**CASUALTY ACTUARIAL SOCIETY**
AND THE
**CANADIAN INSTITUTE OF ACTUARIES**




May 6, 2011

# Exam 3, Segment 3L

**Life Contingencies and Statistics**



**2.5 HOURS**

## INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2011 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

**CONTINUE TO NEXT PAGE OF INSTRUCTIONS**

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 23, 2011.

**END OF INSTRUCTIONS**

# Exam 3L, Spring 2011

1.

You are given:

- The force of mortality, $\mu = 0.10$

Calculate the variance of $T(x)$, the future lifetime of $x$.

A. Less than 70
B. At least 70, but less than 90
C. At least 90, but less than 110
D. At least 110, but less than 130
E. At least 130

CONTINUED ON NEXT PAGE

2.

Given that $s(x) = \left(1 - \frac{x}{10}\right)^{\frac{2}{3}}$ for $0 \le x \le 10$.

Calculate the probability that a life aged (1.5) survives at least six time units.

A. less than 0.441
B. at least 0.441 but less than 0.442
C. at least 0.442 but less than 0.443
D. at least 0.443 but less than 0.444
E. at least 0.444

CONTINUED ON NEXT PAGE

3.

You are given the following information:

- Deaths are uniformly distributed between integer ages.
- $q_{50} = 0.10$
- $q_{51} = 0.20$

Calculate $\overset{o}{e}_{50:\overline{1.25}|}$, the temporary 1.25-year complete expectation of life of a person aged 50.

A. Less than 1.09
B. At least 1.09, but less than 1.12
C. At least 1.12, but less than 1.15
D. At least 1.15, but less than 1.18
E. At least 1.18

CONTINUED ON NEXT PAGE

4.

You are given the following information about the lifetimes of two machines, x and y:

- $\mu_x = \ln(1.05)$, for $x > 0$
- $\mu_y = \dfrac{1}{20-y}$, for $0 < y < 20$
- Survival functions for the two machines are independent.
- Both machines are currently age 2.

Calculate the probability that the first failure occurs when the machines are between ages 4 and 9.

A. Less than 0.32
B. At least 0.32, but less than 0.34
C. At least 0.34, but less than 0.36
D. At least 0.36, but less than 0.38
E. At least 0.38

CONTINUED ON NEXT PAGE

5.

You are given:

- ${}_5p_x = 0.9$
- ${}_5p_y = 0.8$
- $q_{x+5} = 0.2$
- $q_{y+5} = 0.3$

Calculate the probability that exactly one of two independent lives (x) and (y) will die in year six.

A. Less than 0.25
B. At least 0.25, but less than 0.30
C. At least 0.30, but less than 0.35
D. At least 0.35, but less than 0.40
E. At least 0.40

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6

You are given that 1,000 five-year bonds, issued at time 0, face the following three decrements:

1. Default
2. Call (i.e. pre-payment)
3. Maturity

The probabilities of decrement by year and cause are shown in the following table:

| **Year** k | **Default** $q_k^{(1)}$ | **Call** $q_k^{(2)}$ | **Maturity** $q_k^{(3)}$ |
|---|---|---|---|
| 0 | 0.02 | 0.03 | 0.00 |
| 1 | 0.02 | 0.04 | 0.00 |
| 2 | 0.02 | 0.05 | 0.00 |
| 3 | 0.02 | 0.06 | 0.00 |
| 4 | 0.02 | 0.00 | 0.98 |

Calculate ${}_4d_1^{(1)}$.

A. Less than 70
B. At least 70, but less than 75
C. At least 75, but less than 80
D. At least 80, but less than 85
E. At least 85

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7.

You are given the following information:

- A multiple decrement model has an infinite number of distinct causes of decrement.
- ${}_t p_x^{(\tau)} = 0.8^t$, for t = 1, 2, 3, ... , ∞ and for x = 0, 1, 2, 3, ..., ∞
- $q_x^{(j+1)} = 0.9\, q_x^{(j)}$, for j = 1, 2, 3, ..., ∞ and for x = 0, 1, 2, 3, ..., ∞

Calculate the probability that (x) eventually fails due to decrement 3.

A. Less than 0.02
B. At least 0.02, but less than 0.05
C. At least 0.05, but less than 0.08
D. At least 0.08, but less than 0.11
E. At least 0.11

CONTINUED ON NEXT PAGE

8.

You are given the following information:

- Drivers transition between Standard, Preferred or Gold according to the following probability matrix:

$$Q = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}$$

- State 0 = Standard
- State 1 = Preferred
- State 2 = Gold
- Driver A is classified as Preferred at t=0 (time 0)

Calculate the probability that Driver A will transition from Preferred to Gold between t=2 and t=3.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20

CONTINUED ON NEXT PAGE

9.

You are given the following information:

- Buses depart from the bus stop at a Poisson rate of six per hour.
- Today, Jim arrives at the bus stop just as a bus is departing, and will have to wait for the next bus to depart.
- Yesterday, Jim arrived at the bus stop six minutes after the prior bus departed and had to wait for the subsequent bus to depart.

Calculate how much longer Jim's expected wait time is today compared to yesterday.

A. Less than 1 minute
B. At least 1 minute, but less than 2 minutes
C. At least 2 minutes, but less than 3 minutes
D. At least 3 minutes, but less than 4 minutes
E. At least 4 minutes

CONTINUED ON NEXT PAGE

10.

You are given the following information about an insurance policy:

- Claim frequency follows a homogeneous Poisson process.
- The average number of claims reported each month is 10.
- Claim severities are independent and follow an exponential distribution with $\theta = 10{,}000$.
- To monitor the impact of large individual claims on the policy aggregate losses, management receives a large loss report any time a claim occurs that is greater than 30,000.

Calculate the standard deviation of the waiting time (in months) until the second large loss report.

A. Less than 1
B. At least 1, but less than 3
C. At least 3, but less than 6
D. At least 6, but less than 12
E. At least 12

CONTINUED ON NEXT PAGE

11.

For a collection of insured vehicles, windshield cracks are repaired at a Poisson rate of 150 per month.

Windshield crack repairs fall into two categories:

- 90% of the cracks are minor and cost $100 to repair
- 10% of the cracks are major and cost $1,100 to repair

Using the normal approximation, calculate the probability that total windshield crack repair cost in one month is more than $40,000.

A. Less than 0.9%
B. At least 0.9%, but less than 1.0%
C. At least 1.0%, but less than 1.1%
D. At least 1.1%, but less than 1.2%
E. At least 1.2%

CONTINUED ON NEXT PAGE

12.

You are given the following information:

- $a_{x:\overline{2|}} = 0.71$
- ${}_{k|}q_x = (0.5)^{k+1}$ for k = 0, 1, 2, …

Calculate $\ddot{a}_{x:\overline{4|}}$

A. Less than 1.70
B. At least 1.70, but less than 1.75
C. At least 1.75, but less than 1.80
D. At least 1.80, but less than 1.85
E. At least 1.85

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13.

For a fully discrete life insurance policy on two lives aged 50, you are given:

- A benefit of 1,000 is payable at the end of the year of death of the last survivor of the two lives.
- The future lifetimes of the two lives are independent and identically distributed.
- Mortality for both lives follows the Illustrative Life Table.
- $i = 0.06$
- A premium of P is payable at the beginning of each year if both lives are still alive.
- A premium of 0.5P is payable at the beginning of each year if only one of the lives is alive.

Calculate P using the equivalence principle.

A. Less than 12
B. At least 12, but less than 14
C. At least 14, but less than 16
D. At least 16, but less than 18
E. At least 18

CONTINUED ON NEXT PAGE

14.

You are given the following information:

- $P_{40:\overline{30|}} = 0.0161$
- $A^{\,1}_{55:\overline{5|}} = 0.0438$
- $A_{55:\overset{\,1}{\overline{5|}}} = 0.7081$
- $A^{\,1}_{60:\overline{10|}} = 0.1368$
- $A_{60:\overset{\,1}{\overline{10|}}} = 0.4512$
- $\ddot{a}_{55:\overline{5|}} = 4.3837$
- $\ddot{a}_{60:\overline{10|}} = 7.2790$

Calculate ${}_{15}V_{40:\overline{30|}}$.

A. Less than 0.20
B. At least 0.20, but less than 0.40
C. At least 0.40, but less than 0.60
D. At least 0.60, but less than 0.80
E. At least 0.80

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15.

You are given:

- ${}_nV_x = 0.645$
- $P_x = 0.095$
- $P^{1}_{x:\overline{n}|} = 0.085$

Calculate $P_{x:\overset{\;\,1}{\overline{n}|}}$

A. Less than 0.015
B. At least 0.015, but less than 0.016
C. At least 0.016, but less than 0.017
D. At least 0.017, but less than 0.018
E. At least 0.018

CONTINUED ON NEXT PAGE

16.

You are given the following information about a Markov process with two states, state 1 and state 2:

- The transition probability matrix: $\begin{bmatrix} 0.7 & 0.3 \\ 0.8 & 0.2 \end{bmatrix}$

- Transitions occur at the end of the time period.

- The subject starts in state 1.

- There is a cash flow of 10 at the end of the period for each transition from state 1 to state 2 during the next 3 time periods.

- A premium P will be paid at the beginning of each of the next 3 time periods provided the subject is in state 2 at that time.

- The interest rate is 5%.

Calculate P using the equivalence principle.

A. Less than 8.0
B. At least 8.0, but less than 10.0
C. At least 10.0, but less than 12.0
D. At least 12.0, but less than 14.0
E. At least 14.0

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17.

You are given the following information:

- A random variable X is uniformly distributed on the interval $(0, \theta)$.
- $\theta$ is unknown.
- For a random sample of size n, an estimate of $\theta$ is $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.
- The cumulative distribution function. of $Y_n$ is

$$F_{Y_n}(t) = \begin{cases} 1 & t > \theta \\ (t/\theta)^n & 0 < t \leq \theta \\ 0 & t \leq 0. \end{cases}$$

Which of the following is a consistent estimator of $\theta$?

A. $Y_n$
B. $Y_n /(n-1)$
C. $Y_n *(n+1)$
D. $Y_n *(n+1)*(n-1)$
E. $Y_n /(n+1)$

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18.

You are given the following information:

- A distribution has density function:

  $f(x) = (\theta + 1)(1 - x)^{\theta}$ for $0 < x < 1$

- You observe the following four values from this distribution:

  0.05 0.10 0.20 0.50

Calculate the maximum likelihood estimate of the parameter $\theta$.

A. Less than 0.5
B. At least 0.5, but less than 1.5
C. At least 1.5, but less than 2.5
D. At least 2.5, but less than 3.5
E. At least 3.5

CONTINUED ON NEXT PAGE

19.

You are given the following five observations from a single-parameter Pareto distribution:

125 250 300 425 500

The value of the mode, $\theta$, is known in advance to be 100.

Calculate the method of moments estimate for the parameter $\alpha$.

A. Less than 1.1
B. At least 1.1, but less than 1.2
C. At least 1.2, but less than 1.3
D. At least 1.3, but less than 1.4
E. At least 1.4

CONTINUED ON NEXT PAGE

20.

You are testing the hypothesis that the severity of a hurricane (as measured by Peak Hurricane Category) is independent of its ocean of origin.

You are given the following sample of historical hurricane counts originating from each ocean from the same time period:

| Peak Hurricane Category | Ocean A | Ocean B |
|---|---|---|
| 1 | 26 | 48 |
| 2 | 14 | 27 |
| 3 | 13 | 7 |
| Greater than 3 | 7 | 8 |

Using the chi-squared test statistic, calculate the minimum significance level at which you would reject the hypothesis that hurricane category and ocean of origin are independent.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.025
D. At least 0.025, but less than 0.050
E. At least 0.050

CONTINUED ON NEXT PAGE

21.

You are given the following information:

- The Underwriting Department tells you that 35% of homeowners claiming to have alarm systems have canceled their alarm service.
- A survey of 550 homeowners claiming to have alarm systems shows that 213 have canceled their service.
- You wish to test the following hypotheses:

  $H_0$: The probability of cancellation is 35%.
  $H_1$: The probability of cancellation is not 35%.

Calculate the minimum significance level at which you would reject the null hypothesis.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.025
D. At least 0.025, but less than 0.050
E. At least 0.050

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22.

You are given the following information:

- A gambler at a casino believes his probability of winning a game of chance is greater than the probability projected by the casino management, which is 9/19.
- The gambler will make 1,000 consecutive bets and will either win \$10 or lose \$10 on each bet.
- $H_0$: The gambler's probability of winning is 9/19.
- $H_1$: The gambler's probability of winning is greater than 9/19.
- The null hypothesis is rejected if the gambler's net winnings exceed k.

Calculate the lowest value of k at which the probability of a Type I Error is less than 1.0%.

A. Less than \$140
B. At least \$140, but less than \$170
C. At least \$170, but less than \$200
D. At least \$200, but less than \$230
E. At least \$230

CONTINUED ON NEXT PAGE

23.

You are given the following information:

- A random variable X follows a geometric distribution with parameter $\beta$.
- $H_0$: $\beta = 1.5$
- $H_1$: $\beta = 0.5$
- You observe one value of X and reject the null hypothesis if the observed value is equal to 0 or 1.

Calculate the power (i.e. 1 – P(Type II Error)) of this test.

A. Less than 20%
B. At least 20% , but less than 40%
C. At least 40%, but less than 60%
D. At least 60%, but less than 80%
E. At least 80%

CONTINUED ON NEXT PAGE

24.

You are given the following information:

- An actuary suspects that yellow vehicles are driven at higher average speeds than vehicles that are not yellow.
- Speeds for vehicles that are not yellow are normally distributed with mean 64 and standard deviation 6 based on a sample size of 1000 vehicles.
- Speeds for vehicles which are yellow are normally distributed with unknown mean $\mu$ and standard deviation 6.
- A random sample of seven yellow vehicle speeds:

  61 63 63 64 68 80 81

- $H_0$: $\mu = 64$, the mean speed for yellow vehicles equals the mean speed for non yellow vehicles.
- $H_1$: $\mu > 64$, the mean speed for yellow vehicles is greater than the mean speed for non yellow vehicles.

Calculate the minimum significance level at which the actuary would reject the null hypothesis.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.025
D. At least 0.025, but less than 0.050
E. At least 0.050

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25.

You are given the following information:

- Three observations from a uniform distribution on [0,1]
- The order statistics of this sample are $Y_1$, $Y_2$, $Y_3$

Calculate the probability that $Y_2 < 0.5$.

A. Less than 0.25
B. At least 0.25, but less than 0.35
C. At least 0.35, but less than 0.45
D. At least 0.45, but less than 0.55
E. At least 0.55

END OF EXAMINATION

# Exam 3L Answers

| Exam Question Number | Answer |
|---|---|
| 1 | C |
| 2 | C |
| 3 | D |
| 4 | D |
| 5 | C |
| 6 | A |
| 7 | D |
| 8 | B |
| 9 | A |
| 10 | B |
| 11 | D |
| 12 | D |
| 13 | A |
| 14 | B |
| 15 | B |
| 16 | D |
| 17 | A |
| 18 | D |
| 19 | E |
| 20 | E |
| 21 | E |
| 22 | D |
| 23 | E |
| 24 | C |
| 25 | D |