis in early 1997 and the slump in demand for crude oil and nonferrous metals. In addition, Russia's insistence on keeping the exchange rate with the US dollar relatively stable led the Russian central bank to spend over US $27 billion to maintain this exchange rate.

Note that due to default on Russian bonds, Long-Term Capital Management (LTCM) sustained a loss of over 44% of its $125 billion in assets in a single month. The case of LTCM demonstrated that the use of VaR (value-at-risk) has its limits and that stress testing is crucial to identify and hedge against risks with catastrophic repercussions.

9.4.2 European crisis

The European crisis is a series of events that occurred in late 2009, the effects of which are still being felt. First, it is worth mentioning that this crisis resulted from several phenomena. The globalization of finance, easy access to credit and the bursting of the real estate bubble are just a few of the reasons for this crisis. The catalyst, however, was the financial crisis of 2008. This crisis seriously undermined government bonds because investor confidence in the stability of issuing countries plummeted. This fear accentuated the context of an economy with massive government deficits. Note that the acronym PIGS (Portugal, Ireland, Greece, Spain) was widely used to designate the countries hit hardest by the crisis. Here are some important events (by country) related to the crisis.

Portugal

From the 1970s to the financial crisis of 2008, several senior authorities in the public sector received huge sums of cash as bonuses and other compensation. In addition, the government hired more public servants than necessary. The crisis of 2008, followed by a downgrading by Moody's in 2010, led Portugal to seek a €78 billion bailout from the IMF (International Monetary Fund). After major changes within the government and the public sector, the country emerged from the rescue package in May 2014 but still bears a heavy financial burden.

Ireland

The real estate bubble of 2007 is the primary cause of the debt crisis in Ireland. The government guaranteed the six largest banks involved in the bubble. The banks had lost nearly €100 billion, and the country's rating dropped sharply. With bond rates continuing to increase, the Irish government had no choice but to request a bailout package from the EU and the IMF of nearly €70 billion in late 2010. Fortunately for Ireland, its financial woes ended in December 2013, and the country put the rescue measures behind it.

Greece

Greece is the country that suffered the most from this crisis. In the early 2000s, Greece already had a very large deficit. With its main sources of revenue being exports and tourism, the financial crisis of 2008 decimated the country's revenue. The country requested several loans and bailout plans between 2010 and 2015 from the European Commission, European Central Bank and IMF that resulted in downgrades of the country's rating and the deterioration of the Euro and the Greek stock market. Greece went on to announce several austerity measures as a condition for the bailout plans. The austerity measures triggered a recession in 2010-2011. In March 2012, the Greek government defaulted on its obligations. There were even rumors that Greece would be ousted from the euro zone shortly thereafter (an event known as Grexit). After several negotiations and rescue measures, Greece finally rebounded in 2014 (three consecutive quarters of positive economic growth). But in early 2015, the Greek economy deteriorated again, forcing the country to default on a payment to the IMF. Budgetary measures are still in place, and the story continues.

Spain

Despite having the smallest debt of the PIGS countries, Spain's credit rating plunged when the government had to spend massively to save banks following the real estate bubble. In 2012, Spain agreed to be rescued by the ESM (European Stability Mechanism) in exchange for an unlimited bond purchase plan. After several tax measures, Spain emerged from the rescue plan in January 2014.

9.5 List of resources

9.5.1 Books

Presented in alphabetical order.

- Andritzky, J. (2012). Sovereign Default Risk Valuation, Springer.
  - Discusses a model for the evaluation of sovereign obligations.

  - Compares bonds and CDSs during the crisis.
  - Technical/Mathematical level: Very Accessible
- Duffie, D., K.J. Singleton (2003). Credit Risk: Pricing, Measurement, and Management, Princeton University Press.
  - Chapters 6 and 7 discuss a method to determine the price of a sovereign bond and its spread.
  - Technical/Mathematical level: Technical
- Frenkel, M., A. Karmann, B. Scholtens (2004). Sovereign Risk and Financial Crises, Springer.
  - Identifies five factors that explain sovereign credit risk.
  - Qualitative book on sovereign credit risk during financial crises.
  - Technical/Mathematical level: Technical/Accessible
- Gaillard, N. (2012). A Century of Sovereign Ratings, Springer.
  - Discusses rating of sovereign debt.
  - Technical/Mathematical level: Very Accessible
- Kolb, R.W. (2011). Sovereign Debt: From Safety to Default, Wiley.
  - Discusses the global impact of sovereign debt.
  - Technical/Mathematical level: Accessible
- Pepino, S. (2015). Sovereign Risk and Financial Crisis: The International Political Economy of the Eurozone, Palgrave Macmillan.
  - Book on the European crisis.

9.5.2 Website and online report

- IMF website: http://www.imf.org/
- ESM website: http://www.esm.europa.eu/

9.5.3 Database

- IMF Public Debt Database.
- Thomson Reuters Datastream Professional is a “powerful tool that integrates economic research and strategy with cross-asset analysis to seamlessly bring together top down and bottom up in one single, integrated application.”[6]
- Bloomberg: Pricing data for sovereign CDSs.
- GFI “is a globally recognized information hub for credit derivative products.”[7]

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- Bulow, J., K. Rogoff (1989b). Sovereign Debt: Is to Forgive to Forget?, American Economic Review 79, 43–50.

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[6] Commercial flyer from Thomson Reuters.

[7] http://www.gfigroup.co.uk/market-data/fixed-income/credit-derivatives.aspx

- Cole, H.L., T.J. Kehoe (1996). A Self-Fulfilling Model of Mexico's 1994–95 Debt Crisis, Journal of International Economics 41, 309–330.
- Cole, H.L., T.J. Kehoe (2000). Self-Fulfilling Debt Crises, Review of Economic Studies 67, 91–116.
- Dooley, M.P. (2000). A Model of Crisis in Emerging Markets, The Economic Journal 110, 256–272.
- Dooley, M.P., L.E.O. Svensson (1994). Policy Inconsistency and External Debt Service, Journal of International Money and Finance 13, 364–374.
- Duffie, D., L.H. Pedersen, K.J. Singleton (2003). Modeling Sovereign Yield Spreads: A Case Study of Russian Debt, Journal of Finance 58, 119–159.
- Eaton, J., M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis, The Review of Economic Studies, Vol. 48, No. 2, pp. 289–309.
- Edwards, S. (1984). LDC Foreign Borrowing and Default Risk: An Empirical Investigation, 1976–80, American Economic Review. 74, 726–734
- Edwards, S. (1986). The Pricing of Bonds and Bank Loans in International Markets: An Empirical Analysis of Developing Countries' Foreign Borrowing, European Economic Review 30, 565–589.
- Edwards, S. (2002). The Argentine Debt Crisis of 2001–2002: A Chronology and Some Key Policy Issues, Working paper, UCLA.
- Frenkel, M., A. Karmann, B. Scholtens, eds. (2004). Sovereign Risk and Financial Crises, Springer.
- Grossman, H.I., J.B. Van Huyck (1988). Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation, American Economic Review 78, 1088–1097.
- Lane, P. (2012). The European Sovereign Debt Crisis, The Journal of Economic Perspectives, Vol. 26, No. 3, pp. 49–67.
- Longstaff, F.A., J. Pan, L.H. Pedersen, K.J. Singleton (2007). How Sovereign Is Sovereign Credit Risk?, American Economic Journal: Macroeconomics, Vol.3(2), pp.75–103.
- Longstaff, F.A., A. Ang (2013). Systemic sovereign credit risk: Lessons from the US and Europe, Journal of Monetary Economics, vol. 60(5), pages 493–510.
- Pan, J., K.J. Singleton (2007). Default and Recovery Implicit in the Term Structure of Sovereign CDS Spreads, Review of Financial Studies, Vol. 63(5), pp. 2345–2384.
- Zhang, F.X. (2003). What did the Credit Market Expect of Argentina Default? Evidence from Default Swap Data, AFA 2004 San Diego Meetings, Federal Reserve Board.

Chapter 10

Regulatory environment

This section presents the most significant agreements, laws and financial regulations in the world. Section 10.1 covers agreements and laws for banks (from Basel accords and the Federal Reserve), whereas Section 10.2 covers guidelines for insurance companies, both American (NAIC) and European (Solvency).

10.1 Banks

10.1.1 Basel I, II, III

The Basel I Accord was initially drafted in 1988 in Basel, Switzerland, by the Basel Committee on Banking Supervision (BCBS). The accord was intended to provide a basis regarding the capital that banks must hold to hedge against financial and operational risks. Basel I mainly concerned credit risk and risk weighting of assets (RWA). Overall, the accord requires banks with an international presence to hold capital of at least 8% of the sum of their RWA (also known as the Cook ratio).

The simplicity of this ratio means that it can be implemented in countries with different legislation and regulations. The two main objectives of Basel I were consequently achieved: maintain the stability of the banking system and apply internationally. The RWA is calculated by assuming the weight relative to the following risks:

Weight	Category
0%	Liquid cash, gold ingots, national bonds
20%	Securitized assets
50%	Municipal bonds, home mortgages
100%	Corporate bonds
Not measured	Other

It is important to note that the required capital must come from tier 1 or 2. In addition, at least 50% of the capital must be tier 1 capital. The tiers are composed as follows (PRMIA):

1. Tier 1 capital: funds, equity and retained earnings.
2. Tier 2 capital: subordinated debt (long term), and other eligible hybrid instruments and reserves (such as loan loss provisions).

The simplicity of this formula is also its main weakness because the regulation is not directly aligned with institutions' risk characteristics. Thus, the Basel II Accord was created in 2004 to correct the shortcomings of the first accord. Basel II is divided into three pillars: 1) minimum capital requirements, 2) supervisory review and 3) market discipline. The ideas behind Basel I were revisited, and the definition of RWA was changed to better represent the risk sensitivity of the institution. In addition, instead of a single risk measurement method, three more complex methods were introduced. Regarding pillar 2, a series of guiding principles were adopted concerning the revision of the necessary capital. Stress testing and other risk evaluation methods were introduced. Finally, pillar 3 encourages adequate disclosure of capital levels and risk exposure.

Regrettably, the implementation of Basel II was not completed before the financial crisis of 2008. Consequently, stricter laws have since been adopted, and Basel III was ultimately created in 2014 with implementation scheduled for 2019. The general idea behind Basel III is to complete the guidelines of Basel I and II by emphasizing the risk of bank runs. Basel III also includes an important chapter on the obligation to adjust capital to credit quality (credit value adjustment – CVA) because the BCBS observed that during the financial crisis of 2008, most of the losses stemmed from credit downgrades and not necessarily defaults.

CVAs are losses tied to defaults and the downgrading of counterparties for the entire portfolio duration. Note that this is not a reserve but rather a profit and loss (P&L) measure. To understand the impact of CVA on P&L, we must not consider each product individually but rather the set of products traded with the same counterparty. A Monte Carlo method can be used to evaluate CVA, but this method is not optimal given the sometimes very low default probability. The classical method, shown below, may be more effective in some cases:

$$CVA_t = (1 - R) \sum_{k=0}^{\infty} EPE(k) PD(k)$$

where R is the recovery rate, PD the default probability and EPE the expected positive exposure at default. Note that companies may reduce their CVA risk by using a netting agreement (legal agreement recognized by the ISDA) or by collateral arrangements.

10.1.2 Federal Reserve

In the early 2000s, the Federal Reserve introduced a set of measures intended to reduce the insolvency risk of banks and other financial institutions. The main objective of the latter measures is to assess whether the company has sufficient funds to survive an important financial stress or adverse economic conditions. The first measure, performed annually, is known as the Comprehensive Capital Analysis and Review (CCAR), whereas the second measure is the Dodd-Frank Act Stress Test (DFAST).

10.2 Insurance companies

10.2.1 NAIC – RBC

The NAIC (National Association of Insurance Commissioners) is an American organization that facilitates discussions on standard-setting and regulatory support. The organization was founded in 1871 and is governed by all 50 state insurance regulators. The NAIC establishes standards and best practices, which are then adopted, or not, by each state (according to their official website). Since the 1990s, one of the key roles of the NAIC has been to determine the required capital that an insurance company must hold to avoid default risk (risk-based capital – RBC). This is done by determining the quantity of risk that an insurance company may assume. This measure takes the size and risk style of the insurance company into account.

Different models of RBC have therefore been created for the following types of insurance: life, P&C (property & casualty) and health. The possible risk categories and the RBC formulas for each type of company are summarized by the American Academy of Actuaries' Joint Risk Based Capital Task Force (1999) (see references). Note that in Canada, RBC is determined by the Office of the Superintendent of Financial Institutions (OSFI).

The NAIC also defines the SAP (statutory accounting principles), a presentation guide for financial statements that insurance companies must follow. The SAP ensures the homogeneity of insurance companies' financial statements, thereby simplifying the work and increasing the effectiveness of the state insurance department in auditing solvency and company reserves.

10.2.2 Solvency I and II

In 1973, the European commission decided to harmonize the risk management practices of all European insurance companies. The goal of Solvency I was to link the amount of capital required for insurance companies directly to their respective risk. Solvency II went into effect in January 1, 2016, and fills the gaps of Solvency I. It comprises three pillars (European Commission – Solvency II, 2015):

- "Pillar 1 consists of the quantitative requirements (e.g. amount of capital an insurer must hold)
- Pillar 2 sets out requirements for the governance and risk management of the insurer, and for effective supervision of the insurer
- Pillar 3 focuses on disclosure and transparency requirements."

Solvency II's Pillar 1 comprises Solvency and Minimum Capital Requirements (SCR and MCR) with formulas similar to that of RBC. However, Solvency II does not have the same risk separation as RBC. According to the CAS Risk-Based Capital (RBC) Research Working Parties (2012), for a typical P&C insurance company, Solvency II's classification corresponds to the following:

1. "Underwriting Risk: Premium (loss ratio) risk, excluding catastrophe risk, Reserve (loss development) risk, excluding catastrophe risk, Catastrophe risk
2. Default (Counterparty) Risk: Non-diversified counterparties, most significantly reinsurance counterparties, Diversified counterparties, most significantly agents balances and other receivables
3. Market Risk: Interest rate risk, Equity risk, Real estate (property) risk, Spread risk, Currency risk, Concentration risk, Illiquidity risk
4. Operational Risk."

10.4 List of resources

10.4.1 Books

Presented in alphabetical order.

- Barfield, R.E. (2011). A practitioner's guide to Basel III and beyond, Sweet & Maxwell.
  - Complete guide to Basel III written by a team of professionals from Price Waterhouse Coopers.
  - Covers the qualitative and calculation aspects of Basel III.
  - Technical/Mathematical level: Technical/Very Accessible
- Buckham, D., J. Wahl (2010). Stuart Rose, Executives Guide to Solvency II, Wiley.
  - Detailed examination of Solvency II, from qualitative and quantitative perspectives.
  - Technical/Mathematical level: Technical/Very Accessible
- Crouhy, M., D. Galai, R. Mark (2014). The Essentials of Risk Management, Second Edition, McGraw-Hill.
  - Chapter 3 covers various aspects of financial regulation
  - Technical/Mathematical level: Very Accessible
- Cruz, M. (2009). The Solvency II Handbook, Risk Books.
  - This book discusses methods to implement internal models, pillar 1 requirements, etc. It also provides several practical examples of the three pillars.
  - A companion to this book was published in 2014: The Solvency II Handbook: Practical

Approaches to Implementation.
- Technical/Mathematical level: Very Accessible

- Doff, R. (2014). The Solvency II Handbook: Practical Approaches to Implementation, Risk Books
  - This book is mainly intended for professionals but can also be used by consultants and students.
  - It presents a multitude of practical problems that insurance professionals will face throughout their careers.
  - This book is a companion book to the Solvency II Handbook.
  - Technical/Mathematical level: Very Accessible
- Engelmann, B., R. Rauhmeier (2011). The Basel II Risk Parameters: Estimation, Validation, Stress Testing – with Applications to Loan Risk Management (2011), Springer.
  - Covers calculation methods specific to Basel II that banks must use to comply with the standards.
  - Addresses three fundamental elements of Basel: probability of default (PD), loss given default (LGD), and exposure at default (EAD).
  - Technical/Mathematical level: Technical
- Hull, J.C. (2015). Risk Management and Financial Institutions, Wiley Finance
  - Chapters 15–17 cover Basel I, II, III and Solvency II.
  - Technical/Mathematical level: Very accessible.
- NAIC (2006). NAIC Risk-Based Capital Forecasting & Instructions, NAIC
  - Comes in four volumes: Fraternal, Health, Life and Property & Casualty.
  - Note that a CD-ROM is included for the purpose of projecting RBC.
- PRMIA (2004). PRMIA handbook volume III.
  - Good reference for credit risk in general. The other volumes are also very accessible and provide good basic knowledge. Note that a more recent version is available.
  - Chapter B contains information on credit risk.
  - Section 6 of chapter III.B addresses Basel.
  - Technical/Mathematical level: Very Accessible
- Webb, B.L., C.C. Lilly (1994). Raising the Safety Net: Risk-Based Capital for Life Insurance Companies, NAIC, Free book.
  - While not very recent, this free book is recommended by the NAIC and provides a good introduction to the fundamentals of RBC.
  - Many numerical examples are provided.
  - Technical/Mathematical level: Very Accessible

10.4.2 Website and online reports

- Basel accords:
  - Basel II: http://www.bis.org/publ/bcbsca.htm (main document available in English, French, German, Italian and Spanish)
  - Basel III: http://www.bis.org/bcbs/basel3.htm
- NAIC RBC:
  - Overview and links to important resources: http://www.naic.org/cipr_topics/topic_risk_based_capital.htm
- Solvency I and II: Links from the European commission
  - Solvency II Overview – Frequently asked questions
  - Solvency II FAQ
  - All information required for insurance and pension (including technical standards and guidelines)

10.5 Bibliography

- American Academy of Actuaries (1999). Joint Risk Based Capital Task Force, Comparison of the NAIC Life, P&C and Health RBC Formulas – Summary of Differences.
- Casualty Actuarial Society (2012). Solvency II Standard Formula and NAIC Risk-Based Capital (RBC), Report 3 of the CAS Risk-Based Capital (RBC) Research Working Parties Issued by the RBC Dependencies and Calibration Working Party (DCWP), E-Forum Fall 2012.
- PRMIA handbook volume II, PRMIA (2004).
- Sokol, A. (2012). A Practical Guide to Fair Value and Regulatory CVA, Numerix/CompatibL, PRMIA Global Risk Conference.

Chapter 11

Enterprise risk management

Enterprise risk management (ERM) is one of the most important concepts in this compendium. It is covered in the last chapter mostly because it builds on content discussed in previous chapters.

The traditional view of risk management has been to consider each sector of activity as independent (in a silo). ERM responds to companies' desire to integrate all risks into a single risk approach that considers the possible correlation among activity sectors. It creates a link between credit, market, liquidity, operational, information technology and other risks. Without a centralized risk management system, the chief risk officer may not be able to evaluate all of a firm's risks effectively, owing to the use of different valuation techniques in each sector.

The Casualty Actuarial Society (CAS) Committee on ERM defines ERM as (see CAS (2003)) "[...] the <u>discipline</u> by which an organization in <u>any industry</u> assesses, controls, exploits, finances, and monitors <u>risks from all sources</u> for the purpose of <u>increasing the organization's short- and long-term value</u> to its <u>stakeholders</u>." The authors also underline several aspects of this definition:

- <u>Discipline</u>: a process fully supported by the organization to an extent that it becomes part of the company's culture;
- <u>Any industry</u>: not just P&C insurers;
- <u>Increasing value</u>: ERM focuses on both risk mitigation and value creation;
- <u>Stakeholders</u>: includes shareholders, debtholders, management, employees and customers.

CAS (2003) is an excellent source of information for P&C actuaries considering ERM, and some important elements of this document are summarized in this chapter.

11.1 Reasons leading to ERM

What sums up ERM is the view that risk management should aggregate all risks faced by a company, rather than each risk being administered independently. It requires a strong commitment from the firm, and thus, risk management is addressed at a much higher level within the company's structure. According to CAS (2003), there are at least six reasons that enterprises have shifted their views on risk management:

1. Complexifying risks that have arisen from globalization, technology, financial innovation, sophistication of laws, etc.
2. External pressures: the media highlighting corporate mismanagement, along with ratings agencies, regulators, etc., forcing firms to mitigate these risks considerably to limit repercussions on their image, solvency, etc.
3. Portfolio view: companies are increasingly aware of links between departments and want to take this into account (as explained in the previous section) and benefit from these links.
4. Quantification: the growing desire to quantify everything is accentuating the implementation of ERM; companies are increasingly using value-at-risk (VaR) as a risk measure.
5. Information sharing and benchmarking: the flow of information is easier and faster as it now allows a large proportion of risk managers and other stakeholders to read, interpret and compare the financial performance of companies.

6. Alternative view of risk: recent trend in business to perceive risk as an opportunity rather than as a dangerous element. When it is controlled effectively and managed by the implementation of an ERM approach, risk is in fact beneficial for a company and can contribute to projects that generate higher actual net present value (NPV).

Although the implementation of an ERM approach is very beneficial for a company, it requires a significant investment of time and money. In addition, this process may take years to deploy and demands constant management and oversight. Generally, the position of CRO (Chief Risk Officer) must be created when implementing an ERM approach. This person specializes in risk management and reports directly to the CEO (Chief Executive Officer). Finally, ERM allows for better risk analysis during the implementation of new projects and, therefore, allows firms to choose the most profitable endeavors, namely those with the highest RAROC (risk-adjusted return on capital).

11.2 Components of an effective ERM framework

According to Lam (2003), there are seven components of an effective ERM implementation:

- **Corporate governance:** Top management and the Board must define their risk appetite to enable the CRO to work effectively. In addition, they must ensure that the people assigned to each risk are knowledgeable in this area and that the firm's structure is well aligned with the objective of implementing ERM.
- **Line management:** Following up on each line of business is important because it represents the firm's revenue. An exhaustive description of plausible losses must be compiled with the help of each line's managers.
- **Portfolio management:** Firms must diversify their risks with active portfolio management.
- **Risk transfer:** Firms must identify risks that should be transferred to an insurance company. Extreme risks can alter the firm's financial health or trigger bankruptcy. Some risks may be mitigated through the use of derivatives.
- **Risk analytics:** The use of mathematics and statistics to improve future risk management. This can be achieved internally (CRO) or externally with specialized consultants. Complex methods must add value.
- **Data technology and resources:** Centralization of each sector's IT system into a single one. Keeping up to date will surely improve the efficiency of the company in general.
- **Stakeholder management:** Transparency toward stakeholders is crucial to convey good firm management. This lowers credit risk because it eases rating agencies' uncertainty about the business.

11.3 Types of risks and mitigation

When implementing ERM, the CRO must identify and apply a rigorous approach to mitigate risks. According to CAS (2003), there are four major categories of risks:

- **Hazard risks**: fire, theft, vandalism, natural catastrophes, disability, illness, etc.
- **Financial risks**: typical market risks such as asset value, interest rates, foreign exchange rate, credit risk, liquidity risk, inflation, etc.
- **Operational risks**: all risks linked to the failure of a company process such as production, IT, reporting, etc.
- **Strategic risks**: all risks tied to competitors, laws, technological advances, etc.

It is important to note that the ERM Task Force of the Actuarial Standards Board has developed specific Standards of Practice tied to risk evaluation and treatment (see ASOPs 46 and 47) in the context of ERM.

11.4 Modeling

One of the CRO's tasks is to model risks, as discussed in the previous section. There is a large spectrum of tools that can be used to assess these risks and Appendix B of CAS (2003) provides a detailed account of the methods. They can be classified as methods solely based on data, expert opinion, or a combination of both. The categories are summarized as follows:

- **Purely analytical**: In this technique, only data are considered, and the approach is appropriate if there is a large history of data. Examples of tools are direct empirical analysis, estimation of probability density functions or time series (including stochastic differential equations), regressions, extreme value theory, etc.
- **Purely expert opinion**: This technique is often used when the data required are not available or too limited to be deemed reliable (e.g., the creation of a new product). Simple probabilities may be calculated by asking for expert opinions and by creating a distribution of possible results based on their responses. A greater number of scenarios and richer results can be obtained by adding constraints to questions asked to experts.
- **Mix of analytical and expert opinion**: The mix of the two techniques is useful when the quantity of data available is not too large.

11.5 Risk management process

Although the modeling techniques used to represent these risks can be very different, CAS (2003) proposes broad outlines representing the risk management process. The seven steps are as follows:

1. **Establish Context**: Determine the company's internal and external environments.
2. **Identify Risks**: Identify and document the threats that can impact the company.
3. **Analyze/Quantify Risks**: Use the methods listed in the previous section to analyze the financial impact of these risks along with probabilities.
4. **Integrate Risks**: Determine the link between these risks and aggregate the risks to determine the real impact on the company.
5. **Assess/Prioritize Risks**: Prioritize all the risks to identify those with the greatest loss potential, and deal with those first.
6. **Treat/Exploit Risks**: Take the necessary actions to mitigate the potentially most dangerous risks.
7. **Monitor & Review**: This step is the most important. Although the risks were mitigated, it is important to constantly control them and to return to step 1 if the actions taken are not effective or if other risks arise at the company level.

11.6 References

- Casualty Actuarial Society (CAS) (2003). Overview of Enterprise Risk Management, Enterprise Risk Management Committee.
- Lam, J. (2003), Enterprise Risk Management: From Incentives to Controls, Wiley Finance.

11.7 List of resources

11.7.1 Books

Presented in alphabetical order.

- Fraser, J., B. Simkins, K. Narvaez (2014). Implementing Enterprise Risk Management: Case Studies and Best Practices, Wiley Finance.
  - Presents case studies of companies that have adopted ERM.

  - Technical/Mathematical level: Accessible
- Jorion, P. (2006). Value at Risk, 3rd Ed.: The New Benchmark for Managing Financial Risk, McGraw-Hill Education.
  - Good reference for ERM.
  - Deals mainly with VaR (value-at-risk) assessment methods in an ERM context.
  - Technical/Mathematical level: Accessible/Technical
- Olson, D.L., D. Wu (2010). Enterprise Risk Management Models.
  - Discusses the fundamental steps in the ERM process.
  - Uses advanced techniques to model risks related to the supply chain and describes the advantage of using ERM for this purpose.
  - Technical/Mathematical level: Accessible/Technical
- Ross, S., R. Westerfield, J. Bradford (2015). Fundamentals of Corporate Finance, McGraw-Hill.
  - Chapter 23 discusses ERM.
  - Important reference in the corporate finance sphere.
  - Technical/Mathematical level: Accessible
- Sweeting, P. (2011). Financial Enterprise Risk Management, Cambridge University Press
  - Describes a range of qualitative and quantitative techniques to identify, model and measure risks in ERM.
  - Technical/Mathematical level: Accessible/Technical

11.7.2 Websites and online reports

- Casualty Actuarial Society (CAS) (2003). Overview of Enterprise Risk Management, Enterprise Risk Management Committee.
  - Excellent summary of ERM from the point of view of P&C insurers. It also provides an exhaustive list of resources on ERM.
  - A must-read. Very accessible.
- Actuarial Standards of Practice (related to ERM):
  - ASOP 46
  - ASOP 47
- CAS ERM:
  - Articles and press releases written by CAS members who discuss several aspects of ERM.
  - As a practice area

Chapter 12

General resources

In this chapter, we provide an overview of resources that might cover multiple areas of credit risk.

12.1 Research papers

Online repositories:

- Social Sciences Research Network (SSRN) http://papers.ssrn.com/sol3/DisplayAbstractSearch.cfm: online repository for articles in the social sciences, which include finance and economics. The entire library comprises over 500,000 papers, of which over 7,000 relate to credit risk. Note that the articles are freely available but are not peer-reviewed.
- DefaultRisk.com http://www.defaultrisk.com/: online repository for articles specifically focusing on corporate credit risk, with over 1,500 freely available papers. Articles are not peer-reviewed unless linked to a specific journal. This website was very active until 2013. The page http://www.defaultrisk.com/papers.htm groups papers by area of research.

Professional organizations:
Professional organizations for actuaries and other risk professionals conduct research in many areas, and the resulting papers are generally available on their respective websites (public or for members only).

Research papers:

- CAS: http://www.casact.org/research/index.cfm?fa=currentresearch
- SOA: https://www.soa.org/research/research-projects/default.aspx
- CIA: http://www.cia-ica.ca/research/research-projects
- CFA Institute: Financial Analysts Journal http://www.cfainstitute.org/learning/products/Pages/index.aspx
- GARP: White Papers http://www.garp.org/#!/risk-intelligence

Conference proceedings and online repositories:

- CAS: http://www.casact.org/research/index.cfm?fa=researchresources
- SOA: https://www.soa.org/BrowsePublication/BrowsePublication.aspx. The search tool on the upper-right part of the main website also points to the same publications.
- CIA: http://www.cia-ica.ca/publications/search-publications
- CFA Institute: http://www.cfainstitute.org/learning/products/Pages/index.aspx

Peer-reviewed scientific journals:
Generally accessible through subscriptions, but universities usually have access to articles published in these journals. The readership is usually academic researchers, and hence, many articles may not be technically accessible to most practitioners. Journals that also target practitioners are indicated with a *.

- Finance and quantitative finance: Journal of Finance, Journal of Financial Economics, Review of Financial Studies, Journal of Financial and Quantitative Analysis, Journal of Business and Economic Statistics, Journal of Banking and Finance, Journal of Derivatives*, Journal of Fixed Income*, Journal of Credit Risk*, Risk*.

- Actuarial science: North American Actuarial Journal*, Variance*, Insurance: Mathematics & Economics, Scandinavian Actuarial Journal, European Actuarial Journal, ASTIN Bulletin.

12.2 Data

We list the most frequently used databases for various aspects of credit risk modeling and management. The largest databases require a large subscription fee.

- Center for Research in Security Prices (CRSP): maintained by the University of Chicago Booth Business School, the CRSP data contains stock, indexes, mutual fund, Treasury and real estate market prices. http://www.crsp.com/products/research-products
- Datastream (Thomson Reuters): "Datastream is a global financial and macroeconomic database covering equities, stock market indices, currencies, company fundamentals, fixed income securities and key economic indicators for 175 countries and 60 markets." (European University Institute) Prices of credit derivatives can be found in Datastream (http://financial.thomsonreuters.com/en/products/data-analytics/market-data/evaluated-pricing-data.html)
- Compustat (Standard & Poor's): Database of market and accounting information for US and international companies.
- Markit: provides prices of credit default swaps, corporate and municipal bonds and equity volatility data. They maintain the well-known CDX indices. https://www.markit.com/Product/
- Bloomberg http://www.bloomberg.com/enterprise/data/reference-data-services/
- Moody's Analytics http://www.moodysanalytics.com/
- Standard & Poor's Capital IQ https://www.spcapitaliq.com/client-solutions/data

12.3 Computer programs

The online programming community can share its codes on repositories. It might be possible to find credit risk valuation tools or other mathematical tools useful for credit risk modeling, pricing and risk management. It is important to note that those are user-generated code and are not necessarily validated (accuracy, bugs, etc.).

- Matlab: Matlab Central File Exchange http://www.mathworks.com/matlabcentral/fileexchange/
- R: CRAN Packages https://cran.r-project.org/web/packages/
- Python: Python Package Index https://pypi.python.org/pypi
- GitHub: online code repository service https://github.com/. Typing keywords provides available codes, sorted by programming language. Credit risk returns 34 programs, including 6 in R, 2 in Python, 2 in Java and 2 in C#.

12.4 Other resources

Material available online about credit risk can be found in the following:

- Master's and PhD theses: most universities post electronic versions of their students' theses. This can be a valuable, free peer-reviewed resource. Searching each university's website can be tedious. However, Canadian and US theses are available (or can be requested) through national libraries.

- US Library of Congress http://www.loc.gov/rr/main/alcove9/education/theses.html: links to several portals
- Library and Archives Canada, Theses Canada Portal: http://www.bac-lac.gc.ca/eng/services/theses/Pages/theses-canada.aspx

• University course material: many professors put their teaching material online, which can be accessed through a standard Google search. Massively online open courses (MOOC) can also be a very useful resource:

- https://www.coursera.org/: collaborates with hundreds of universities around the world including top universities such as Princeton, Stanford, and Columbia.
- https://www.edx.org/: founded by MIT and Harvard, collaborates with other large US and Canadian universities.
- https://www.class-central.com/: aggregates MOOCs from Coursera, edX, and many other providers of MOOCs. Therefore, readers can search for courses on various topics from various MOOC providers.

• Online courses and webinars (webcasts): professional organizations often provide webinars for their members and non-members that can be viewed online.

- CAS: http://www.casact.org/education/webinar/
- SOA: https://www.soa.org/professional-development/archive/webcast-recordings.aspx
- CIA: http://www.cia-ica.ca/professional-development/webcasts
- PRMIA: http://www.prmia.org/webinars (webinars)
  http://www.prmia.org/training/online (online courses)
  - o Note that there is an online course on credit risk management on this website: http://www.prmia.org/online-course-group/credit-risk-management
- GARP: https://www.garp.org/#!/risk-intelligence
- CFA Institute: http://www.cfainstitute.org/learning/products/Pages/index.aspx. The CFA Institute offers online courses and webcasts.

• Presentation online repository: speakers at conferences, professors, and so forth can share their presentations on these specialized repositories.

- SlideShare: the most well-known online service, owned by LinkedIn. http://www.slideshare.net