Notes on Using Property Catastrophe Model Results

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Abstract

This article will discuss the use of results from popular Property Catastrophe models. It will explain common terms like Occurrence Exceedance Probability (OEP) and Aggregate Exceedance Probability (AEP) and show how these are related to event count and event size ideas. Simulation and the use of multiple models (blending) will also be discussed.

Keywords. Catastrophe Modeling, Occurrence Exceedance Probability, OEP, Aggregate Exceedance Probability, AEP, Probable Maximum Loss, PML.

1 Introduction

A reinsurance or insurance actuary will frequently need to work with the results of commercial Property Catastrophe models. This work may include incorporating results such as an average annual loss (AAL) into a pricing exercise or making additional calculations such as simulating catastrophes in a capital model. This article will provide an introduction to some of the simpler uses of commercial catastrophe models, including common terms, basic calculations and simulation. Combining or blending of models will also be discussed.

1.1 Popular Cat Models

Two models will be reviewed along with their standard formats. These are Risk Management Solution's RMS platform and Verisk's AIR platform.

1.1.1 AIR

The AIR output is provided in the form of sample data and some capital models refer to this as pre-simulated data. Table 1 provides an example. The table values are illustrative and don't represent any particular exposures to actual losses. Columns are provided for simulation number or year, event id, and claim size. This format is relatively easy to use because it looks like a historical loss listing. This table is sometimes referred to as a year-event loss table (YELT) because it provides loss detail by year and event. Note that Table 1 is missing year 2 and that year 3 has multiple events.

The mean and standard deviation of the annual loss from an AIR YELT created from n simulated years with n_y events in year y (which could be zero) are

$$\mu = (\sum_{y=1}^{n}\sum_{e=1}^{n_y} \text{loss}_{y,e})/n \quad (1)$$

$$\sigma = \sqrt{\frac{\sum_{y=1}^{n}\left(\sum_{e=1}^{n_y} \text{loss}_{y,e}\right)^2}{n} - \mu^2} \quad (2)$$

Table 1: AIR-style Year Event Loss Table (4 Years)

Year	Event ID	Loss
1	1	100
3	2	500
3	3	300
4	4	100

The events within each year are summed by year before computing the annual mean and standard deviation. For the annual mean, this is equivalent to a straight sum of all the event-year data divided by the number of years n. For Table 1 we have

$$\mu = 250 = (100 + 500 + 300 + 100)/4 \quad \text{and} \tag{3}$$

$$\sigma = 320 = \sqrt{(100)^2 + (500 + 300)^2 + (100)^2)/4 - 250^2}. \tag{4}$$

1.1.2 RMS

The RMS output is provided in the form of a list of parameters for each event. Table 2 provides a brief description of each column. There are two columns that need additional comment, *Sdi* and *Sdc*. These two columns represent an approximation that RMS uses to represent the standard deviation of the loss for a given event. The standard deviation for the event loss is the sum of an independent component, Sdi, and a correlated component, Sdc. This split facilitates RMS calculations as the event loss is built up from components whose individual losses are partially dependent on one another. Later on we will discuss events that are split into subcomponents like Personal lines and Commercial lines. The Sdi-Sdc split will be important then, but for now we can just think of their sum as the standard deviation of the event loss.

Table 2: RMS-style Event Loss Table Parameters

Column Name	Description
Event ID	Unique identifier of the event
Rate	Annual event frequency
Mean	Average Loss if the event occurs
Sdi	Independent component of the spread of the loss if the event occurs.
Sdc	Correlated component of the spread of the loss if the event occurs.
Exposure	Total amount of limits exposed to the event (Maximum loss)

Table 3 provides an example of RMS output. This table is often referred to as an event loss table (ELT) because it provides event details. As before, the table values are illustrative and don't represent any particular exposures to actual losses.

The mean and standard deviation of the annual loss described by an RMS ELT with m event rows are

$$\mu = \sum_{e=1}^{m} (\text{Rate}_e)(\text{Mean}_e) \tag{5}$$

Table 3: RMS-style Event Loss Table

Event ID	Rate	Mean	Sdi	Sdc	Exposure
1	.10	500	500	500	10,000
2	.10	300	400	800	5,000
3	.50	200	300	400	4,000

$$\sigma = \sqrt{\sum_{e=1}^{m}(\mathrm{Rate}_e)((\mathrm{Sdi}_e + \mathrm{Sdc}_e)^2 + \mathrm{Mean}_e^2)}. \tag{6}$$

For Table 3 we have

$$\mu = 180 = (.1)(500) + (.1)(300) + (.5)(200) \tag{7}$$

$$\begin{aligned}\sigma = 737 = \; & [(.1)((500+500)^2 + 500^2) + \\ & (.1)((400+800)^2 + 300^2) + \\ & (.5)((300+400)^2 + 200^2)]^{1/2}.\end{aligned} \tag{8}$$

These formulae can be derived by assuming each event is an independent collective risk model (CRM) [1] with Poisson mean Rate_e and a severity distribution with mean Mean_e and standard deviation $\mathrm{Sdi}_e + \mathrm{Sdc}_e$.

A common use for these tables is to apply reinsurance terms and then estimate prices or distributions net of reinsurance. This is straightforward with AIR-style data (Table 1) and a bit more difficult with RMS-style data (Table 3). So it is common to use the parameters from RMS-style data to simulate individual events and then work with the simulated data directly. Simulating from RMS-style ELTs will be discussed in section 3.2.

1.2 OEP, Return Period, AEP and PML

The terms Occurrence Exceedance Probability (OEP), Return Period, Aggregate Exceedance Probability (AEP), and Probable Maximum Loss (PML) are commonly used and commonly confused. Sometimes OEP and AEP will be abbreviated as EP (Exceedance Probability) [GK05]. We will step through each term and explain it. To begin, it is useful to think of these definitions in the context of a collective risk model with an annual event count distribution and an event size distribution. Imagine simulating a set of losses from these distributions and presenting the results in a form similar to Table 1. The statistics that we want, like OEP and AEP, can then be computed from that table.

1.2.1 OEP

The Occurrence Exceedance Probability(OEP) curve $O(x)$ describes the distribution of the *largest* event in a year. In particular, $O(x)$ is the probability that the largest event in a year exceeds x.

[1] A collective risk model assumes a claim count N and claim sizes $X_i, i = 1, ..., N$ with each X_i independent and identically distributed and each X_i indepdendent from N.

The distribution of the largest event in the year is not the same as the distribution of the event size.

Consider our AIR-style (Table 1) losses. The empirical claim size distribution $F_X(x)$ shown in Table 4 reflects all event losses. In contrast, the OEP curve is derived from the largest event in each year, which is shown in Table 5. Table 6 presents our empirical OEP curve.

Table 4: Empirical Claim Count and Severity Distribution Derived from Table 1

x	$\Pr(X = x)$	n	$\Pr(N = n)$
100	50%	0	25%
300	25%	1	50%
500	25%	2	25%

Table 5: Largest Events by Year Derived from Table 1

Year	Largest Event
1	100
2	0
3	500
4	100

Table 6: Empirical OEP Curve Derived from Table 1

PML_{occ} x	OEP $O(x)$	Return Period $r = 1/O(x)$
0	75%	1.33
100	25%	4.00
500	0%	∞

The OEP is often used by primary insurers to help select their catastrophe reinsurance program limits and retentions.

1.2.2 Return Period

It is common to talk about a return period r instead of the OEP, where $r = 1/O(x)$. It is the expected number of years between events that exceed x.

1.2.3 PML

The dollar amount of loss x is often called the Occurrence Probable Maximum Loss (PML) at return period r, or simply the PML for the return period r. Thus,

$$1/r = O(x) = O(\text{PML}_{\text{occ}}) \tag{9}$$

or

$$\text{PML}_{\text{occ}}(r) = O^{-1}(1/r), \tag{10}$$

where $O^{-1}(x)$ is the inverse OEP function. The OEP and the PML are linked. Sometimes actuaries will refer to an OEP curve or a PML curve; they refer to the same thing. Table 6 represents an OEP or PML curve. The PML column shows dollars and the OEP column shows probabilities, though often OEP is supplemented or replaced with its reciprocal, the return period. As part of their rating process, AM Best asks companies for their Occurrence PML losses for the 100-year return period for wind and for the 250-year return period for earthquake. [Irw16]

1.2.4 AEP

The Aggregate Exceedance Probability(AEP) curve $A(x)$ describes the distribution of the *sum* of the events in a year. In particular, $A(x)$ is the probability that the sum of the events in a year exceeds x.

The AEP is not the same thing as the OEP, but is often confused with it.

The AEP can be very different from the OEP when the probability of two or more events is significant. The AEP and OEP can be similar when the probability of two or more events is very small; they are identical when there is zero probability of two or more events. (See appendix A.) The AEP is used to help consider the total volume of catastrophe events in a year. The total losses by year from Table 1 are shown in Table 7 and used to compute an empirical AEP in Table 8.

Table 7: Total Losses by Year Derived from Table 1

Year	Total Losses
1	100
2	0
3	800
4	100

Table 8: Empirical AEP Curve Derived from Table 1

PML_{agg} x	AEP $A(x)$	Return Period $r = 1/A(x)$
0	75%	1.33
100	25%	4.00
800	0%	∞

Note that our empirical OEP and AEP curves are not the same. However, our OEP and AEP curves would be identical if year 3 did not have a second event.

Sometimes modelers will use the term aggregate PML which is defined in a manner similar to the occurrence PML but with the aggregate distribution. The aggregate PML is essentially the inverse function of the AEP.

$$\text{PML}_{\text{agg}}(r) = A^{-1}(1/r). \tag{11}$$

It should be noted that PML is often used informally and its meaning is not always clear. Usually PML used by itself is understood to mean Occurrence PML, but it can also refer to an Aggregate PML. It may simply refer to an intuitive notion of a large loss without a well defined statistical meaning.

2 OEP and the Collective Risk Model

Sometimes a reinsurance actuary will receive an OEP table or even part of one and be asked to apply reinsurance terms for pricing. In these situations it is helpful to be able to reverse engineer a claim count distribution $F_N(n)$ and a severity distribution $F_X(x)$ from the OEP curve. Using the claim count and severity distributions one can then simulate individual losses and apply reinsurance terms to the simulated data. It is easy to start with detailed event loss data and compute the OEP curve as we did with Table 1 and Table 6, and just a bit harder to go the other way.

Conversely, there may be situations where an actuary starts with the claim count distribution and claim size distribution and it may be convenient to compute the OEP curve directly, without simulating.

These tasks are relatively easy if we assume that the vendor models can be represented by a collective risk model with independent claim counts and independent and identically distributed claim sizes. This is probably an oversimplication, but it provides a convenient and useful framework.

2.1 Converting OEP Curves to Claim Count/Severity Curves

There is substantial information contained in the OEP and it is tightly connected to the distribution of the number of events in a year and the distribution of the size of an event. Given the cumulative distribution function (cdf) $F_X(x)$ for the claim size X and the probability function $P_N(n)$ for the claim count N, $O(x)$ can be written explicitly.

$$\begin{aligned} O(x) &= \Pr(M > x) \text{ where } M = \max(X_1, ..., X_N) && (12)\\ &= 1 - \Pr(X_i \leq x \text{ for } i = 1, ..., N) && (13)\\ &= 1 - E_N(F_X(x)^N) = 1 - \text{PGF}(F_X(x)) && (14) \end{aligned}$$

where $\text{PGF}(x)$[2] is the probability generating function for N. The claim size cdf $F_X(x)$ may then be derived from this equation. For some claim count distributions $\text{PGF}(t)^{-1}$ is easily expressed and we obtain

$$F_X(x) = \text{PGF}^{-1}(1 - O(x)). \tag{15}$$

This process does not generally produce a unique size distribution $F_X(x)$ because we need to select the claim count distribution $F_N(n)$ and its parameters. A different $F_N(n)$ will yield a different $F_X(x)$. However, the size distributions computed this way will be consistent with the starting OEPs and the claim count assumption.

[2]The PGF of a discrete distribution is defined as $\text{PGF}(t) = E(t^N)$.

2.1.1 OEP Conversion Example

Let's take our empirical OEP (Table 6) and using the empirical PGF

$$\text{PGF}(t) = 0.25 + .5t + .25t^2, \tag{16}$$

estimate $F_X(100)$. In practice, we usually use the Poisson claim count assumption, but it is convenient here to stick with the empirical figures. We don't have a closed form for the inverse function of the empirical PGF, but since it is a quadratic we can solve for the roots.

$$O(100) = .25 = 1 - (.25 + .5F_X(100) + .25(F_X(100))^2). \tag{17}$$

or

$$0 = -.25(F_X(100))^2 - .5F_X(100) + (1 - .25 - .25) \tag{18}$$

The negative root yields

$$F(100) = .73 = \frac{.5 - \sqrt{.5^2 - (4)(-.25)(.5)}}{(2)(-.25)} \tag{19}$$

Table 9 completes this process. The inverted claim size curve is a coarse approximation (we are only working with three points), but it is entirely consistent with the starting OEP.

Table 9: Claim Size Distribution from OEP

x	$O(x)$	Inverted $F_X(x)$	Table 4 $F_X(x)$
0	75%	0%	0%
100	25%	73%	50%
500	0%	100%	100%

2.1.2 OEP and the Poisson Distribution

For the Poisson claim count distribution we have

$$\begin{aligned}
\text{PGF}(t) &= \exp(\lambda(t-1)) & (20)\\
O(x) &= 1 - \exp(\lambda(F(x) - 1)) & (21)\\
F(x) &= 1 + \frac{\log(1 - O(x))}{\lambda}. & (22)
\end{aligned}$$

Equation 22 can be used to convert an OEP to an event size distribution if an estimate of λ is available. This is very convenient.

In theory, λ may be estimated directly from $O(x)$. [3]

$$\begin{aligned}
\exp(-\lambda) &= \Pr(0) = 1 - O(0) & (23)\\
\Longrightarrow \lambda &= -\log(1 - O(0)). & (24)
\end{aligned}$$

[3] The OEP can "pack" both claim count and severity information if the count distribution is Poisson and $F_X(0) = 0$. See appendix B

This may be difficult to apply in practice since it is common to receive only a partial OEP curve without an entry for zero or Pr(0) will be very nearly zero when the catastrophe distribution includes frequent losses.

A common practice is to take the smallest claim size entry $x_{\min}$ of interest and compute

$$\lambda = -\log(1 - O(x_{\min})). \tag{25}$$

In this case, λ represents the Poisson mean for claims greater than $x_{\min}$ and equation 22 is applied to produce a claim size distribution for claims greater than $x_{\min}$. Note, $F(x_{\min}) = 0$.

2.1.3 OEP and the Negative Binomial Distribution

Using the mean-contagion form [HM83] for the Negative Binomial claim count distribution we have

$$\text{PGF}(t) = (1 - c\lambda(t-1))^{-1/c} \tag{26}$$

$$O(x) = 1 - (1 - c\lambda(F(x) - 1)) - 1/c \tag{27}$$

$$F(x) = 1 + \frac{1 - (1 - O(x))^{-c}}{c\lambda}. \tag{28}$$

3 Aggregation and Simulation of Cat Losses

A common use for catastrophe modeling output is to feed it into capital models to be mixed with other sources of loss. Randomly drawn catastrophe losses are combined with randomly drawn losses from other sources. In order to do this the capital model has to have a mechanism for using the catastrophe output. In the case of AIR-style output it is often as simple as randomly drawing a year and then looking that year up in a table like Table 1. In the case of RMS-style output, the capital model needs to use the parameter table to perform its own simulation.

3.1 AIR

The YELT produced by AIR is essentially pre-simulated data and can be used directly or re-sampled. One should be careful with re-sampling if the results are to be combined with other AIR model results (for example, merging two companies' cat results) because the dependencies among events can be destroyed by independently sampling two separate YELTs that share perils. Two AIR-style tables should be joined by common years and events. Alternatively, one can draw a single random year and use the same year to extract losses from both tables.

3.2 RMS

The RMS ELT contains parameters for both the number of events and the size of each event.

3.2.1 Number of Events

The number of events N can be simulated from a Poisson with mean λ set to the sum of the ELT rates.

$$\lambda = \sum \text{Rate}_i \tag{29}$$

$$N \sim \text{Poisson}(\lambda). \tag{30}$$

3.2.2 Size of Events

A claim size can be simulated for each claim in two steps. First, we determine which event occurs, that is, which ELT row are we using. This is done by drawing a random row/event R from the ELT with each row/event having a probability in proportion to its rate.

$$U \sim \text{Uniform}(0,1) \tag{31}$$

$$R = \min\{r : U \leq \sum_{i=1}^{r} \text{Rate}_i/\lambda\} \tag{32}$$

Second, now that we know which row or event we are using, we use the event parameters to draw a random claim size X from a scaled Beta distribution. The Beta distribution parameters a and b are computed as follows:

$$a_R = \left(\frac{\text{Mean}_\text{R}}{\text{Sdi}_\text{R} + \text{Sdc}_\text{R}}\right)^2 \left(1 - \frac{\text{Mean}_\text{R}}{\text{Exposure}_\text{R}}\right) - \frac{\text{Mean}_\text{R}}{\text{Exposure}_\text{R}} \tag{33}$$

$$b_R = a_R\left(\frac{\text{Exposure}_\text{R}}{\text{Mean}_\text{R}} - 1\right) \tag{34}$$

$$X \sim (\text{Exposure}_R)(\text{Beta}(a_R, b_R)). \tag{35}$$

The cdf for the Beta distribution is the incomplete Beta function

$$F(x) = \beta(a,b;x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\int_0^x t^{a-1}(1-t)^{b-1}dt. \tag{36}$$

In EXCEL one can generate a scaled beta variate with

$$= \texttt{BETA.INV(RAND()},a_R, b_R)*\text{Exposure}_\text{R}. \tag{37}$$

Table 10 illustrates RMS-style simulation using the parameters from our RMS-style ELT (Table 3).

Table 10: RMS-Style Simulation using Table 3-ELT

Trial	Poisson Count	Uniform Draw	Row	Beta Parameter	Beta Parameter	Scaled Beta
	N	U	R	a	b	X
1	1	0.70	3	11.5875	220.1625	204
2	0	–	–	–	–	–
3	2	0.15	2	14.9800	234.6867	272
3	–	0.40	3	11.5875	220.1625	168
4	1	0.05	1	3.3750	23.6250	268

This procedure works if the ELT does not have event parameters sub-divided by region or line of business. When each event is sub-divided by region or line of business the simulation process requires additional steps to preserve dependencies between sub-divisions. Table 11

Table 11: RMS-Style ELT with Two Sub-categories

		Personal Lines				Commercial Lines			
Event ID	Rate	Mean	Sdi	Stc	Exposure	Mean	Sdi	Sdc	Exposure
1	0.1	300	400	300	3000	200	300	200	1000
2	0.1	100	371	267	1000	200	150	533	4000
3	0.5	100	224	200	2000	100	200	200	2000

provides an example of an ELT with sub-divisions. In order to simulate from Table 11 we need to aggregate it to make it look like Table 3. The following approximation has worked well for the authors.

1. Aggregate the event parameters as follows

$$\text{Mean}_R = \sum_k \text{Mean}_{R,k} \tag{38}$$

$$\text{Exposure}_R = \sum_k \text{Exposure}_{R,k} \tag{39}$$

$$\text{Sdi}_R = \sqrt{\sum_k \text{Sdi}^2_{R,k}} \tag{40}$$

$$\text{Sdc}_R = \sum_k \text{Sdc}_{R,k}. \tag{41}$$

2. Apply equations 33–35 to the results of step 1 (equations 38–41) to simulate the total event loss X.

3. Allocate X to the sub-divisions X_k.

$$X_k = X\frac{\text{Mean}_{\text{R,k}}}{\text{Mean}_{\text{R}}}. \tag{42}$$

This allocation is not perfect but it assures that the sub-categories sum to the simulated total and preserves much of the component dependencies. The values in Table 11 can be aggregated across Personal and Commercial Lines using equations 38–41 to reproduce our simpler RMS-style ELT (Table 3).

4 Model Blending

The catastrophe models available in the market can produce a wide range of loss results. Companies that use these models need to understand the model differences and determine the best model(s) to manage their catastrophe risk. A common practice of using multiple models is to blend the models together. For example, Florida Hurricane Catastrophe Fund uses a weighted average of five models (RMS, AIR, EQE, ARA, FPM) in their ratemaking [Inc16]. The benefit of model blending is that it reflects elements of a range of models, stabilizes changes in individual models across time and yields a single set of results.

It is beyond the scope of this paper to discuss how to determine the weights that should be used to blend the models. We will focus on the technical approaches that can be used to blend the models, assuming the weights have already been determined. The model blending approaches can become quite complicated if we consider breaking down the models into various components and blending them at the component level. However, those approaches need to be supported by tremendous amount of independent research and the practical difficulties have limited their applications. Below we will discuss two more straightforward and much more widely used approaches for model blending: ELT/YELT blending and OEP blending.

4.1 ELT/YELT Blending

When the YELT results are provided, we can blend them together by following the steps below. For the sake of simplicity, we assume that the RMS (ELT) and AIR (YELT) loss results are provided and the blending weights to be used are ω RMS and (1-ω) AIR, which can be easily generalized to other cases if necessary.

1. Convert RMS ELT to YELT format using Monte Carlo simulation described in section 3.2.
2. Sample from a uniform distribution. For a given year, if the sampled value is less than ω, take the losses from the RMS YELT, otherwise take the losses from the AIR YELT.
3. Repeat the above for year 1 to year 10k to create a 10k blended YELT.

This process is illustrated in Table 12 for a 50/50 weighting. In terms of the OEPs of the component models, the theoretical OEP derived from blending the simulations is

Table 12: 50/50 Blending of Models Using AIR-Style Table 1 and RMS-Simulation Table 10

Trial	Model Uniform	Model Selected	Event Count	Loss
1	0.599	AIR	1	100
2	0.041	RMS	0	–
3	0.401	RMS	2	168
3	–	–	–	268
4	0.925	AIR	1	100

$$O_{\text{mix}}(x) = \omega O_{\text{rms}}(x) + (1-\omega)O_{\text{air}}(x), \tag{43}$$

where ω is the weight given to the RMS model. The advantage of this approach is that it produces a blended set of results comprised of specific modeled events, it is simple to implement, and it can be used to model dependencies with other portfolio results as long as the same uniform draw and technique is used to blend the other portfolio results. Its disadvantage is that the blended results with this approach could be different from the blended OEPs that are usually presented to the management teams, regulatory bodies and rating agencies.

4.2 OEP Blending

In practice, the modeling results are often presented to various interested parties as summaries like the OEPs instead of the underlying ELTs or YELTs. The weighting factors are usually directly applied to the dollar amounts x for a fixed return period r or exceedance probability $1/r$. Table 13 shows a 50/50 blend of RMS and AIR results derived from Tables 1 and 10. The Table 13 AIR PML for return period 2 was interpolated from Table 6 and

Table 13: 50/50 OEP Blending Using AIR-OEP Table 6 and RMS-Simulation Table 10

Return Period	AIR PML	RMS PML	50/50 PML
1.33	0	0	0
2	50	204	127
4	100	268	186
∞	500	272	384

the RMS PML column was constructed from Table 10 RMS simulated losses.

The theoretical OEP for this approach, in terms of the occurrence PMLs, is

$$\text{PML}_{\text{mix}}(r) = \omega \text{PML}_{\text{rms}}(r) + (1-\omega)\text{PML}_{\text{air}}(r) \quad \text{or} \tag{44}$$

$$O_{\text{blend}}(\text{PML}_{\text{mix}}) = 1/r. \tag{45}$$

This is not equivalent to ELT/YELT Blending. The ELT/YELT blending essentially weights probabilities at fixed amounts while the OEP blending weights dollars amounts at fixed probabilties. A better name for OEP blending might be *PML blending*.

The OEP blending is certainly very intuitive and it has become a common practice to present results this way. However, this only provides a high level summary of blended results. For some calculations, actuaries need the underlying loss details by event. The OEP conversion technique introduced in section 2.1 can be applied to the blended OEPs to produce claim count and severity distributions that can be used in simulation models and will yield the "blended" OEP curve.

5 Conclusion

It is helpful to understand the various terms used by consumers of catastrophe modeling and their relationship with the traditional claim count/severity collective risk model (CRM).

In particular, one can avoid common areas of confusion:

1. *The OEP and AEP are not the same.*

   The OEP and AEP keep track of different random variables, respectively, the largest event each year versus the total of each year's events.

2. *The probable maximum loss (PML) can be associated with the OEP or the AEP.*

   PML is often used informally and usually refers to the dollar amount x associated with a particular return period r or exceedance probability $1/r$, that is, the inverse OEP function,

$$\text{PML}(r) = x = O^{-1}(1/r). \tag{46}$$

Sometimes PML refers to the AEP, where it might be called the *aggregate* PML, and it becomes the inverse AEP function (A^{-1}),

$$\text{PML}(r) = x = A^{-1}(1/r). \tag{47}$$

3. *Blending OEPs is not the same as blending simulated results.*

   It is a relatively simple experiment to take two cat models and compare a 50/50 weighting of their Occurrence PML curves (OEP blending) with the OEP curve produced by simulating from one model half the time and the other model the rest of the time (ELT/YELT Blending). These are not the same. The former weights dollar amounts for a fixed exceedance probability or return period, while the latter weights probabilities for a fixed dollar amount. It is an unfortunate use of language that OEP blending actually refers to weighting PMLs or dollar amounts while ELT/YELT blending actually refers to weighting probabilities (or OEPs).

The OEP contains substantial information and it can be used to infer information about all events. Catastrophe modeling can be viewed in the context of a collective risk model (CRM) with a claim count distribution and a severity distribution. Understanding the connection between the OEP and an underlying CRM allows one to go back and forth between the two forms.

One can convert OEP output from a vendor model into claim count and severity distributions that can then be included in a capital model that uses claim count/severity inputs. Conversely, one can compute OEPs from a custom model, built from a traditional claim count/claim size approach, and compare the custom results with vendor models.

The trick is to understand what the components are and how they are different.

Acknowledgment

The authors would like to thank Bradley Andrekus, James Chang, Kara Kemsley, and Mohsen Rahnama for numerous thoughtful and helpful suggestions.

A When are the AEP and the OEP alike?

The AEP is generally not the same as the OEP. However, the two can be similar when the probability of 2 or more claim counts is very small. They are identical when the probability of 2 or more claim counts is zero. To see this, recall that for a collective risk model with size cdf $F_X(x)$ and count probabilities $P_N(n)$ the aggregate distribution for

$$Z = X_1 + ... + X_N \tag{48}$$

is

$$F_Z(x) = \sum_n P_N(n) F_X^{(n)}(x), \tag{49}$$

where $F_X^{(n)}(x)$ is the nth convolution of F_X with itself. Therefore, the AEP which is the probability of annual losses Z exceeding a given amount x is $1 - F_Z(x)$ or

$$A(x) = 1 - \sum_n P_N(n) F_X^{(n)}(x). \tag{50}$$

Compare this to the OEP

$$O(x) = 1 - \sum_n P_N(n)(F_X(x))^n. \tag{51}$$

When $P_N(n) = 0$ for $n > 1$ then $A(x) = O(x)$ because $F_X^{(1)} = F_X$. Similarly, $A(x) \approx O(x)$ if

$$A(x) - O(x) = \sum_{n=2}^{\infty} P_N(n)(F_X^{(n)}(x) - (F_X(x))^n) \tag{52}$$

is sufficiently small.

B OEP packing

Generally speaking, we need to add information about the claim count distribution to our knowledge about the OEP in order to compute a size distribution consistent with the OEP. However, if we can make two specific assumptions then we can compute a size distribution solely from the OEP.

1. There is no point mass at zero in the claim size distribution, $F_X(0) = 0$.
2. The claim count distribution is Poisson.

We can imagine all the claim count and severity information as packed into the OEP when these two conditions are met. The claims size distribution $F(x)$ is extracted as described in section 2.1.2 and the Poisson mean is extracted from $O(0)$. Recall from equation 22 for Poisson counts that

$$F(x) = 1 + \frac{\log(1 - O(x))}{\lambda}. \tag{53}$$

Adding that $F(0) = 0$ implies

$$\lambda = -\log(1 - O(0)). \tag{54}$$

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