CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

May 11, 2012

Exam 3, Segment 3L

Life Contingencies and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2012 and that the exam number is 3L.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3L" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 28, 2012.

END OF INSTRUCTIONS

1.

You are given the following information:

- $f_X(x) = \dfrac{x^2}{3c}$, for $0 < x \leq c$

Calculate $\mu(2.3)$.

A. Less than 1.05
B. At least 1.05, but less than 1.15
C. At least 1.15, but less than 1.25
D. At least 1.25, but less than 1.35
E. At least 1.35

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2.

You are given the following information:

- $p_k = 0.8$, for $k \leq 7$
- $p_k = 0.7$, for $7 < k \leq 8$
- $e_9 = 1.5$

Calculate e_6.

A. Less than 2.40
B. At least 2.40, but less than 2.50
C. At least 2.50, but less than 2.60
D. At least 2.60, but less than 2.70
E. At least 2.70

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3.

You are given the following information:

- Mortality follows the Illustrative Life Table.
- Assume the Uniform Distribution of Deaths between integer ages.

Calculate the median future lifetime for a life aged 24.5 years.

A. Less than 52.50
B. At least 52.50, but less than 52.75
C. At least 52.75, but less than 53.00
D. At least 53.00, but less than 53.25
E. At least 53.25

CONTINUED ON NEXT PAGE

4.

You are given the following information:

- The future lifetimes of (30) and (50) are independent and identically distributed.
- The probability that the first death occurs more than 20 years from now is 0.30.
- ${}_{10}p_{60} = 0.40$

Calculate ${}_{30}p_{30}$.

A. Less than 0.605
B. At least 0.605, but less than 0.655
C. At least 0.655, but less than 0.705
D. At least 0.705, but less than 0.755
E. At least 0.755

CONTINUED ON NEXT PAGE

5.

You are given the following information:

- Individual X is 50 years old and has a family history of cancer.
- Individual Y is 60 years old and has no family history of cancer.
- Mortality for individuals with no family history of cancer follows De Moivre's law with $\omega = 120$.
- An individual with a family history of cancer is subject to a force of mortality equal to three times the force of mortality of an individual with no family history of cancer.
- The future lifetimes of X and Y are independent.

Calculate the probability that both individuals X and Y live more than three years and at least one dies within the following five years.

A. Less than 0.232
B. At least 0.232, but less than 0.242
C. At least 0.242, but less than 0.252
D. At least 0.252, but less than 0.262
E. At least 0.262

CONTINUED ON NEXT PAGE

6.

You are given the following information for a double decrement model:

- $\mu_x^{(1)}(t) = \dfrac{1}{5}, \quad t \geq 0$
- $\mu_x^{(2)}(t) = \dfrac{1}{20}, \quad t \geq 0$
- T is the time-until-decrement random variable for (x).
- J is the cause-of-decrement random variable for (x).

Calculate E [T | J = 1], the expected time to failure from decrement 1..

A. Less than 3.5
B. At least 3.5, but less than 6.0
C. At least 6.0, but less than 8.5
D. At least 8.5, but less than 11.0
E. At least 11.0

CONTINUED ON NEXT PAGE

7.

You are given the following double decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$p_x^{(\tau)}$	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
61	0.015		0.935	100,000		
62	0.016		0.944			
63		0.030			1,589	
64		0.020	0.960		1,681	

Calculate ${}_{2|2}q_{61}^{(2)}$.

A. Less than 0.040
B. At least 0.040, but less than 0.042
C. At least 0.042, but less than 0.044
D. At least 0.044, but less than 0.046
E. At least 0.046

CONTINUED ON NEXT PAGE

8.

You are given the following information:

- Workers transition through the labor force according to the following matrix:

$$Q = \begin{vmatrix} 0.90 & 0.07 & 0.03 \\ 0.05 & 0.80 & 0.15 \\ 0.15 & 0.15 & 0.70 \end{vmatrix}$$

- State 0: Employed full-time
- State 1: Employed part-time
- State 2: Unemployed

- Individual Y is employed full-time at time t = 0.
- Individual Z is employed part-time at time t = 0.
- Individuals Y and Z transition through the labor force independently.

Calculate the probability that either Individual Y or Individual Z, but not both, will be unemployed at t = 2.

A. Less than 0.254
B. At least 0.254, but less than 0.256
C. At least 0.256, but less than 0.258
D. At least 0.258, but less than 0.260
E. At least 0.260

CONTINUED ON NEXT PAGE

9.

Claims reported for a group of policies follow a non-homogeneous Poisson process with rate function:

$\lambda(t) = 100/(1+t)^3$, where t is the time (in years) after January 1, 2011.

Calculate the expected number of claims reported after January 1, 2011 for this group of policies.

A. Less than 45
B. At least 45, but less than 55
C. At least 55, but less than 65
D. At least 65, but less than 75
E. At least 75

CONTINUED ON NEXT PAGE

10.

You are given the following information:

- An insurance policy has aggregate losses according to a compound Poisson distribution.
- Claim frequency follows a Poisson process.
- The average number of claims reported each year is 200.
- Claim severities are independent and follow an Exponential distribution with $\theta = 160{,}000$.

Management considers any claim that exceeds 1 million to be a catastrophe.

Calculate the median waiting time (in years) until the first catastrophe claim.

A. Less than 1
B. At least 1, but less than 2
C. At least 2, but less than 3
D. At least 3, but less than 4
E. At least 4

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11.

In a manufacturing company, work-related accidents occur at a constant Poisson rate of 10 per month. This company purchases a Workers Compensation policy from an insurance company.

The insurance company assumes that the payment for each claim follows an Exponential distribution and the average payment is 500 per claim.

Using the Normal approximation, calculate the probability that the insurance company will pay more than 70,000 within a year.

A. Less than 0.0960
B. At least 0.0960, but less than 0.0970
C. At least 0.0970, but less than 0.0980
D. At least 0.0980, but less than 0.0990
E. At least 0.0990

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12.

You are considering two survival models for calculating the benefit premium for a fully discrete whole life policy sold to a policyholder who is at age 0:

- Model F assumes ${}_{k|}q_0 = 0.5$ for k = 0, 1
- Model G assumes ${}_{k|}q_0 = (0.5)^{k+1}$ for k = 0, 1, …
- Assume i = 10%
- P_F is the benefit premium calculated under Model F assumptions.
- P_G is the benefit premium calculated under Model G assumptions.

Calculate the absolute value of P_F - P_G.

A. Less than 0.12
B. At least 0.12, but less than 0.13
C. At least 0.13, but less than 0.14
D. At least 0.14, but less than 0.15
E. At least 0.15

CONTINUED ON NEXT PAGE

13.

You are given the following information:

- A two-year term insurance policy is offered to a randomly chosen member of a population of 25-year-olds.
- Death benefit is 150,000 payable at the end of the year of death.
- The population mix is 25% are smokers and 75% are nonsmokers.
- The hazard rate functions are:

$$\mu_x^{(s)} = \frac{1}{75-x}, \text{ for } 0 \le x \le 75 \text{ for smokers}$$

$$\mu_x^{(n)} = \frac{1}{100-x}, \text{ for } 0 \le x \le 100 \text{ for nonsmokers}$$

- Interest rate $i = 0.06$

Calculate the actuarial present value of this insurance policy.

A. Less than 4,000
B. At least 4,000, but less than 4,200
C. At least 4,200, but less than 4,400
D. At least 4,400, but less than 4,600
E. At least 4,600

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14.

You are given the following information:

- $\ddot{a}_{110} = 1.3223$
- $P_{110} = 0.6997$
- $P_{90} = 0.2175$

Calculate ${}_{20}V_{90}$.

A. Less than 0.00
B. At least 0.00, but less than 0.25
C. At least 0.25, but less than 0.50
D. At least 0.50, but less than 0.75
E. At least 0.75

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15.

You are given the following information:

- The premium for a fully discrete whole life insurance policy for a life aged 27 is calculated using a benefit of 1,000,000 and the equivalence principle.
- The actual benefit that will be paid out is 980,000.
- Mortality follows the Illustrative Life Table.
- $i = 6\%$
- Let a be the nearest integer age where the probability that the benefits will have been paid out is 18%.
  That is ${}_{a-27}q_{27} = 18\%$.

Calculate ${}_{a-27}V_{27}$

A. Less than 320,000
B. At least 320,000, but less than 330,000
C. At least 330,000, but less than 340,000
D. At least 340,000, but less than 350,000
E. At least 350,000

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16.

You are given the following information:

- An entity can be in any of three states: State 1, State 2, or State 3.
- Transitions and cash flows occur at the end of each period.
- The cash flow when moving from State 1 to State 2 is 0.8.
- There are no other costs associated with transitions.
- There is a cash flow of -0.25 if the entity does not change states at the end of a period.
- $i = 10\%$
- The following matrices show the probabilities of moving between states at time $t = 1$ and $t = 2$.

$$Q_1 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.2 & 0.5 \\ 0.1 & 0.5 & 0.4 \end{bmatrix} \qquad Q_2 = \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.6 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

Calculate the expected present value of the cash flow for the entity starting in State 1 at $t = 1$ over the next two periods.

A. Less than 0.265
B. At least 0.265, but less than 0.270
C. At least 0.270, but less than 0.275
D. At least 0.275, but less than 0.280
E. At least 0.280

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17.

You are given the following information:

- Claim severity follows an Inverse Exponential distribution with parameter θ.
- One claim is observed, which is known to be between 50 and 500.

Calculate the maximum likelihood estimate of θ.

A. Less than 60
B. At least 60, but less than 90
C. At least 90, but less than 120
D. At least 120, but less than 150
E. At least 150

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18.

You are given a distribution with the following probability density function where α is unknown:

$$f(x;\alpha) = \left(1+\frac{1}{\alpha}\right)x^{\frac{1}{\alpha}},\ 0 < x < 1,\ \alpha > 0$$

You are also given a random sample of four observations:

0.2 0.5 0.6 0.8

Estimate α by the maximum likelihood method.

A. Less than 2.9
B. At least 2.9, but less than 3.0
C. At least 3.0, but less than 3.1
D. At least 3.1, but less than 3.2
E. At least 3.2

CONTINUED ON NEXT PAGE

19.

You are given the following information:

- The number of claims follows a Poisson distribution with mean λ.
- Observations other than 1 and 2 have been deleted from the data.
- In the remaining data set, 75% of the observations are of 1 and 25% are of 2.

Calculate the maximum likelihood estimate of λ.

A. Less than 0.15
B. At least 0.15, but less than 0.30
C. At least 0.30, but less than 0.45
D. At least 0.45, but less than 0.60
E. At least 0.60

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20.

You are given the following information:

- Acorns from two different species of oak trees, white and red, were collected by scientists and planted in an open field with the plots arranged to ensure independence between samples.
- The probability of an acorn germinating is p.
- 1000 acorns were planted for each species.
- For white oak (species 1), 600 acorns germinated.
- For red oak (species 2), 550 acorns germinated.
- $H_0 : p_1 = p_2$
- $H_1 : p_1 \neq p_2$

Calculate the significance level at which one could reject the null hypothesis.

A. Less than 1.0%
B. At least 1.0%, but less than 1.5%
C. At least 1.5%, but less than 2.0%
D. At least 2.0%, but less than 2.5%
E. At least 2.5%

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21.

You are given the following information:

- You are conducting a hypothesis test using three claims.
- Claim severity follows the uniform distribution on [0, k].
- H_0: k = 2,000
- H_1: k > 2,000
- The null hypothesis is rejected if the smallest of the three claims is greater than 1,250.

Calculate the probability of a Type I error.

A. Less than 2.5%
B. At least 2.5%, but less than 5.0%
C. At least 5.0%, but less than 7.5%
D. At least 7.5% but less than 10.0%
E. At least 10.0%

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22.

A company has introduced a policyholder education course and wishes to determine whether the course has any effect on loss ratios. The matched loss ratios prior to and following the education course are recorded for 5 policies:

Loss Ratio Prior to the Course	Loss Ratio Following the Course
0.62	0.35
0.41	0.44
0.35	0.17
0.81	0.70
0.50	0.45

The null hypothesis is that the education course has no impact (positive or negative) on the loss ratios.

Calculate the minimum significance level at which the null hypothesis can be rejected.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.05
D. At least 0.05, but less than 0.10
E. At least 0.10

CONTINUED ON NEXT PAGE

23.

You are given the following information about a random sample:

- $X_1,\ldots,X_n$ is a random sample where n =100 and the X_i are distributed $\text{N}(\theta,4)$
- $H_0 : \theta = 1$
- $H_1 : \theta \neq 1$
- In this problem, the likelihood ratio test is defined as the likelihood function for the null hypotheses divided by the likelihood function for the alternative hypotheses. The likelihood function in the denominator uses $\hat{\theta}$, an estimate of μ from the random sample, calculated as the maximum likelihood estimate for the normal distribution
- $\overline{X} = 2$
- The formula for the Normal Distribution is

$$\text{f(x)} = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right],\ \text{for } -\infty < x < \infty$$

Calculate the test statistic that is a function of the likelihood ratio defined above in terms of the Chi-Square distribution and select the range below that contains the significance level for this test.

Hint: $\left(\dfrac{\overline{X}-\theta}{\sigma/\sqrt{n}}\right)^2 \approx \chi^2(1)$

A. Less than .005
B. At least .005, but less than .01
C. At least .01, but less than .025
D. At least .025, but less than .05
E. At least .05

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24.

Let $Y_1 < Y_2 < \ldots < Y_5$ be the order statistics associated with five independent observations X_1, X_2, …, X_5 from a continuous distribution with the cumulative density function $F(x)$, where $0 < x < 1$.

The exact form of function $F(x)$ is unknown, but you are given

$$F(0.2) = 0.5$$
$$F(0.5) = 0.6$$

Calculate the probability that $Y_3 < 0.5$.

A. Less than 0.55
B. At least 0.55, but less than 0.60
C. At least 0.60, but less than 0.65
D. At least 0.65, but less than 0.70
E. At least 0.70

CONTINUED ON NEXT PAGE

25.

You are using the simple linear model:

$$Y_i = \alpha + \beta X_i + \epsilon_i$$

Where

- Y_i, X_i are the dependent and independent variables, respectively.
- The ϵ_i's are independent, identically distributed normal random variables with $E(\epsilon_i) = 0$.

The parameters are estimated by:

$$\hat{\beta} = \frac{\Sigma(X_i - \bar{X})(Y - \bar{Y})}{\Sigma(X_i - \bar{X})^2}$$

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

Which of the following characteristics apply to $\hat{\alpha}$ and $\hat{\beta}$?

I. Least-squares estimators
II. Maximum-likelihood estimators
III. Biased estimators

A. None
B. I and II
C. I and III
D. II and III
E. I, II and III

END OF EXAMINATION

Exam 3L

Answer Key

Question_Num	Answer
1	B
2	C
3	B
4	D
5	A
6	A or B
7	C
8	D
9	B
10	B
11	D
12	D
13	B
14	D
15	D
16	C
17	D
18	D
19	E
20	D
21	C
22	D
23	A
24	D
25	B