CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES



October 27, 2011

# Exam 3, Segment 3L

## Life Contingencies and Statistics



**2.5 HOURS**

### INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2011 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

**CONTINUE TO NEXT PAGE OF INSTRUCTIONS**

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2011.

**END OF INSTRUCTIONS**

1.

You are given the following:

- For a group of lives ABC, $p_{30} = 0.8$
- For a second group of lives XYZ which is subject to an additional risk, the force of mortality is $\mu_x(t) + k$, where $\mu_x(t)$ is the force of mortality for group ABC and $k$ is a positive constant.
- The probability of someone age (30) dying within 1 year is 50% higher for group XYZ than it is for group ABC.

Calculate $k$.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20


EXAM CONTINUED ON NEXT PAGE

2.

You are given the following information:

$$F(x) = \frac{x^2}{81}, \; for \;\; 0 \le x \le 9$$

Calculate $\overset{o}{e}_{4:\overline{4}|}$, the expectation of life over the next four years for a life age four.

A. Less than 1.7
B. At least 1.7, but less than 2.0
C. At least 2.0, but less than 2.3
D. At least 2.3, but less than 2.6
E. At least 2.6


EXAM CONTINUED ON NEXT PAGE

3.

You are given the following information:

| $x$ | $q_x$ |
|---|---|
| 50 | 0.10 |
| 51 | 0.13 |
| 52 | 0.17 |
| 53 | 0.21 |
| 54 | 0.28 |
| 55 | 0.37 |

- All lives are independent.
- Deaths between integral ages follow a uniform distribution.

Calculate $_{0.5}q_{51.5:53.75}$

A. Less than 0.14
B. At least 0.14, but less than 0.16
C. At least 0.16, but less than 0.18
D. At least 0.18, but less than 0.20
E. At least 0.20


EXAM CONTINUED ON NEXT PAGE

4.

You are given the following information:

- ${}_{25}p_{15:40} = 0.25$
- ${}_{10}q_{15} = 0.05$
- Assume that individual lifetimes are independent.

Calculate ${}_{40}q_{25}$.

A. Less than 0.25
B. At least 0.25, but less than 0.45
C. At least 0.45, but less than 0.65
D. At least 0.65, but less than 0.85
E. At least 0.85


EXAM CONTINUED ON NEXT PAGE

5.

You are given the following information:

- $s(x) = e^{-0.05x}$, for x > 0
- $s(y) = e^{-0.02y}$, for y > 0
- (x) is aged 20
- (y) is aged 50
- Assume that the future lifetimes of (x) and (y) are independent.

Calculate $\overset{o}{e}_{\overline{20:50}}$, the complete life expectancy for the last survivor status.

A. Less than 52
B. At least 52, but less than 53
C. At least 53, but less than 54
D. At least 54, but less than 55
E. At least 55

6.

You are given the following information for a double-decrement model:

- $\mu_{30}^{(1)}(t) = \dfrac{3}{50+t}, \quad t \geq 0$
- $\mu_{30}^{(2)}(t) = \dfrac{1}{50+t}, \quad t \geq 0$
- T is the time-until-decrement random variable for (30).
- J is the cause-of-decrement random variable for (30).

Calculate $f_{T,J}(10,1)$, the joint p.d.f. of T and J evaluated at $t = 10$ and $j = 1$.

A. Less than 0.025
B. At least 0.025, but less than 0.050
C. At least 0.050, but less than 0.075
D. At least 0.075, but less than 0.100
E. At least 0.100


EXAM CONTINUED ON NEXT PAGE

7.

A four-year college program has 1000 new students and faces the following two decrements:

1. Academic failure
2. Withdrawal for all other reasons

The probabilities of decrements by year and cause are shown in the following table:

| $x$ | $q_x^{(1)}$ | $q_x^{(2)}$ |
|---|---|---|
| 0 | 0.10 | 0.20 |
| 1 | 0.08 | 0.15 |
| 2 | 0.05 | 0.10 |
| 3 | 0.00 | 0.05 |

Calculate the expected number of those who will withdraw for all reasons other than academic failure during the four-year program.

A. Less than 300
B. At least 300, but less than 325
C. At least 325, but less than 350
D. At least 350, but less than 375
E. At least 375


EXAM CONTINUED ON NEXT PAGE

8.

You are given the following information:

- An insurance company is established on January 1.
- The initial surplus is $2.
- On the 7th of every month a premium of $2 is collected.
- On the 14th of every month, the company pays a random claim amount, x, with a distribution as follows:

| x | Pr(x) |
|---|---|
| $1 | 0.40 |
| $2 | 0.20 |
| $3 | 0.40 |

- The company goes out of business if its surplus is $0 or less at any time.
- If the company's surplus is greater than $2 at the end of a given month, then the company pays a dividend to shareholders, which reduces the surplus back to $2.
- $i = 0$ (the interest rate is zero)

Calculate the probability that the company will be out of business on or before March 14.

A. Less than 10%
B. At least 10%, but less than 20%
C. At least 20%, but less than 30%
D. At least 30%, but less than 40%
E. At least 40%


EXAM CONTINUED ON NEXT PAGE

9.

A multiple-life Markov Process model has the following four states:

1. Both (x) and (y) are alive.
2. (x) is alive, but (y) has died.
3. (y) is alive, but (x) has died.
4. Both (x) and (y) have died.

You are given that (x) and (y) are dependent lives.

The symbol, ${}_kQ_n^{(i,j)}$, is the probability that (x) and (y) are in state j at time (n + k), given that they are in state i at time (n).

Select the expression that equals the probability ${}_3Q_2^{(1,4)}$.

A. ${}_{3|2}q_{\overline{x:y}}$

B. ${}_{3|2}q_{x:y}$

C. ${}_3q_{\overline{x+2:y+2}}$

D. ${}_3q_{x+2:y+2}$

E. ${}_3q_{x+2}\ {}_3q_{y+2}$

10.

You are given the following information:

- An insurance policy covers claims arising from two independent perils, Fire and Wind.
- Claim frequency for each peril follows a Poisson process.
- For Fire, the average number of claims reported each year is 700. Fire claim severities are independent and follow an exponential distribution with $\theta = 35{,}000$.
- For Wind, the average number of claims reported each year is 2. Wind claim severities are independent and follow an exponential distribution with $\theta = 200{,}000$.

A reinsurance contract provides coverage for any individual claim amount in excess of 250,000.

Calculate the variance of the annual number of claims covered by the reinsurance contract.

A. Less than 0.25
B. At least 0.25, but less than 0.50
C. At least 0.50, but less than 0.75
D. At least 0.75, but less than 1.00
E. At least 1.00


EXAM CONTINUED ON NEXT PAGE

11.

You are given the following information:

- Claims are reported according to a homogeneous Poisson process.
- Starting from time zero, the expected waiting time until the second claim is three hours.

Calculate the standard deviation of the waiting time until the second claim.

A. Less than 1.75
B. At least 1.75, but less than 2.00
C. At least 2.00, but less than 2.25
D. At least 2.25, but less than 2.50
E. At least 2.50

12.

You are given the following information:

- $T(x)$ is the future lifetime random variable.
- Mortality follows the Illustrative Life Table.
- $i = 6\%$

Calculate the probability that $\ddot{a}_{\overline{T(40)|}}$ will exceed $\ddot{a}_{40}$

A. Less than 0.35
B. At least 0.35, but less than 0.45
C. At least 0.45, but less than 0.55
D. At least 0.55, but less than 0.65
E. At least 0.65


EXAM CONTINUED ON NEXT PAGE

13.

You are given the following:

- Independent lives (30) and (40)
- Mortality follows the Illustrative Life Table for single lives and for the joint life status.
- $i = 6\%$

Calculate $A_{\overline{30:40}}$

A. Less than 0.060
B. At least 0.060, but less than 0.065
C. At least 0.065, but less than 0.070
D. At least 0.070, but less than 0.075
E. At least 0.075


EXAM CONTINUED ON NEXT PAGE

14.

You are given the following information about a fully continuous whole life policy:

- Mortality follows De Moivre's law with $\omega = 100$
- $\delta = 0.04$
- (x) is age 20.

Calculate the variance of the reserves at $t = 5$.

A. Less than 0.12
B. At least 0.12, but less than 0.13
C. At least 0.13, but less than 0.14
D. At least 0.14, but less than 0.15
E. At least 0.15

15.

You are given the following information:

- $\ddot{a}_x = 14.81660$
- $A_{x+k} = 0.24905$
- $i = 6\%$

Calculate ${}_kV_x$, the benefit reserve at time t = k.

A. Less than 0.10
B. At least 0.10, but less than 0.11
C. At least 0.11, but less than 0.12
D. At least 0.12, but less than 0.13
E. At least 0.13


EXAM CONTINUED ON NEXT PAGE

16.

You are given the following:

- A company's degree of profitability transitions according to the following matrix:

$$Q = \begin{vmatrix} 0.75 & 0.22 & 0.03 \\ 0.05 & 0.90 & 0.05 \\ 0.00 & 0.25 & 0.75 \end{vmatrix}$$

- State 0 = Very Profitable
- State 1 = Profitable
- State 2 = No Profits

The profitability status for the year is determined at year end, just before the dividend is paid.

- A dividend of \$5 is paid for State 0, Very Profitable for the year
- A dividend of \$2 is paid for State 1, Profitable for the year
- No dividend is paid for State 2, No Profits for the year
- A dividend of \$5 was paid on December 31, 2010, which is the starting point.
- $i = 4\%$

Calculate the actuarial present value of the dividend payment to be made on December 31, 2012.

A. Less than 3.30
B. At least 3.30, but less than 3.40
C. At least 3.40, but less than 3.50
D. At least 3.50, but less than 3.60
E. At least 3.60


EXAM CONTINUED ON NEXT PAGE

17.

You are given the following:

- There are three available estimators for $\theta$ : X, Y, and Z.
- $\theta$ is a parameter in a probability distribution function.
- The variance of the estimators are
  - Var(X) = 1.9;
  - Var(Y) = 1.0;
  - Var(Z) = 2.1;
- The difference between the expected value of the estimator and the true parameter are
  - Bias(X, $\theta$) = 0.5
  - Bias(Y, $\theta$) = -1.0
  - Bias(Z, $\theta$) = 0.0
- You wish to rank the estimators in order to minimize the mean square error (MSE).
- Hint: MSE(X) = E[(X-$\theta$)^2]

Choose the ranking of MSE(X), MSE(Y), and MSE(Z) from lowest to highest.

A. MSE(X) < MSE(Y) < MSE(Z)
B. MSE(X) < MSE(Z) < MSE(Y)
C. MSE(Y) < MSE(Z) < MSE(X)
D. MSE(Z) < MSE(Y) < MSE(X)
E. MSE(Z) < MSE(X) < MSE(Y)


EXAM CONTINUED ON NEXT PAGE

18.

You are given the following information:

- Mortality follows the survival function $S(x) = (1-\frac{x}{90})^k$ , $0 \leq x \leq 90$ , $k > 0$

- For a sample size of two, the deaths are recorded as one at age 10 and one at age 50.

Calculate the maximum likelihood estimate of k.

A. Less than 1.0
B. At least 1.0, but less than 1.5
C. At least 1.5, but less than 2.0
D. At least 2.0, but less than 2.5
E. At least 2.5

19.

You are given the following five observations:

2.3 3.3 1.2 4.5 0.7

A uniform distribution on the interval [a,b] is fit to these observations using maximum likelihood estimation.

This produces parameter estimates $\hat{a}$ and $\hat{b}$.

Calculate $\hat{b} - \hat{a}$.

A. Less than 4.0
B. At least 4.0, but less than 4.2
C. At least 4.2, but less than 4.4
D. At least 4.4, but less than 4.6
E. At least 4.6

20.

You are given the following hypothesis test:

- A random variable has a normal distribution with a known variance of 9.
- 10 observations are drawn from this distribution.
- $H_0$: $\mu \leq 50$
- $H_A$: $\mu > 50$
- The significance level of the test is $\alpha = 0.05$.

When $\mu = 52$, calculate the probability of a Type II error.

A. Less than 31%
B. At least 31%, but less than 32%
C. At least 32%, but less than 33%
D. At least 33%, but less than 34%
E. At least 34%


EXAM CONTINUED ON NEXT PAGE

21.

You are given the following information:

- A sample of 1,000 independent lives at age 65.
- $H_0$: Mortality follows the Illustrative Life Table.
- $H_1$: Mortality does not follow the Illustrative Life Table.
- You reject the null hypothesis if more than 310 of the 1,000 lives die within the next 10 years.

Using a normal approximation with no continuity correction, calculate the probability of a Type I error.

A. Less than 1%
B. At least 1%, but less than 2%
C. At least 2%, but less than 3%
D. At least 3%, but less than 4%
E. At least 4%


EXAM CONTINUED ON NEXT PAGE

22.

You are given the following information:

- There are two types of claims reported for a given line of business: Class 1 and Class 2.
- $H_0$: Probability of a claim being reported under Class 1 is 25%
- $H_1$: Probability of a claim being reported under Class 1 is not equal to 25%
- Sample size is 1000 claims.
- Significance level is 1% for a two tailed test.

Calculate the maximum number of claims reported for Class 1 at which one would accept the null hypothesis.

A. Less than 290
B. At least 290, but less than 295
C. At least 295, but less than 300
D. At least 300, but less than 305
E. At least 305


EXAM CONTINUED ON NEXT PAGE

23.

Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of a random sample of size n = 5 from a Pareto distribution with p.d.f.: $f(x) = \dfrac{18}{(x+3)^3}$. $(\alpha = 2, \theta = 3)$

Calculate $\Pr(Y_4 < 4)$.

A. Less than 0.65
B. At least 0.65, but less than 0.70
C. At least 0.70, but less than 0.75
D. At least 0.75, but less than 0.80
E. At least 0.80


EXAM CONTINUED ON NEXT PAGE

24.

Wait times between calls at a claims office are exponentially distributed with mean 10 minutes.

Five wait times will be recorded.

Calculate the expected value of the shortest wait time.

A. Less than 1 minute
B. At least 1 minute, but less than 4 minutes
C. At least 4 minutes, but less than 7 minutes
D. At least 7 minutes, but less than 10 minutes
E. At least 10 minutes


EXAM CONTINUED ON NEXT PAGE

25.

You are modeling your client's annual revenue as a function of time using the linear model:

- $y = \alpha + \beta x$
- y represents revenue (millions of dollars)
- x represents the number of years since the company started doing business

You have calculated the following statistics:

$n = 12$

$\sum x_i = 78$ $\quad\quad$ $\sum y_i = 324$

$\sum (x_i - \bar{x})^2 = 143$ $\quad\quad$ $\sum (y_i - \bar{y})^2 = 1839$

$\sum (y_i / x_i) = 56$ $\quad\quad$ $\sum (y_i - \bar{y})(x_i - \bar{x}) = 503$

Calculate the least squares estimate of $\beta$.

A. Less than 1.00
B. At least 1.00, but less than 3.00
C. At least 3.00, but less than 5.00
D. At least 5.00, but less than 7.00
E. At least 7.00



END OF EXAMINATION

| Question # | Answer |
|---|---|
| 1 | C |
| 2 | E |
| 3 | D |
| 4 | D |
| 5 | E |
| 6 | A |
| 7 | E |
| 8 | C |
| 9 | C |
| 10 | E |
| 11 | C |
| 12 | E |
| 13 | C |
| 14 | C |
| 15 | B |
| 16 | B |
| 17 | C |
| 18 | D |
| 19 | A |
| 20 | C |
| 21 | D |
| 22 | A |
| 23 | D |
| 24 | B |
| 25 | C |