Exam 8

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

October 25, 2017

Exam 8

Advanced Ratemaking

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 53.75 point examination consists of 20 problem and essay questions.

2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors or correction fluid/tape.

   - Write your Candidate ID number and the examination number, 8, at the top of each answer sheet. For your Candidate ID number, four boxes are provided corresponding to one box for each digit in your Candidate ID number. If your Candidate ID number is fewer than 4 digits, begin in the first box and do not include leading zeroes. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER**. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
   - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have received the reference materials:

   a. National Council on Compensation Insurance, Experience Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2017 Study Kit*).
   b. Insurance Services Office, Inc., Commercial General Liability Experience and Schedule Rating Plan.
   c. National Council on Compensation Insurance, Retrospective Rating Plan Manual for Workers Compensation and Employers Liability Insurance (*Excerpt from 2017 Study Kit*).

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 8, 2017.

END OF INSTRUCTIONS

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1. (8.5 points)

An actuary has constructed a pure premium model using the Tweedie distribution with parameter $1 < p < 2$ to determine manual rates for a Workers' Compensation book of business. The output of the model is pure premium per $100 of payroll.

The following variables were considered for inclusion in the model:

Variable	Description or Source of Data	p-value
Industry	Construction, Manufacturing, All Other	0.002
Return to Work Program	Yes or No	0.008
Employee Age	Average Age in Years	0.003
Employee Tenure	Average Number of Years of Employment	0.005
Location	State of Jurisdiction	0.080
Employee Morale	Based on Results of Annual Company Survey	0.150
Number of Back Injuries	Supplied by Employer	0.010

The actuary has decided to use the following variables in the model: Industry, Employee Tenure, and Return to Work Program.

a. (1.5 point) Discuss the statistical and non-statistical considerations of including each of the three variables (Industry, Employee Tenure and Return to Work Program) in the model.

b. (2 points) Discuss the statistical and non-statistical considerations of excluding each of the remaining four variables (Employee Age, Location, Employee Morale and Number of Back Injuries) from the model.

c. (1.5 points) The actuary fits the log link GLM model using the three selected variables. Given the fitted model parameters below and the following information for a Manufacturing Workers' Compensation risk, calculate the standard premium for this risk for an annual policy period.

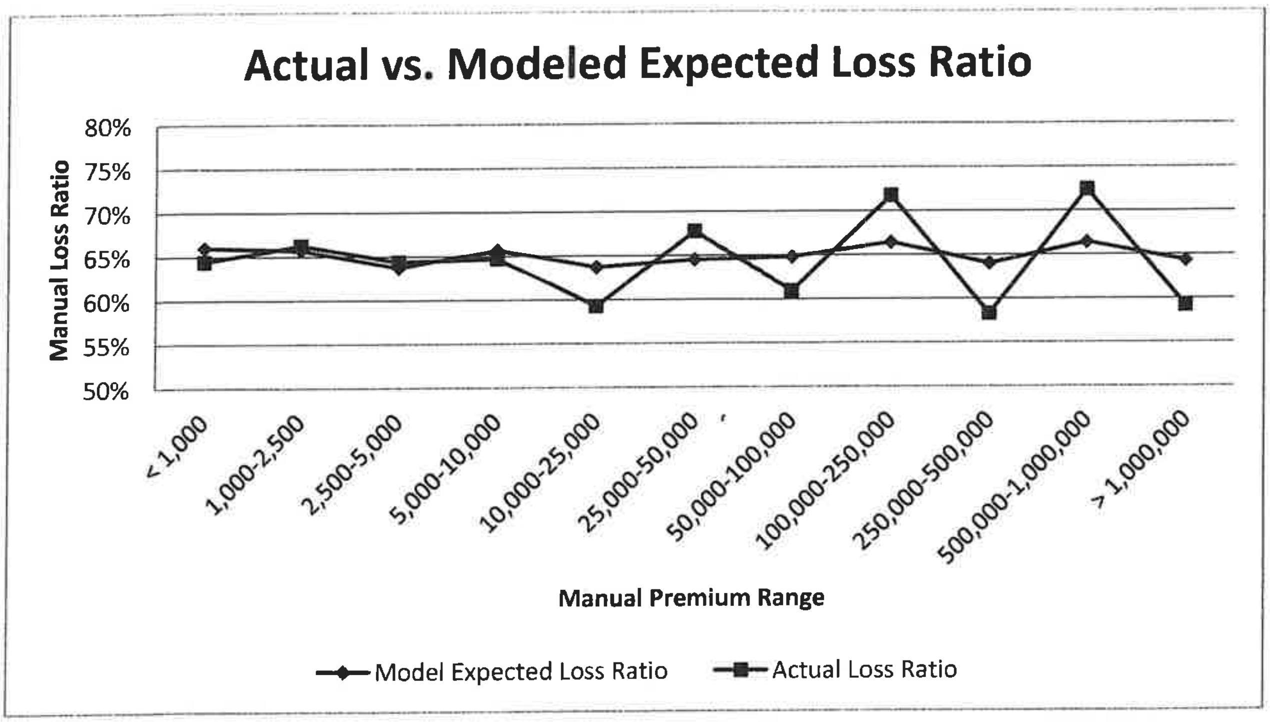
Parameter	Coefficient
Intercept	-0.631
Employee Tenure	-0.040
Return to Work Program: Yes	-0.200
Industry Type: Construction	0.350
Industry Type: All Other	-0.550

<<QUESTION 1 CONTINUED ON NEXT PAGE>>

Data for Manufacturing Risk	Value
Payroll	$1,000,000
Employee Tenure	5 Years
Return to Work Program	No
Actual Losses for Experience Rating	$12,500
Fixed Expenses	$1,500
Variable Expenses (as % of premium)	20%
Experience Rating Constant (K)	$10,000

d. (1.5 points) The actuary has graphed actual vs. modeled expected loss ratios by size of risk for the company's entire Workers' Compensation book, which is shown below.

In order to improve the model fit for larger risks, the actuary is considering incorporating the variable "Latest 3 Year Historical Losses" into the model. Explain three reasons against doing so.



e. (2 points) The actuary is now developing a quote for a new Construction risk with $50,000,000 of payroll using this rating plan.

i. (1 point) Describe two potential issues in developing a premium for this risk under this rating plan.

ii. (1 point) Provide an alternative rating approach for this risk and briefly discuss the advantages of this alternative for the insured as well as the insurance company.

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2. (2.0 points)

An actuary wants to cluster five Workers' Compensation classes based on excess ratios at two limits: 500,000 and 1,000,000. The actuary decides to use a weighted k-means algorithm with two clusters. Given the following:

Class	On-Leveled Earned Premium ($ Thousands)	Normalized Excess Ratio at 500,000 Limit	Normalized Excess Ratio at 1,000,000 Limit	Initial Cluster
1	6,500	0.240	0.080	A
2	5,000	0.350	0.200	A
3	4,000	0.210	0.080	A
4	3,000	0.110	0.030	A
5	5,000	0.180	0.070	B

- Distance will be measured using the L^2 (Euclidean) norm.
- At the start of the algorithm, the actuary randomly assigns each class to a cluster.

a. (1.75 points)

Determine the cluster for each class after the first iteration of the weighted k-means algorithm.

b. (0.25 point)

Briefly describe one advantage of using the L^1 measure rather than L^2 when computing clusters.

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3. (1.5 points)

The following data shows the experience of a merit rating plan for private passenger vehicles. The merit rating plan uses multiple rating variables, including territory.

Number of Accident-Free Years	Earned Car Years (000s)	Earned Premium ($000s)	Number of Incurred Claims
5 or More	250	500,000	15,000
3 and 4	100	90,000	13,500
1 and 2	80	60,000	8,000
0	70	50,000	10,500
Total	500	700,000	47,000

Territory	Frequency	Average Premium
A	0.05	1,500
B	0.10	2,000
C	0.15	1,250

a. (0.75 point)

Recommend and justify an exposure base for this merit rating plan.

b. (0.75 point)

Calculate the relative credibility of an exposure that has been three or more years accident-free using the exposure base from part (a) above.

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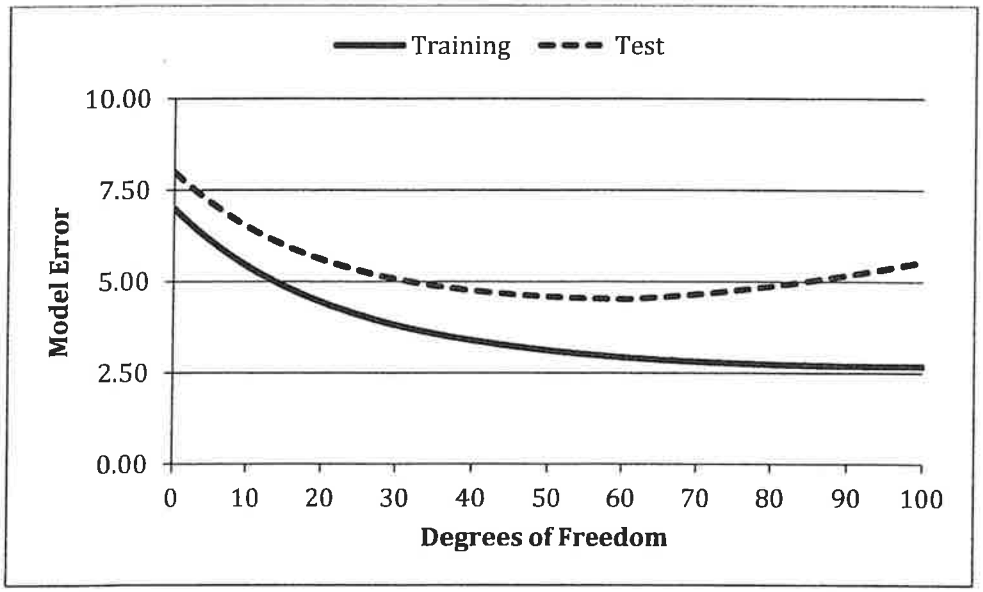
4. (1.75 points)

An actuary has split data into training and test groups for a model. The chart below shows the relationship between model performance and model complexity. Model performance is represented by model error and model complexity is represented by degrees of freedom.



a. (0.5 point)

Briefly describe two reasons for splitting modeling data into training and test groups.

b. (0.75 point)

Briefly describe whether each of the following model iterations has an optimal balance of complexity and performance.

i. Model iteration 1: 10 degrees of freedom
ii. Model iteration 2: 60 degrees of freedom
iii. Model iteration 3: 100 degrees of freedom

c. (0.5 points)

Identify and briefly describe one situation where it is an advantage to split the data by time rather than by random assignment.

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5. (1.75 points)

An analyst has fit several different variations of a logistic GLM to a dataset containing 1,000 records of fraudulent claims and 9,000 records of legitimate claims. For each model variation listed below, draw a quintile plot based on the training data. Label the axes and identify each data series.

i. A saturated model
ii. A null model
iii. A model that could be used in practice

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6. (3.5 points)

A logistic model was built to predict the probability of a claim being fraudulent. Consider the predicted probabilities for the 10 claims below to be a representative sample of the total model.

Claim Number	Actual Fraud Indicator	Predicted Probability of Fraud
1	Y	11%
2	N	23%
3	N	15%
4	N	70%
5	Y	91%
6	Y	30%
7	N	11%
8	Y	75%
9	N	58%
10	N	27%

a. (1 point)

Construct confusion matrices for discrimination thresholds of 0.50 and 0.25.

b. (1.5 points)

Plot the Receiver Operating Characteristic (ROC) curve with the discrimination thresholds of 0.50 and 0.25. Label each axis and the coordinates and discrimination threshold of each point on the curve.

c. (0.5 point)

Describe an advantage and a disadvantage of selecting a discrimination threshold of 0.25 instead of 0.50.

d. (0.5 point)

Describe whether a discrimination threshold of 0.25 or 0.50 is more appropriate for a line of business with low frequency and high severity.

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7. (1.75 points)

Given the following information:

- Allocated loss adjustment expense is 15% of the indemnity amount.
- The variance method has been selected to include a risk load in the Increased Limits Factors (ILFs) with $k = 0.000064$ and $\delta = 0$.

Limit, l	$E[X;l]$	$E[X^2:l]$
1,000	840	790,123
5,000	2,485	9,467,456

a. (1 point)

Calculate the following:

- The risk loads for each limit.
- The ILF with and without risk load for the 5,000 limit.

b. (0.75 point)

Assuming portfolio weights of 75% for a 1,000 limit and 25% for a 5,000 limit, determine the overall impact on premium by using the ILFs with the risk loads instead of the ILFs without risk loads.

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8. (1.75 points)

An insurer sells coverage with an attachment point of 5,000 and a layer limit of 5,000. The following table represents the expected cumulative severity distribution and limited expected severity for the current year at various limits:

l	$F(l)$	$E[X;l]$
4,545	0.842	1,807
5,000	0.859	1,875
5,500	0.875	1,941
9,091	0.938	2,256
10,000	0.947	2,308
11,000	0.954	2,357

The actuary expects a 10% severity increase for all claim sizes next year.

a. (0.75 point)
Calculate the percent change in frequency of claims in the layer.

b. (1 point)
Calculate the percent change in pure premiums in the layer.

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9. (3.25 points)

Consider the following claims-made commercial general liability policy:

- The insurance contract was originally written January 1, 2011, and has been renewed annually as a claims-made policy.
- The annual Premises/Operations basic limits manual premium is $200,000 and there is no products exposure.
- The expected loss ratio is 70%.
- Loss experience is evaluated as of June 30, 2016.

Claim Number	Policy Year	Indemnity	ALAE
1	2011	$5,000	$5,000
2	2012	$15,000	$25,000
3	2013	$58,000	$0
4	2013	$20,000	$85,000
5	2014	$118,000	$82,000
6	2015	$8,000	$5,000

Calculate the experience modified premium for the policy effective January 1, 2017.

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10. (1 point)

An actuary is evaluating a risk to determine the appropriateness of applying any schedule credits or debits. In order to reduce expenses the insured removed a safety program 2 years ago and in the next month will be reducing its staff.

Range of Modifications:

Risk Characteristic	Description	Credit		Debit
Cooperation	Safety Program	10%	to	10%
Employees	Selection, training, supervision, experience	5%	to	5%

Recommend and defend an appropriate schedule modification.

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11. (2.5 points)

A workers' compensation insurer is facing an increasingly competitive market. Its management is concerned about customer retention, premium growth, and loss ratio deterioration. The insurer's actuaries have proposed an updated experience rating plan. After grouping insureds for the purposes of an efficiency test, the projected impact of this proposal is below (values in thousands):

Current Plan

Quintile	Manual Premium	Loss	Standard Premium	Loss Ratio to Manual	Loss Ratio to Standard
A	4,750	1,978	3,278	42%	60%
B	4,825	2,824	4,005	59%	71%
C	4,450	2,915	4,450	66%	66%
D	4,845	3,608	5,378	74%	67%
E	4,400	3,520	5,500	80%	64%
Total	23,270	14,845	22,610	64%	66%

Proposed Plan

Quintile	Manual Premium	Loss	Standard Premium	Loss Ratio to Manual	Loss Ratio to Standard
A	4,220	1,494	2,068	35%	72%
B	5,100	2,922	4,233	57%	69%
C	4,150	3,088	4,109	74%	75%
D	4,950	3,689	5,346	75%	69%
E	4,850	3,652	5,723	75%	64%
Total	23,270	14,845	21,478	64%	69%

It is estimated that the upfront cost to adopt the new rating plan will be $500,000.

a. (1.5 points)

Perform an efficiency test and evaluate the proposed experience rating plan relative to the current plan.

b. (1 point)

In light of management's concerns, evaluate the merits of adopting the new rating plan versus keeping the current plan, and provide a recommendation.

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12. (3.25 points)

A policy has a flat dollar deductible of M and a maximum payout on a loss by the insurer of N.

a. (1.0 point)

Draw a Lee diagram representing the expected amount of loss incurred by this policy. Label the following:

i. The axes
ii. The deductible amount
iii. The policy limit
iv. The expected insured loss.

b. (0.5 point)

Assume cumulative losses follow a distribution $F(x)$. Write the formula for covered losses for this policy using:

i. The layer method
ii. The size method

c. (0.5 point)

Briefly describe when the layer method may be preferred and when the size method may be preferred.

d. (1.25 points)

Use a Lee diagram to demonstrate the consistency test of ILFs.

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13. (2.5 points)

A risk is written using a balanced retrospective rating plan with the following characteristics:

- Losses at the minimum premium = $50,000
- Losses at the maximum premium = $300,000
- Loss conversion factor = 1.05
- e = $10,000

The following table shows actual experience from a representative sample of risks that are similar to the risk in question:

Risk	Actual Aggregate Loss	Risk	Actual Aggregate Loss
1	$25,000	6	$175,000
2	$50,000	7	$200,000
3	$100,000	8	$300,000
4	$100,000	9	$350,000
5	$150,000	10	$550,000

a. (2 points)

Determine the maximum premium that the insured can be charged.

b. (0.5 point)

An actuary has acquired actual aggregate data from a new book consisting of risks within the same industry. The loss experience of five representative risks from this book is shown below:

Risk	Actual Aggregate Loss
1	$275,000
2	$300,000
3	$500,000
4	$700,000
5	$800,000

The actuary proposes to combine the data from the two books, stating that the combined data will result in a more accurate calculation of the insured's retrospective premium. Assess the validity of the actuary's statement.

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14. (3.0 points)

A reinsurer has been supplied the following information from a large insurance company:

Claim Size Range	Expected Number of Claims	Expected Ultimate Losses (000s)
$0 to $1,000,000	19,000	$6,750,500
$1,000,001 to $2,000,000	359	$525,300
$2,000,001 to $3,000,000	230	$566,500
$3,000,001 to $4,000,000	147	$507,700
above $4,000,001	264	$1,650,000
TOTAL	**20,000**	**$10,000,000**

The reinsurer is entering into an excess of loss contract with the primary insurance company. The reinsurer will pay all losses above a $5,000,000 per claim retention.

a. (1.5 points)

Construct a graph of the excess severity function for claim sizes of $1,000,000, $2,000,000, $3,000,000, and $4,000,000.

b. (1.5 points)

Calculate the reinsurer's expected losses under the proposed contract.

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15. (3.25 points)

The following applies to an incurred loss retrospectively rated policy effective January 1, 2017.

Per occurrence limit	$150,000
Maximum ratable loss	$600,000
ULAE as a percentage of loss & ALAE	8%
Expected Loss & ALAE limited to $150,000	$400,000
Basic Premium	$375,000
Tax Rate	3%

The expected loss & ALAE limited to $150,000 is used to determine the initial premium. Retrospective rating adjustments start at 18 months and occur every twelve months thereafter.

The following loss experience occurs during the policy period. All claims are closed at 42 months.

	Maturity (months)		
	18	30	42
Unlimited Incurred Loss & ALAE	$425,000	$650,000	$950,000
Incurred Loss & ALAE Limited to $150,000	$375,000	$475,000	$700,000
Incurred Loss & ALAE Excess of $150,000	$50,000	$175,000	$250,000

a. (1.75 points)
Determine the amount and timing of each incremental cash flow payment made by the insured for this policy beginning with time 0.

b. (1.0 point)
Propose and justify an alternative policy that would increase the insured's cash flow benefit without the insured retaining any additional excess loss risk.

c. (0.5 point)
Identify and briefly describe one disadvantage to the insurer of the proposed plan in part b. compared to the existing retrospectively rated plan.

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16. (1.75 points)

An insurance company generally uses two different methods to price excess layer insurance contracts:

- Empirical construction of Table M; or
- Approximating the distribution of aggregate losses with a continuous approximation model.

a. (0.5 point)
Briefly describe two potential disadvantages of using a continuous approximation model.

b. (1.25 points)
Fully describe the process of constructing an empirical Table M and estimating the aggregate loss cost of an excess layer using the table.

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17. (2.0 points)

A workers' compensation policy has the following limits and expected losses:

Per-Occurrence Limit	\$300,000
Expected Unlimited Aggregate Losses	\$700,000
Expected Limited Aggregate Losses	\$500,000
Aggregate Deductible Limit	\$800,000
State Hazard Group Differential	0.95

Table of Expected Loss Groups

Expected Loss Group	Range of Values
31	630,000-720,000
30	720,001-830,000
29	830,001-990,000
28	990,001-1,180,000
27	1,180,001-1,415,000
26	1,415,001-1,744,000

Table of Insurance Charges

	Expected Loss Group					
Entry Ratio	31	30	29	28	27	26
0.43	0.5675	0.5626	0.5584	0.5543	0.5497	0.5456
0.60	0.4865	0.4799	0.4737	0.4676	0.4613	0.4553
1.14	0.2658	0.2552	0.2448	0.2341	0.2234	0.2128
1.60	0.1644	0.1537	0.1367	0.1322	0.1215	0.1107
2.67	0.0822	0.0769	0.0684	0.0661	0.0608	0.0554

a. (1.5 points)

Calculate the total expected loss cost for this policy using the Insurance Charge Reflecting Loss Limitation (ICRLL) adjustment procedure.

b. (0.5 points)

Describe how the ICRLL procedure is used to adjust expected losses for a workers' compensation policy.

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18. (3.25 points)

A homogenous group of property risks exhibit the following risk profile:

Exposure Distribution	Losses as a percent of the Maximum Possible Loss (MPL)
80%	0%
6%	25%
8%	50%
4%	75%
2%	100%

a. (2 points)

Plot the exposure curve, G(x). Label the axes and the points on the curve.

b. (0.75 point)

Determine the parameters b and g for the 2 parameter MBBEFD distribution that fits this exposure curve using the following information:

Calculated value of parameter b									
Parameter g	μ								
	40.0%	42.5%	45.0%	47.5%	50.0%	52.5%	55.0%	57.5%	60.0%
50.0	0.0023	0.0017	0.0013	0.0009	0.0005	0.0003	0.0002	0.0001	0.0001
25.0	0.0096	0.0073	0.0055	0.0041	0.0022	0.0015	0.0010	0.0006	0.0004
16.7	0.0239	0.0181	0.0137	0.0103	0.0057	0.0039	0.0026	0.0017	0.0010
12.5	0.0483	0.0365	0.0276	0.0209	0.0118	0.0081	0.0055	0.0036	0.0023
10.0	0.0877	0.0659	0.0497	0.0375	0.0212	0.0147	0.0100	0.0067	0.0044
8.3	0.1498	0.1116	0.0836	0.0628	0.0354	0.0245	0.0168	0.0113	0.0075
7.1	0.2470	0.1817	0.1347	0.1004	0.0561	0.0389	0.0267	0.0181	0.0121
6.3	0.4000	0.2892	0.2116	0.1561	0.0861	0.0594	0.0408	0.0277	0.0185
5.6	0.6446	0.4557	0.3277	0.2385	0.1291	0.0885	0.0605	0.0411	0.0276
5.0	1.0469	0.7190	0.5055	0.3616	0.1910	0.1297	0.0882	0.0597	0.0400

c. (0.5 point)

A reinsurer uses this exposure curve to price a property excess of loss treaty where the underlying limit is \$100 million. Calculate the proportion of total losses in the layer \$25 million excess of \$25 million.

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19. (3.0 points)

A primary insurer wants to institute a swing plan with the following terms:

- Layer: $500,000 xs $500,000 per claim
- Premium = Layer Losses * 1.3
- Minimum Premium = $100,000
- Maximum Premium = $325,000

The reinsurer has analyzed the insurer's experience. All claims are closed and trend has been applied.

Accident Year	On-Level Premium	Claim #1	Claim #2	Claim #3	Claim#4
2012	400,000	465,899	230,567	512,017	
2013	420,000	240,785	125,766	80,154	470,666
2014	1,200,000	704,895	106,528		
2015	2,000,000	45,988	349,145	376,011	1,246,852
2016	2,200,000	437,100	654,158	250,000	156,750

a. (1.5 points)

Calculate the expected loss ratio for the reinsurer based on the empirical distribution model using equal weights for each year.

b. (0.5 point)

Determine if an empirical distribution model is appropriate for this account and give one reason in support of your answer.

c. (1 point)

The use of a collective risk model is an alternative approach. Explain two concerns when using this type of model.

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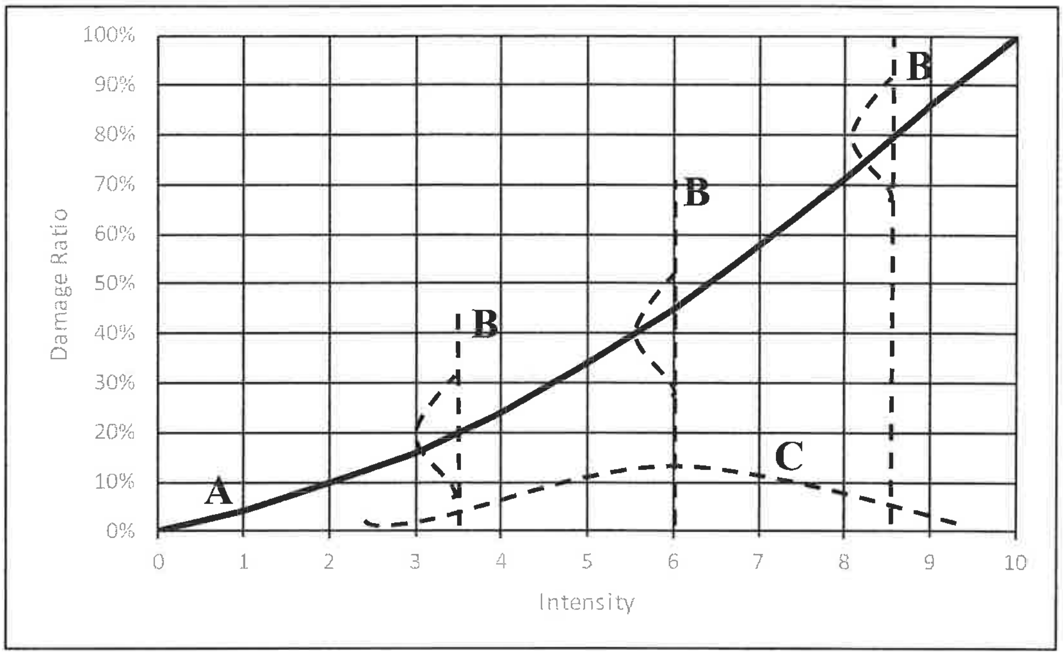
20. (3 points)

An earthquake model produces the following damage function for a $1,000,000 home in California:



The probability of an event occurring with a given intensity is:

Intensity	Probability
3.5	10%
6.0	5%
8.5	2%

a. (0.75 points)

Identify what A, B, and C represent in the above graph.

b. (1.75 points)

Determine the premium for this home given the following information:

- The insurer's expense load is 20% of premium
- The risk load is set to 8% of the standard deviation of the loss

END OF EXAMINATION

Exam 8
Advanced Ratemaking

POINT VALUE OF QUESTIONS

QUESTION	VALUE OF QUESTON	SUB-PART OF QUESTION (a)	(b)	(c)	(d)	(e)	(f)	(g)
1	8.50	1.50	2.00	1.50	1.50	2.00		
2	2.00	1.75	0.25					
3	1.50	0.75	0.75					
4	1.75	0.50	0.75	0.50				
5	1.75							
6	3.50	1.00	1.50	0.50	0.50			
7	1.75	1.00	0.75					
8	1.75	0.75	1.00					
9	3.25							
10	1.00							
11	2.50	1.50	1.00					
12	3.25	1.00	0.50	0.50	1.25			
13	2.50	2.00	0.50					
14	3.00	1.50	1.50					
15	3.25	1.75	1.00	0.50				
16	1.75	0.50	1.25					
17	2.00	1.50	0.50					
18	3.25	2.00	0.75	0.50				
19	3.00	1.50	0.50	1.00				
20	2.50	0.75	1.75					
TOTAL	53.75							

SAMPLE ANSWERS AND EXAMINER'S REPORT

GENERAL COMMENTS:

- Candidates should note that the instructions contained in the exam explicitly say to show all work; Graders expect to see enough support on the candidate's answer sheet to be able to follow the calculations performed. While graders make every attempt to follow poorly documented calculations, there were many instances that candidates did not receive full credit because their calculations could not be followed or did not have sufficient support.
- Candidates should be reminded that writing clearly and legibly is critical for graders to be as generous as possible in awarding points for a given question. There were responses that the graders and others could not read and therefore no credit or less than full credit was awarded.
- Questions are asked in a deliberate way to indicate the level and depth of response that is required to earn full credit. There were numerous examples of lengthy responses to questions that asked candidates to 'briefly describe' an example. These responses do not generate any additional credit beyond full credit and take up valuable time during the exam itself.
- Questions are also very clear in terms of whether the candidate should provide a specific term or terms as part of their answer. For example, 'Identify and briefly describe' requires candidates to actually list (i.e. identify) the response asked for, and then provide a brief description about that response. On certain questions there were examples of candidates providing clear explanations, but not specifically identifying the item(s) asked for, and credit was lost.
- This exam contained a number of questions that could be interpreted in different ways. On each of these questions, graders were flexible in accepting multiple answers for full credit. In cases where the candidate feels there is ambiguity in a question, they should clearly state any assumptions made in order to resolve any ambiguity. Graders will take those assumptions (to the extent they are valid) into account when grading the candidate's response.
- Candidates should note that the sample answers provided in the examiner's report are not an exhaustive representation of all responses given credit during grading, but rather the most common correct responses.
- Incorrect responses in one part of a question did not preclude candidates from receiving credit for correct work on subsequent parts of the question that depended upon that response.
- In cases where a given number of items was requested (e.g., "three reasons" or "two scenarios"), the examiner's report provides more sample answers that the requested number. The additional responses are provided for educational value, and would not have resulted in any additional credit for candidates who provided more than the requested number of responses.

EXAM STATISTICS:

- Number of Candidates: 945
- Available Points: 53.75
- Passing Score: 37.5
- Number of Passing Candidates 376
- Raw Pass Ratio: 39.8%
- Effective Pass Ratio: 41.7%

QUESTION 1	
TOTAL POINT VALUE: 8.5	LEARNING OBJECTIVE(S): A1a, A1d, A3a, B3a, B3b, B5a, B6c
SAMPLE ANSWERS	
Part a: 1.5 points	

Industry – industry has a low p-value indicating that this variable is statistically significant and a good predictor of future losses.

Additionally, there is a connection between industry and overall loss levels (construction companies would be expected to have more losses than florists as an example)

Industry is a variable that is practical and easy to collect and easy to verify (via SIC code)

Employee Tenure – employee tenure has a low p-value indicating that this variable is statistically significant and a good predictor of future losses.

Employee tenure is directly connected to future loss activity – new employees may not be as safe as employees who know the building, the machinery, or have experience with other loss control method (i.e. know how to set safety guards, etc.)

Employee tenure can likely be easily collected by asking the customer or agent, but on the other hand some customers may not know it or may feel like it is easy to 'game'

Return to Work Program – return to work program has a lower p-value indicating that this variable is statistically significant and a good predictor of future losses.

There is a direct connection to the amount an employer spends on employee injuries – the quicker an employee returns to work, the less amount of time the employer needs to pay wage replacement. Additionally, this is an easy piece of information to obtain and since it is a discrete variable (yes or no) it should be easy to verify.

Part b: 2 points

Employee Age – Employee age is has a lower p-value indicating that this variable is statistically significant and a good predictor of future losses.
However, ASOP 3.2.5 discusses compliance with applicable law. Using age could be considered to be discriminatory and not in compliance with Federal laws.
(Alternatively, some candidates argues that employee age could be correlated with employee tenure and should be excluded so that the model converges. Credit was given for this response.)

Location – Location of the company would likely be an acceptable variable – companies that are located in areas with plenty of hospitals and doctors would likely have overall lower loss costs than companies in rural areas. However, the p-value for this variable is high indicating that it is not a good predictor of future losses.

Employee Morale – Employee morale does not appear to be predictive of future losses based on the high p-value being generated from the model. Furthermore, the variable is very subjective, difficult to collect and difficult to verify so it also does not meet the standards set in ASOP 12.

Number of Back Injury Claims - # of back injury claims has a relatively small p-value so it may have some predictive power. However, it fails many of the ASOP 12 considerations including causality – it's not clear that the # of back injuries is directly related to losses – injuries could be small or large. Additionally, since the information is supplied by the employer, it may be tough to verify and be subject to gaming.

Part c: 1.5 points

There are five different elements where there are two valid assumptions for each one:

i. Candidates could interpret Manufacturing to be part of All Other Industry type OR the base class
ii. Candidates could take the Employee Tenure variable either directly or by taking the natural log
iii. Candidates could assume the given actual experience includes one year of experience OR three years of experience
iv. Candidates could apply the experience modification factor either before or after expense loading
v. Candidates could calculate the experience modification factor directly or using a credibility formula. These two approaches are numerically identical.

The following are a sample of acceptable full credit approaches

Sample 1 – Using Manufacturing as the base industry class / actual experience includes 1 year / employee tenure is not transformed / experience mod calculated directly / experience mod applied after expense load

ln(u) = -0.631 + (-.040*5) + (-0.20 * 0) + (-.55*1)
ln(u) = -1.381
u = .251 or $0.251 per $100 of

Manual Premium =
$0.251 * ($1,000,000/$100) = $2,513

P = Losses + Fixed Expenses + Variable Expenses
P = $2,513 + $1,500 + 0.2P
0.8P = $4,013
P = $5,017

M = (A + K) / (E + K)
= ($12,500 + $10,000) / ($2,513 + $10,000) = $22,500 / $12,513
=1.798

Standard Premium = $5,017 * 1.798 = $9,020

*Sample 2 - **Using Manufacturing as the All Other industry class** / actual experience includes 1 year / employee tenure is not transformed / experience mod calculated directly / experience mod applied after expense load*

ln(u) = -0.631 + (-.040*5) + (-0.20 * 0)
ln(u) = -0.831
u = .436 or $0.436 per $100 of

Manual Premium =
$0.436 * ($1,000,000/$100) = $4,356

P = Losses + Fixed Expenses + Variable Expenses
P = $4,356 + $1,500 + 0.2P
0.8P = $5,856
P = $7,320

M = (A + K) / (E + K)
= ($12,500 + $10,000) / ($4,356 + $10,000) = $22,500 / $12,513
=1.567

Standard Premium = $7,320 * 1.567 = $11,470

*Sample 3 - Using Manufacturing as the base industry class / actual experience includes 1 year / **employee tenure IS transformed** / experience mod calculated directly / experience mod applied after expense load*

u = -0.631 + -.040*ln(5) + (-0.20 * 0)
ln(u) = -0.6954
u = .4989 or $0.4989 per $100 of

Manual Premium =
$0.4989 * ($1,000,000/$100) = $4,989

P = Losses + Fixed Expenses + Variable Expenses
P = $4,989 + $1,500 + 0.2P
0.8P = $6,489
P = $8,111

M = (A + K) / (E + K)
= ($12,500 + $10,000) / ($4,989 + $10,000) = $22,500 / $14,989
=1.501

Standard Premium = $8,111 * 1.501 = $12,175

*Sample 4 - Using Manufacturing as the base industry class / **actual experience includes 3 years** / employee tenure is not transformed / **experience mod calculated via a credibility approach** / experience mod applied after expense load*

ln(u) = -0.631 + (-.040*5) + (-0.20 * 0)
ln(u) = -0.831
u = .436 or $0.436 per $100 of

Manual Premium =
$0.436 * ($1,000,000/$100) = $4,356

P = Losses + Fixed Expenses + Variable Expenses
P = $4,356 + $1,500 + 0.2P
0.8P = $5,856
P = $7,320

E = 4356*3 = 13,068

Z = 13,068/(13,068 + 10,000) = .566
E Mod = (12,500 * .566 + 13,068 * (1 - .566))/13,068 = .975
$7,320 * .975 = $7,137

Part d: 1.5 points

Candidates responding with any three of the following would receive full credit:

- Historical loss experience already impacts premium through experience rating. They should not be double considered.
- "Latest 3 year losses" is highly correlated to premium range. If we want to use historical loss experience we'd better use loss ratio.
- The bad fit on the higher range might be caused by sparse data of large premium insureds. So it is normal and maybe how much effort to improve fit can be a waste of time
- Small risks have volatile experience and incorporating latest 3 years historical losses will cause fluctuations in their premium
- This variable will most likely be highly correlated with other variables which may lead to an unstable model. If multicolinearity is present, the model may not converge at all or lead to irrational outputs.
- The latest 3 yr losses won't be fully developed. So, if the reporting lag varies by company it would be over or under predicting the loss experience.

- The variable would be very correlated with risk size since larger insureds are more likely to have higher historical losses so including the variable without stating it as a ratio to exposure, manual premium or expected loss could skew results by size.
- It would be unfair as not all losses are predictive of future – for example random large losses – they should be capped and credibility weighted with expected.
- 3 year historical losses does not separate frequency from severity. May be better to build variables that separate these impacts.
- There may be recent changes to the risk's safety procedures or WC benefit levels that would not yet be fully reflected in 3 yr historical loss.

Part e: 2 points

Sample Responses for part i)

- Since risk is new, there won't be any actual loses with which to do experience rating, so it is difficult for insurer to know whether manual premium is adequate and not excessive for this risk. (new – no loss)
- High payroll amount puts the risk in the high manual premium range, where model is less successful at predicting the actual losses – so there's additional uncertainty in manual premium due to this. (large fit)
- Based on the graph in part C, I assume that the experience shown in the larger premium range is thin so the company may not have enough data to feel confident on pricing risks of this size. (thin)
- The experience rating constant K is likely too low for this risk as it is giving very high credibility to actual experience of a risk of $50M. (k low)
- The experience rating plan does not seem to have a cap or apply a split between primary and excess losses which may lead to oversensitivity to large loss events. (no split)
- Large companies may have more effective return to work programs or other unique characteristics that are not contemplated by this model that doesn't consider variables for large size risks separately (large unique)
- The variable expenses of 20% may be appropriate for smaller risks but could be too high for a risk of this size and the plan does not include any expense discount to account for this. (expenses)
- Having only one group in GLM for all construction likely insufficiently addresses differences in WC loss potential between different hazard levels of construction work such as roofing vs. drywall installation.
- Assuming the question implies rate is set only using GLM without experience mod, then the risk has little incentive to control losses.

Sample Responses for part ii)

- They should enter in to a retro rating agreement. Since the new risk has no prior experience to base price on, the retro can allow adjustments to premium based on current period experience. For the insured, the retro plan may result in cash flow advantages if the initial premium is set lower. For the insurer, one advantage is a lower capital requirement.
- A large deductible plan could be considered as a loss sensitive rating option where the insured is responsible for claims below the deductible. Benefit to the insured: Would see

a more immediate reflection of good loss experience. Benefit to the insurer: the insured has a financial incentive for loss control.

- Base the experience rating component of pricing on the NCCI method of splitting primary and excess losses so that premium is not inappropriately impacted by large losses with less credibility. The advantage for the insured is that their premium will not see large swings year to year from large loss experience. The advantage for the insurer is that the experience mods from a split plan for WC are generally shown to be more accurate in estimating expected costs so they are less likely to underprice risk.
- A new GLM could be developed to estimate pure premium that focuses on variables more relevant to construction risks. This could remove some of the premium uncertainly for larger construction policies. For the insurer, the model will provide more insights into drivers of expected loss. For the insured, the premium would be more likely to appropriately reflect their existing characteristics and therefore be more equitable.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of both statistical and non-statistical considerations for the inclusion or exclusion of variables in a model. Candidates were also expected to demonstrate knowledge of experience rating along with their knowledge of GLMs.

Part a

Candidates were expected to provide statistical and non-statistical reasons for including the given variables in the model. Additional responses that related back to the Actuarial Standards of Practice were also accepted for the non-statistical responses.

Common mistakes included:

- Omitting either a statistical or non-statistical consideration for any of the variables
- Not arguing for inclusion in the model

Part b

Candidates were expected to provide statistical and non-statistical reasons for excluding the given variables in the model. Additional responses that related back to the Actuarial Standards of Practice were also accepted for the non-statistical responses.

Common mistakes included:

- Omitting either a statistical or non-statistical consideration for any of the variables
- Not arguing for exclusion from the model

Part c

Candidates were expected to calculate a standard premium based on model parameters and other data provided. There were many possible answers that received full credit based on different interpretations of several elements to this question. Any correctly calculated answer based on the varying selected assumptions in i. through iv. below received full credit.

i. Candidates could interpret Manufacturing to be part of All Other Industry type OR the base class
ii. Candidates could take the Employee Tenure variable either directly or by taking the natural log

iii. Candidates could assume the given actual experience includes one year of experience OR three years of experience
iv. Candidates could apply the experience modification factor either before or after expense loading
v. Candidates could calculate the experience modification factor directly or using a credibility formula. These two approaches are numerically identical.

Common mistakes included:
- Not calculating an experience mod at all
- Calculation errors

Part d

From LO/KS B3a, candidates were expected to be able to draw on their knowledge of experience rating for WC and recognize that the standard premium calculation would include the application of an experience mod based on recent loss experience which would have some overlap with the experience rating plan leading to 'double counting' of experience. Furthermore, the candidate should be able to pick up some of the following reasons against using total three year losses directly in the model:
- Use of total losses could result in volatile pricing outcomes from large loss events
- For smaller risks, 3 year losses may not be predictive of future outcomes since their year-to-year actual results are expected to have more variation than large risks
- Losses should be scaled by exposure or expected loss otherwise the correlation of total loss with manual premium will distort the fit
- To provide an appropriate prediction for future loss, past 3 year losses may need to be developed and adjusted for changes in benefit levels or changes in the risk's size, operations, or safety practices over the three year period vs. prospective period

Although the question was designed to focus on concepts of experience rating, alternative responses related to knowledge of GLMs based on LO 4 were also accepted. This could include:
- Adding more parameters or degrees of freedom to the model could lead to over-fitting
- Correlation between variables in a model can lead to erratic coefficients and the prior loss could be highly correlated with in-model predictors
- In practical application it is typical when fitting commercial models for there to be fewer risks in the large premium bucket so the lack of fit may still be within acceptable variation for actual results

Credit was not awarded for the following types of responses:
- Comments that correlation of variable with target were given no credit because we are trying to find variables that are correlated with the target.
- Comments that 3 year historical losses lack credibility or that GLM gives full credibility were not given credit unless they included specifics about large losses or small risks.
- Comments that large loss experience was volatile were not given credit – the lack of fit in the higher range does not necessarily imply more volatile experience for larger risks – more likely there are risk differences not captured by the GLM.

- Comments that there could be difficulty collecting prior 3 year losses were not given credit because it was already given in part c of the question that they would be collected for experience rating anyway.
- Comments that it is not industry practice to include prior losses in a GLM or that there might not be public acceptance or regulatory approval were not given credit because this is something that is commonly done in the industry.
- Comments related to appropriateness of use of Tweedie model were not given credit because although the reading states that Tweedie assumptions are violated when frequency and severity do not move together, it is very common practice for commercial pricing models with a wide array of frequency and severity profiles to use Tweedie models. Note the reading also states, "However, Tweedie GLMs can be quite robust against such violations of its assumptions and still produce very strong models." ("Generalized Linear Models for Insurance Rating," CAS Monograph #5, pg 23)

Common mistakes included:

- Interpreting the chart as a quintiles test output or lift chart, not a graph of model fit by premium range or, similarly, thinking that the groups on the X axis of the chart were predicted loss ratios, not manual premium ranges
- Believing that loss ratios should be either monotonically increasing or decreasing by size of risk, when in reality if the prediction is fairly good they should be fairly flat other than not capturing experience component that is captured in standard premium.

Part e

Candidates were expected to recognize that the fit of the pricing model for larger risks was not doing a good job of predicting losses for larger accounts so the premium could be too high or too low. Additionally, candidates were expected to recognize that there would be some sort of issue with experience rating this risk, either making the assumption that "new Construction Risk" indicated a new operation with no prior loss experience or recognizing that the experience rating plan was offering too much credibility or should split excess vs. primary in some way. However, assuming that the risk was only new to the company could also result in full credit answers.

To get full credit a response needed to include sufficient description of the issue as in the sample responses given above. Responses that briefly identified an acceptable issue but did not describe it only received partial credit.

Credit was not given for responses that said larger risks should be experience rated and not rely on manual rating – in part (c) of the problem we are told that manual premium is subject to experience rating to get to standard premium in this pricing plan.

Credit was not given for responses stating that variables other than prior loss experience would be unknown or cause problems for a new risk. It is expected that candidates should assume underwriters would evaluate the risk and collect data required for manual rating.

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 2	
TOTAL POINT VALUE: 2	**LEARNING OBJECTIVE(S): A1**

SAMPLE ANSWERS

Part a: 1.75 points

Determine centroid of each initial cluster

R_A(500K) = (6500*0.24+5000*0.35+4000*0.21+3000*0.11)/(6500+5000+4000+3000) = 0.242
R_B(1M) = (6500*0.08+5000*0.2+4000*0.08+3000*0.03)/(6500+5000+4000+3000) = 0.104

R_B(500K) = 0.18
R_B(1M) = 0.07

Now determine Euclidean distance between each class and centroids of A & B, and assign class to cluster with smaller distance.

Class	Distance to R_A	Distance to R_B	New Cluster
1	0.024 = [(0.24-0.242)^2 + (0.08-0.104)^2] ^ (1/2)	0.061 = [(0.24-0.18)^2 + (0.08-0.07)^2] ^ (1/2)	A
2	0.144	0.214	A
3	0.040	0.032	B
4	0.151	0.081	B
5	0.07	0	B

Part b: 0.25 point

- Many small errors would have the same effect as one large error which results in outliers having less of an impact on the result.
- L^1 minimizes the relative error PLR*|R_C(L)-R_{HG}(L)|; L2 does not necessarily.
- L^1 minimizes the relative error when calculating the premium.
- L^1 minimizes the relative error when calculating the excess ratio.
- The unit of L^1 is the same as expected loss costs, i.e. in dollars, whereas L^2 has unit in dollar^2

EXAMINER'S REPORT

Candidates were expected to know how to perform the first iteration of a clustering analysis – determine centroid coordinates, calculate L^2 distance from each class to each centroid and assign each class to the closest centroid. Many candidates struggled with the calculation of the L^2 distance measure.

Part a

Candidates were expected to calculate the centroid coordinates (500K, 1M) of each cluster as they were assigned in the question. With those centroids, candidates were then expected to calculate the Euclidean distance (L^2) of each class from each of the two clusters. Finally, candidates needed to assign each class to a cluster based on the smaller of the two distances.

Common mistakes included: • Taking a straight average rather than a weighted average of the centroid A excess ratios • Taking an average of the 500K and 1M excess ratios for each class individually and using that as the centroid coordinate • Only calculating centroids and distances for one limit • Using an incorrect distance formula (such as L^1) • Calculating separate distances and assigning separate clusters for each limit • Not showing complete supporting work on how clusters were assigned
Part b
Candidates were expected to state that: • The L^2 measure gives undue weight to outliers, or • The L^1 measure minimizes the relative error in excess premium (or excess ratios). Common mistakes included: • Stating that L^1 is easier to calculate/requires less computation • Stating that L^1 is easier to explain • Stating that L^1 considers negative/positive direction of the distance

QUESTION 3	
TOTAL POINT VALUE: 1.5	**LEARNING OBJECTIVE(S): A1c**
SAMPLE ANSWERS	
Part a: 0.75 point	

Sample 1
Using earned prem as exposure base only correct the maldistribution due to correlation between freq & territory when:

1. Territory differentials are proper, and
2. High freq territory are also high premium terr

Here: Terr C has highest frequency but not highest prem → (2) not satisfied

Hence, using EP as base does not make an improvement

- ➔ Use earned care year as expo base.

Sample 2
Since high frequency territories are not also high avg premium territories, using EP as a base will not correct for maldistribution. Therefore, I choose ECY as my base.

Sample 3
I would use earned car years. A premium base is appropriate when high frequency territories are also high premium territories. And when territorial differentials are proper. Here, C is the highest frequency but the lowest premium territory. So use earned car years instead.

Sample 4
Territory is a variable that tends to be correlated with other risk characteristics so it would be advisable to use earned premium as an exposure base to correct for exposure correlation, but only if high frequency territory are also high average premium.

This doesn't seem to be the case (i.e. terr C is highest freq, but lowest avg premium) but prem could reflect other vars' impact, so use EP as exposure base.

Part b: 0.75 point

If using car years as exposure base:

Sample 1
3+: (13,500+15,000)/(100+250) = 81.43
Total: 47,000/500 = 94
Rel Freq: 81.43/94 = 0.866
Z= 1 – 0.866 = 0.134

1+: (13,500+15,000+8,000)/(250+100+80) = 84.88
Rel Feq: 84.88/94 = 0.903
Z = 1-0.903 = 0.097

Rel credibility = 0.134/0.097 = 1.38

Sample 2
Mod = (28,500/350)/(47,000/500) = 0.866
Mod = 1 – Z so Z=0.1337

Assume freq = Possion
λ = 47,000/500,000 = 0.94
R 0 years claim free = 1/(1-e^(- λ)) = 11.146
Mod 0 years claim free = (10500/70)/(47,000/500) = 1.596
Mod = RZ+(1-Z) → Z = (Mod – 1)/(R-1) = (1.596-1)/(11.146-1) = 0.0587

Rel cred 3+/Cred 0 = 0.1137/0.0587 = 2.28

If using earned premium as exposure base:

Sample 3
3 or more years claim frequency: (13,500+15,000)/(500,000+90,000) = 0.048
Total claim frequency: 47,000/700,000 = 0.067
Relative claim Frequency of 3 or more years: 0.048/0.067 = 0.72
Z= 1 – 0.72 = 0.28

1 or more years claims frequency: (13,500+15,000+8,000)/(500,000+90,000+60,000) = 0.056
Relative claim frequency of 1 or more years = 0.056/0.067 = 0.84
Z = 1-0.84 = 0.16

Rel credibility = 0.28/0.16 = 1.75

Sample 4
Mod(3+) = ((13.5+15)/(500+90))/(47/700) = 0.719
Cred(3+)= 1 – Mod(3+) =0.281

Mod(0) = (10.5/50)/(47/700) = 3.128
λ = 47,000/500,000 = 0.94
R 0 years claim free = 1/(1-e^(- λ)) = 11.15
Mod(0) = Z_0 * R + (1- Z_o)
3.128 = 11.15*Z_0 + (1-Z_o)
Z_0 = 0.209

Z_{3+} / Z_0 = 1.34

EXAMINER'S REPORT

Candidates were expected to be able to demonstrate knowledge of the potential causes of distortion in the choice of an exposure base and then use that exposure base to determine relative credibility.

In general, candidates did well with part (a) but were confused with the relative credibility for part (b), with many candidates only calculating the 3+ credibility and not the relative credibility to 1+. Note that full credit was given for calculating credibility relative to 0+ as well.

Part a

Candidates were expected to discuss that the maldistribution of rating variables can be corrected by earned premium, but only under two circumstances:

- High frequency territories are also high premium territories
- The territory differentials are properly priced

Candidates needed to use the data provided to build a case for the exposure base that they selected (premium or car years). This could be in the form of:

- Arguing that because the high frequency territories are not also high premium territories the first condition is not met and thus car years must be used
- Arguing that while the frequencies do not appear to be in-line with premiums by territory, that premium may still be a better choice as it addresses some maldistribution and should be still used as the exposure base

Common mistakes included:

- Selecting an exposure base without including the reasoning behind the selection

Part b

Candidates were expected to calculate the relative credibility for the class of operators who have been accident-free for 3 or more years relative to those who have been accident-free for 1 or more years. Full credit was given for using 0+ years accident-free instead, and full credit was given for using either car years or premium as the exposure base.

Common mistakes included:

- Failing to calculate a ratio of credibilities (many candidates simply calculated the credibility for 3 or more accident-free years)

QUESTION 4	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): A3b, A2d**

SAMPLE ANSWERS

Part a: 0.5 point

Sample 1

- Splitting data into training and test groups prevents overfitting of the model because the model will always fit the training better with more parameters but it also could pick up random variation as a predictive variable.
- It also allows for the testing of the predictive power of the model, if it doesn't fit the test data well it likely won't predict future outcomes well either.

Sample 2

To test the predictive power of the model & to avoid overfitting to the noise of the training set. We use the training set to fit the model but then we test this on the test set (which is data that is "unseen" by the model) to make sure we have not overfit & that the model is predictive.

Part b: 0.75 point

Sample 1

- Model iteration 1: The model does not have an optimal balance here as the model error for both training and test data is relatively high and can be decreased.
- Model iteration 2: This model has an optimal balance as the training error is lower which is expected with more variables. However the test error is also at it lowest point, meaning adding more parameters has not lead to overfitting.
- Model iteration 3: This model does not have an optimal balance. Adding more parameters has lowered the training data errors but increased the error for the test data. This model has been overfit.

Sample 2

i. not optimal because we can still improve model performance on the test set by adding additional degrees of freedom

ii. This appears to be the optimal balance because this is right around where the test set has lowest model error

iii. not optimal, too much complexity (ie degrees of freedom) in the model, which has caused the model error on the test set to actually increase (ie we have overfit to the training set).

Part c: 0.5 point

Sample 1

Splitting data by time would be advantageous when weather events occur. This keeps the entire event in one section of data and prevents overfitting.

Sample 2

When data is affected by a large weather event. In which case, this event will only be included in one of the two sets & we won't get overly optimistic results that would occur if the event affected both sets.

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to have a high-level understanding of how to segment a dataset for construction and evaluation of a predictive model, how to assess the quality of modeled output, and to comment on strengths and weaknesses of alternative data segmentation strategies.
Part a
Candidates were expected to clearly identify and describe two reasons for using a holdout dataset. In particular, acceptable reasons fell into two distinct groups: to avoid overfitting, and to assess the predictive power of the model. Common variations that received full credit for the first of these categories (avoid overfitting) were: • Discussion of the fact that the addition of variables will always improve the fit on train data, but that it may not improve the fit on test data • Discussion of k-fold cross-validation methods (or similar techniques) when referencing parameters, quality of fit, etc. Common variations that received full credit for the second of these categories (assess predictive power) were: • Discussion of the fact that "attempting to test the performance of any model on the same set of data on which the model was built will produce overoptimistic results" (from the GLM paper) • Selecting a best model from multiple alternative models • Discussion of validating a model on an "unseen" dataset • Discussion of assessing model stability • Discussion of k-fold cross-validation methods (or similar techniques) when referencing model stability, lift, etc. Common mistakes included: • Discussing two variations of the same thing: either preventing overfitting or assessing predictive power • Essentially re-stating the question: for example, simply stating that a test group is used to test a model (without being specific regarding what is being tested)
Part b
Candidates were expected to assess the balance of complexity and performance based on a graph of model error as related to model degrees of freedom (number of free parameters). The question specifically referred to three candidate model iterations. While full-credit responses varied significantly in length, in general to receive full credit, candidates needed to recognize and convey an understanding of each of the following: • A training data curve will always decrease monotonically with addition of degrees of freedom • Model performance on test data will improve until the model has been overfit • An optimal balance of complexity and performance occurs at/near the minimum of the test data curve

Common mistakes included:

- Drawing conclusions based primarily on training set error rather than test set error
- Thinking that the gap between the training data curve and the test data curve is meaningful in itself, without addressing the actual magnitude of these curves
- Not addressing the trade-off between complexity and performance (most candidates implicitly did this by selecting an optimal model iteration)

Part c

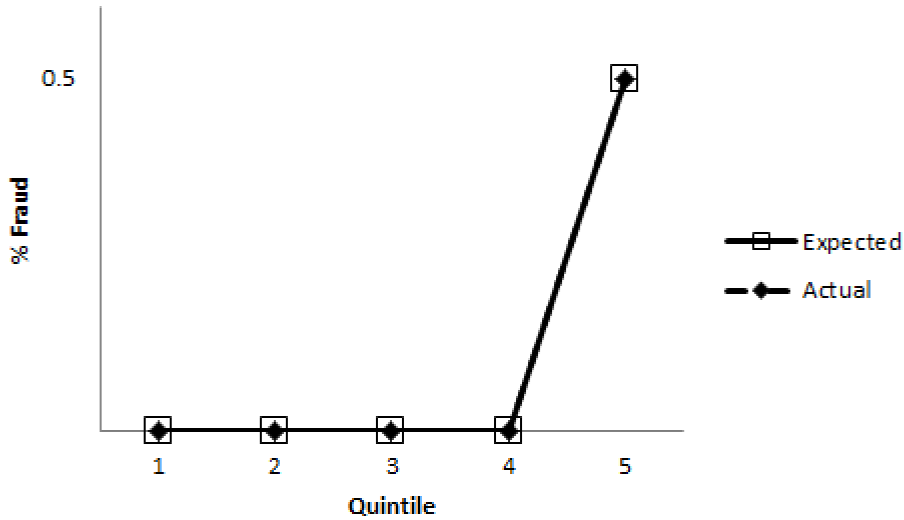
Candidates were expected to know that it is an advantage to have an out-of-time validation dataset when many records are influenced by a single event. The most common examples were a catastrophe or other weather event. To receive full credit, candidates were expected to identify such a situation, to note that splitting by time would result in claims from the same event being assigned to either the training dataset or the testing dataset (not both), and would therefore lessen the chance for overfitting or producing overly optimistic validation results.

Common mistakes included:

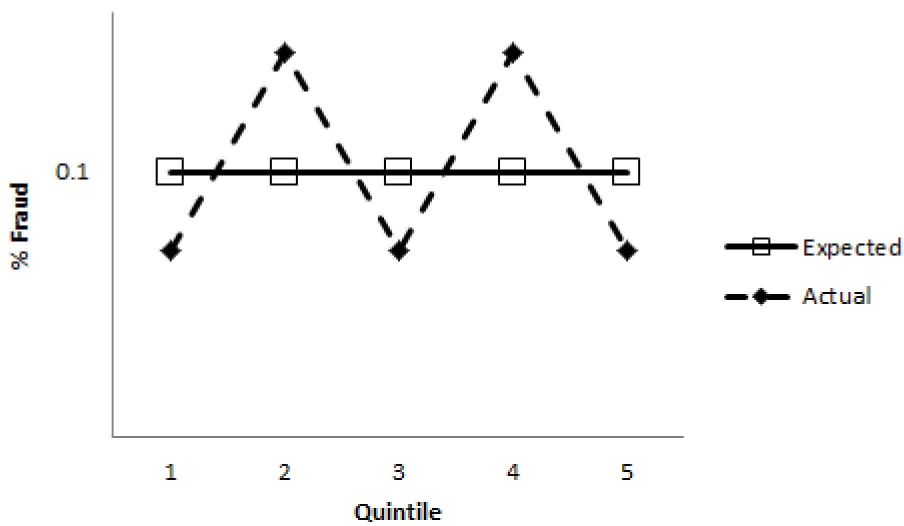
- Correctly identifying the situation (CAT or other large event), but not explaining why a time-based split is preferred
- Identifying "weather" as a situation, rather than specifying "weather events" (which implies correlated claims, and which is a fundamental reason to prefer an out-of-time split)
- Arguing that because a random split *__might__* result in an imbalanced test or train dataset (most candidates identified "seasonal effects" as the cause, but other reasons were given), therefore splitting by time (even/odd years, for example) is more appropriate than randomly splitting (note: credit was given in this case to candidates who demonstrated specific cases in which this might be reasonable)
- Claiming that splitting by time would alleviate the problem of correlation due to insureds having more than one policy period included in the dataset
- Arguing that time-dependent signals in the data (for example, underlying trends in the data, shifts in the mix of business, or shifts of fitted parameters over time) could be either identified or adjusted for by using a *__single__* time-based train/test split of the data (note: using multiple train/test splits may help in these situations – for example, using a k-fold out-of-sample out-of-time validation – and credit was given if the candidate made this point)

QUESTION 5	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE(S): A4c
SAMPLE ANSWERS	

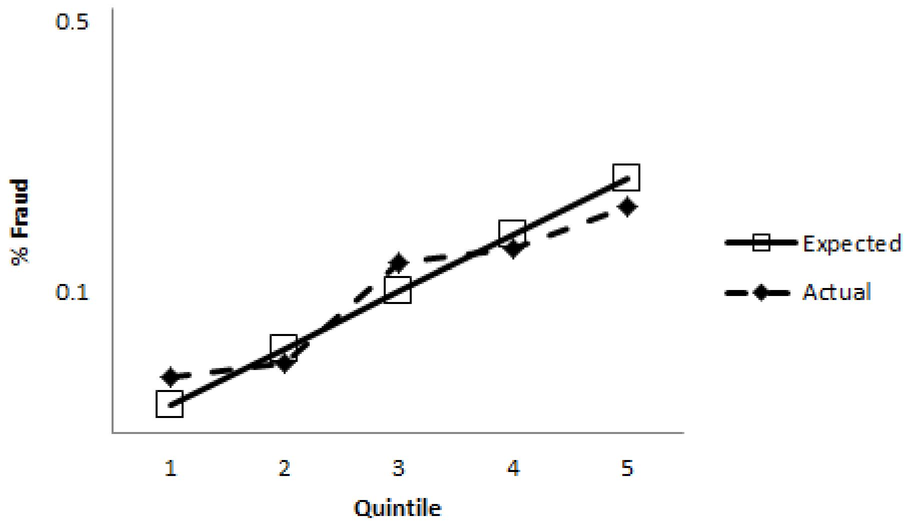
i. Saturated Model



ii. Null Model



iii. Model in Practice



EXAMINER'S REPORT

Candidates were expected to create 3 separate quintile plots, which demonstrated how saturated, null and practical models compared to actuals for a logistic model. The response of each model was expected to identify the percentage of fraudulent (or alternatively percentage of non-fraudulent) records that were identified with the model.

Common mistakes included:

- Not plotting actuals on the graph. The purpose of a quintile plot is to compare how well predicted values compare to actual values
- Plotting the same actuals on each graph. Since the records are ordered by predicted values, the records in each bucket change for each graph. Thus, actuals are not the same for each graph.
- Plotting all models on one graph. Actuals are not the same for each model.
- Sorting records by Actuals. Quintile plots are sorted by predicted values from smallest to largest value. Thus the predicted values must be monotonically increasing. Actuals need not be.
- Plotting loss ratios, pure premiums, loss costs, or losses. The response for this logistic model is either % fraudulent or % non-fraudulent claims.
- Not labeling the axes or not including scale. The average % fraud of all graphs is 0.1.
- A null model plots the grand mean of the data. The mean of this dataset is 1,000/10,000 = 0.1, not 1/9, 0.5 or 0. A null model does not mean there is no prediction (i.e. predicted value of 0), nor does it mean that half the records are predicted to be fraud (i.e. predicted value of 0.5).
- A saturated model means there is an equal number of predictors as there are records in the dataset, not that all variables are used. Thus a saturated model would perfectly predict every historical outcome in the training data.
- Not plotting the practical model between the saturated and null models. Most common was for quintile 5 of the practical model to have a larger value than the saturated model.

- Plotting something other than a quintile plot. Partial credit was given for other graphs, including, but not limited to, ROC, Actual vs. Predicted, and QQ plots, as long as an understanding of saturated, null and practical models was demonstrated.

QUESTION 6	
TOTAL POINT VALUE: 3.5	**LEARNING OBJECTIVE(S): A4c,d**
SAMPLE ANSWERS	
Part a: 1 point	

Sample 1

Discrimination Threshold: 50%

Claim	Actual Fraud	Predicted Fraud	True Pos	False Pos	True Neg	False Neg
1	Y	N	0	0	0	1
2	N	N	0	0	1	0
3	N	N	0	0	1	0
4	N	Y	0	1	0	0
5	Y	Y	1	0	0	0
6	Y	N	0	0	0	1
7	N	N	0	0	1	0
8	Y	Y	1	0	0	0
9	N	Y	0	1	0	0
10	N	N	0	0	1	0
		Totals:	**2**	**2**	**4**	**2**

Discrimination Threshold: 25%

Claim	Actual Fraud	Predicted Fraud	True Pos	False Pos	True Neg	False Neg
1	Y	N	0	0	0	1
2	N	N	0	0	1	0
3	N	N	0	0	1	0
4	N	Y	0	1	0	0
5	Y	Y	1	0	0	0

6	Y	Y	1	0	0	0
7	N	N	0	0	1	0
8	Y	Y	1	0	0	0
9	N	Y	0	1	0	0
10	N	Y	0	1	0	0
		Totals:	**3**	**3**	**3**	**1**

Discrimination Threshold: 50%

	Predicted		
Actual	Fraud	No Fraud	Total
Fraud	2	2	4
No Fraud	2	4	6
			10

Discrimination Threshold: 25%

	Predicted		
Actual	Fraud	No Fraud	Total
Fraud	3	1	4
No Fraud	3	3	6
			10

Sample 2

Claim	Actual Fraud	Above .50?	Above .25?
1	Y	N	N
2	N	N	N
3	N	N	N
4	N	Y	Y
5	Y	Y	Y
6	Y	N	Y
7	N	N	N
8	Y	Y	Y
9	N	Y	Y
10	N	N	Y

Discrimination Threshold 0.50

True Positive	False Negative
2	2
False Positive	True Negative
2	4

Discrimination Threshold 0.25

True Positive	False Negative
3	1
False Positive	True Negative
3	3

Part b: 1.5 points

Sensitivity = True Positives / Total Positives

Specificity = True Negatives / Total Negatives

False Positive Rate = 1 - Specificity

Discrimination Threshold: 50%

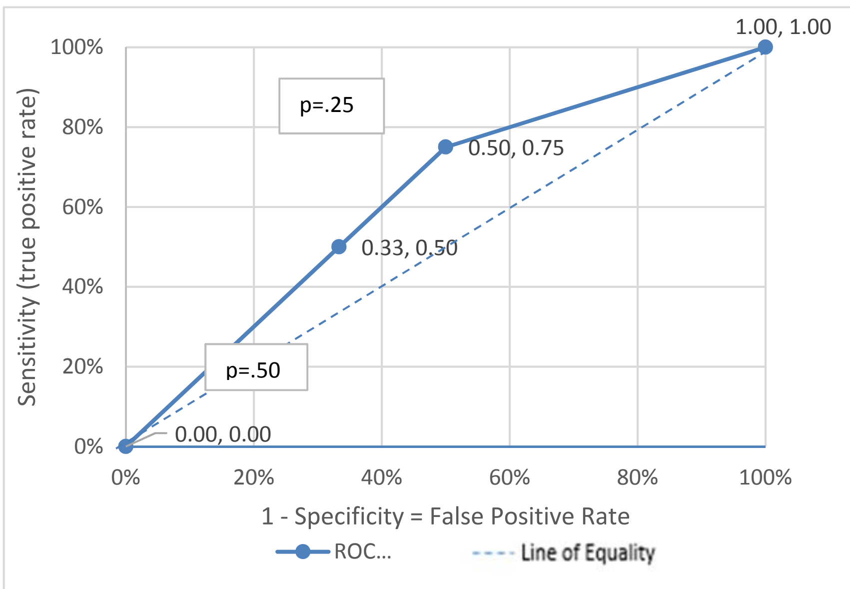
Sensitivity	0.500	=2/(2+2)
Specificity	0.667	=4/(2+4)
False Pos Rate	0.333	= 1 – 0.667

Discrimination Threshold: 25%

Sensitivity	0.750	=3/(3+1)
Specificity	0.500	=3/(3+3)
False Pos Rate	0.500	= 1 – 0.500

<u>ROC Curve</u>

false pos rate	**sensitivity**
x	**y**
0%	0%
33.33%	50.00%
50.00%	75.00%
100.00%	100.00%



Part c: 0.5 point

- An advantage of a lower threshold is that we will correctly identify more of the fraudulent claims. A disadvantage is that more of the non-fraudulent claims will be identified as fraudulent, so more time will be spent investigating.

- Advantage: You will catch more actual fraud claims because you will have a higher true positive rate. Disadvantage: You will have a higher false positive rate as well, which means you will waste resources to review claims that are not fraudulent.

Part d: 0.5 point

- A threshold of 0.25 is more appropriate. The high severity makes the cost of not investigating a fraudulent claim very high. The low frequency means that the number of additional claims that will need to be investigated is not very large. The cost of investigating these few additional claims is far less than the cost of potentially missing a few fraudulent claims at a higher discrimination threshold.
- Select 0.25 threshold. The benefit of detecting even one more fraud case is high (due to severity) and would outweigh the relatively low expense of inspecting a few extra false positives.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of confusion matrices, Receiver Operating Characteristic (ROC) curves, and the appropriateness of selecting various discrimination thresholds. This was a fairly straightforward question, and most candidates performed well. The question was very similar to the example from the reading.

Part a

Candidates were expected to construct two confusion matrices, one for each of the two discrimination thresholds. To receive full credit, candidates needed to show their work, such as creating a table of observations for true/false positive/negative occurrences, identifying the claims which fall into each matrix cell, or a sentence describing how the matrix was populated.

Common mistakes included:

- Not showing work and simply writing the correct values in the matrices

Part b

Candidates were expected to construct the ROC curve from (0,0) to (1,1) and passing through the points plotted at the two thresholds. Candidates did not have to plot every point along the curve; credit was given for piecewise linear and interpolated curved segments. The most common mistake was not including important components of the graph, such as axis labels, plotted points, or even the curve itself.

Part c

Candidates were expected to demonstrate knowledge of the advantages and disadvantages of selecting a particular discrimination threshold. To receive full credit, a sufficient level of detail was required. Simply stating that the 0.25 threshold increases sensitivity and decreases specificity, while true, does not adequately demonstrate the impact of the selection and only received partial credit.

Part d

Candidates were expected to discuss the reasoning behind selecting a particular threshold. To receive full credit, candidates needed to discuss the incentive for selecting a lower threshold, given the cost of missing a fraudulent claim due to high severity. The most common mistake was selecting the lower threshold without explaining why this made sense.

QUESTION 7	
TOTAL POINT VALUE: 1.75	LEARNING OBJECTIVE(S): B1b

SAMPLE ANSWERS

Part a: 1 point

Risk load = k[E[x^2;l] + $\delta[E[X;l]^2]$
δ = 0
risk load @ 1000 = 0.000064 [790,123] = 50.568
risk load @ 5000 = 605.92

ILF(5k) w/o risk load = $\frac{2485(1.15)}{840(1.15)} = \mathbf{2.958}$

ILF(5k) w/ risk load = $\frac{2485(1.15)+605.92}{840(1.15)+50.568} = \mathbf{3.4072}$

Part b: 0.75 point

Sample 1

$$\frac{.75X + .25(3.407X)}{.75X + .25(2.958X)} - 1 = +\mathbf{7.54\%}\ \boldsymbol{increase\ in\ premium}$$

Sample 2

Average ILF w/o risk loads = [1*(.75) + 2.9583*(.25)] = 1.4896
Average ILF w/ risk loads = [1*(.75) + 3.4072*(.25) = 1.6018

Overall Impact (dollar)
1.6018 – 1.4896 = .1122

Sample 3
Assume base premium of 1000
Average ILF w/o risk loads = [1000*(.75) + 1000*2.9583*(.25)] = 1489.575
Average ILF w/ risk loads = [1000*(.75) + 1000*3.4072*(.25) = 1601.80

1601.80 / 1489.575 – 1 = 0.0753

EXAMINER'S REPORT

Candidates were expected to understand the properties of ILFs, how to calculate an ILF when provided with an ALAE provision and how to calculate it when accounting for risk loading.

Part a

Candidates were expected to know how to calculate an ILF both with and without a risk load. Given the parameters k and δ, they were expected to first calculate the risk load for each limit.

Common mistakes included:

- Not multiplying the expected ground-up losses by the ALAE percentage in the ILF with risk load calculation

- Multiplying both expected ground-up losses and the risk loads by the ALAE percentage when calculating the risk-loaded ILF
- Using the wrong definition of the risk load function by multiplying the k-multiplier with the variance of the ground-up loss distribution

Part b

Candidates were expected to know how to calculate an average ILF given portfolio weights and the premium impact of using ILFs with risk loads versus ILFs without risk loads.

No base premium was given, and therefore any base premium used was given full credit. However, the base premium assumed must be consistent when calculating both average ILFs.

Common mistakes included:

- Calculating a weighted average of premium (with and without risk load) and determined the impact (this was the most common mistake)
- Applying the ILF to the 75% of the portfolio at the 1,000 limit (as opposed to applying it to the 25% of the portfolio that is at the 5,000 limit)
- Calculating an ILF with risk load at the 1,000 limit (this should be 1.000)

QUESTION 8	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): B1**
SAMPLE ANSWERS	
Part a: 0.75 point	

Sample 1

Current Layer Frequency = F(10,000)-F(5,000) = 0.947 – 0.958 = 0.088
Future Layer Frequency = F(10,000/1.1) – F(5,000/1.1) = 0.938 – 0.842 = 0.096
% Change Frequency = 9.09%

Sample 2

$$t_n = \frac{S\left(\frac{a}{t}\right)}{S(a)} = \frac{S(4545)}{S(5000)} = \frac{1-.842}{1-.859} = 1.12 \quad => 12.1\%$$

Part b: 1 point

Sample 1

$$Current\ Layer\ Severity = \frac{E[X;10000]-E[X;5000]}{F(10000)-F(5000)} = \frac{2308-1875}{0.088} = 4920$$

$$Future\ Layer\ Severity = \frac{1.1(E[X;9091]-E[X;4545])}{F(9091)-F(4545)} = \frac{1.1(2256-1807)}{0.096}$$
$$= 1.1 \times 4677 = 5145$$

$$\%\ Change\ severity = \frac{5145}{4920} - 1 = 4.57\%$$

$$\%\ Change\ PP = (1+0.099)(1+0.0457) - 1 = 14.06\%$$

Sample 2

$$t_s = \frac{t(E[X;10000/1.1]-E[X;5000/1.1])}{E[X;10000]-E[X;5000]} - 1 = \frac{1.1(2256-1807)}{2308-1875} - 1 = 14.06\%$$

Sample 3

$$Current\ Layer\ Severity = \frac{E[X;10000]-E[X;5000]}{1-F(5000)} = \frac{2308-1875}{1-.859} = 3070.9$$

$$Future\ Layer\ Severity = \frac{1.1(E[X;9091]-E[X;4545])}{1-F(4545)} = \frac{1.1(2256-1807)}{1-.842} = 3125.9$$

$$\%\ Change\ severity = \frac{3125.9}{3070.9} - 1 = 1.8\%$$

$$\%\ Change\ PP = (1.121)(1.018) - 1 = 14.1\%$$

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to apply frequency and severity distributions to determine expected losses by layer. Credit was given for 2 interpretations of "claims in the layer": 1. All the losses above 5,000 2. Only the losses that fell in the layer 5,000 to 10,000
Part a
Candidates were expected to calculate the effect of inflation on the frequency. Most candidates were able to calculate this correctly and received full marks. A common mistake was multiplying by the trend factor (1.1).
Part b
Candidates were expected to calculate the effect of inflation on the severity and pure premium. Common mistakes included: • Multiplying the pure premium trend by the frequency trend • Forgetting to multiply the severity trend by the frequency trend • Forgetting to multiply the future layer severity by the trend factor (1.1)

QUESTION 9	
TOTAL POINT VALUE: 3.25	**LEARNING OBJECTIVE(S): B3b, B3c**
SAMPLE ANSWERS	

Sample 1

Using ISO CGL Experience Rating Plan

(1)	(2)	(3)	(4)	(5)	(6)	(7) = (1)*(2)*(3)*(4)*(5)*(6)
Year	**Prem Ops Prem**	**ELR**	**PAF 1**	**PAF 2**	**Detrend**	**Co Subj LC**
2015	200K	0.7	1.03	0.94	0.907	122942.04
2014	200K	0.7	1.03	0.91	0.864	113375.81
2013	200K	0.7	1.03	0.88	0.823	104435.41
						340,753.25

Z = 0.54
EER = 0.94
MSL = 173,150

Act. Loss:

Clm	Lim Ind (K)	ALAE	Lim Ind + ALAE (K)
1	Not in exp pd		
2	Not in exp pd		
3	58	0	58
4	20	85	105
5	100	82	173.15
6	8	5	13
			349,150

AER = (349,150 + 0) / 340,753.25 = 1.0246

Mod = Z * (AER – EER) / (EER) = 0.049

Mod. Prem = 200K * 1.049 = **209,800**

Sample 2

PY	BLEL	PAF1	PAF2	Detrend	CSLC	EER	ILF	Exp. Dev.
2015	140	1.03	0.94	0.907	122,942	0.94	-	-
2014	140	1.03	0.91	0.864	113,376	0.94	-	-
2013	140	1.03	0.88	0.823	104,435	0.94	-	-
					340,753			0

CSLC = 340,753 → Z = 0.54   EER = 0.940   MSL = 173,150

Clm	PY	BL Indemnity	ALAE	MSL
1	2011	-	-	-
2	2012	-	-	-
3	2013	58k	0	58k
4	2013	20k	85k	105k
5	2014	100k	80k	173.15k
6	2015	8k	5k	13k

Ratable losses = 58k + 105k +173.15k + 13K = 349.15k

AER = 349.150/340.753 = 1.025

Mod = 0.54 x (1.025 – 0.940)/0.940 = 0.049

Hence the Mod Prem = 200,000 x (1+0.049) = **209,725**

Sample 3

Experience Period 1/1/13 – 12/31/15
BLEL = 200,000(0.7) = 140,000

PY	BLEL	x PAF 13B	x PAF 13C	x Detrend	= CSLC
2015	140,000	1.03	0.94	0.907	122,942
2014	140,000	1.03	0.91	0.864	113,376
2013	140,000	1.03	0.88	0.823	104,435
					340,753

	2011	2012	2013	2014	2015	2016	2017
CLM YID	1	2	3	4	5	6	7

Lookup: Z=0.54 EER=0.940 MSL=173,150
Basic Limit = 100,000

Clm	BLL	BLL + ALAE	Lim by MSL
3	58,000	58,000	58,000
4	20,000	105,000	105,000
5	100,000	182,000	173,150
6	8,000	13,000	13,000
			349,150

No development since claims made polices.

AER = 349,150/340,753 = 1.025 Mod = 0.54((1.025 – 0.94)/0.94) = 0.05

Modified Premium = 200,000 (1+0.05) = **210,000**

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
Candidates were expected to follow the ISO rating procedure to calculate the modified premium. Table 13B, 13C, detrend factors, and lookups from the Credibility and Maximum Single Loss (MSL) table can be referenced directly from the ISO rating manual. This was a relatively straightforward question, which candidates performed very well on. Candidates did not receive full credit if they did not limit to BLL even if they calculated the correct premium due to the BLL+ALAE being limited to the MSL (claim #5). Common mistakes included: • Looking up incorrect values in tables PAF 13B and PAF 13C • Forgetting to calculate the modified premium • Misidentifying which claims were in the experience period • Calculating IBNR and adding IBNR into the numerator to calculate AER. Some candidates calculated IBNR but then stated that because we are looking at a claims-made policy, no loss development would be expected. Candidates who did this received credit for stating that no development would take place. • Forgetting to limit BLL+ALAE at the MSL • Forgetting to multiply by Z in the Mod factor calculation • Using the higher detrend factors for later years (flipping the order in which detrend factors should be applied)

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 10	
TOTAL POINT VALUE: 1.0	**LEARNING OBJECTIVE(S): B3**

SAMPLE ANSWERS

- Experience used is skip the prior year (the second year of no safety program) and use the 3 years prior to that. So the safety prog was in place for 2 of 3 years in experience but not going forward. So I will apply a debit of 2/3 x 10% (2/3 of the max) to account for increase in future losses over historical.
  Will be reducing staff next month so won't be in experience at all. Reduction in staff does not mean remaining staff is any less experienced, needs more training or more supervision. Hence, I don't see a need (or ability) to schedule rate based solely on fewer employees. 0%
- The experience of the 1st of the 2 yrs the safety program was removed will be reflected/incorporated in experience rating will not have the arguably poorer loss experience from removal of safety program so can give +10%(2/3) = +6% debit
  Assuming insured reduces its staff by most reckless ones, perhaps can give small credit of say -2% credit since this will occur in the next month and is definitely not reflected in 3yr experience window but will impact prospective loss experience.
- I assume that this risk is also experience rated using the standard 3 years of data w/ a lag.
  Cooperation – safety program elimination should partially show in claims exp but not fully. I recommend a debit at +5% for the piece not yet in data.
  Employees – upcoming change so not in data. This could lead to understaffing or improper supervision. Recommend the full 5% debit in this category.
- Cooperation range should be 2%. As it's removed 2 years ago it will be partially reflected in experience rating. So I recommend 1% debit to reflect the other part that hasn't be reflected.
  Reducing staff will reduce the payroll so will be reflected in prem credit due to change of exposure. However, staff reducing might lead to overload for other works. Then risk will be higher. So I recommend 1% debit to reflect potential riskier workers.
- I recommend a 5% debit mod for safety program. They removed it so they'll be less safe and losses will increase in future. Since it was 2 years ago, I assume it is only partially reflected in experience so still need some debit, but not full.
  I recommend a 5% credit for reduction in staff, assuming they will keep the most experienced and highly trained employees and let the less experienced employees with less training go. This will reduce future losses.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of experience rating and its interaction with schedule rating. To receive full credit, candidates needed to be able to recommend a specific debit/credit based upon the information given.

Candidates were expected to understand that the safety program removal was only partially reflected in experience and choose a specific debit for the modification. They were also expected to realize that the mix of employees might change and this was not reflected in past experience, explain how that might occur, and give an appropriate debit or credit amount to go with the explanation.

SAMPLE ANSWERS AND EXAMINER'S REPORT

Note that some candidates assumed this was an ISO G/L policy. Credit was given accordingly for reasonable responses using this assumption.

Common mistakes included:

- Assuming that the experience period fully held experience of no safety program
- Not specifically mentioning that the experience period was the reason for the debit on the safety program
- Assuming that the employer received a credit in the employees characteristic due to exposure
- Confusing the credit/debit label or not giving a specific credit/debit at all

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 11	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): B4**
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

Efficiency Stat = Var(Standard LRs)/Var(Manual LRs)

Current Plan:
Sample Var (std LR) = [(0.6-0.66)^2 +……..+(0.64-0.66)^2]/4 = 0.00165
Sample Var (Man LR) = 0.021725

Efficiency Stat = 0.00165/0.021725 = 0.0759

Proposed Plan:
Sample Var (std LR) = 0.00175
Sample Var (Man LR) = 0.0308

Efficiency Stat = 0.00175/0.0308 = 0.0568

The proposed plan has a lower efficiency stat => the proposed is better.

Sample 2

Manual Variance Current =[(0.42-0.64)^2+……..+(0.8-0.64)^2]/5 = 0.01738
Standard Variance Current =[(0.6-0.66)^2+………+(0.64-0.66)^2]/5 = 0.00132

Efficiency stat = 0.00132/0.01738 = 0.076

Manual Variance Proposed =[(0.35-0.64)^2+……..+(0.75-0.64)^2]/5 = 0.02464
Standard Variance Proposed =[(0.72-0.69)^2+………+(0.64-0.69)^2]/5 = 0.0014

Efficiency stat = 0.0014/0.02462= 0.057

The proposed is better as it has a lower efficiency stat.

Part b: 1.0 point

Sample 1

Even though proposed has a better efficiency test, I would not choose the new plan. Its separation from min to max manual LRs is only a slight improvement (75% -35%) vs (80%- 42%) but at the cost of 3% on the standard LRs. And the implementation is expensive.

Sample 2

- Customer retention – changing plans may give big hikes to some customers, hurting retention as they go somewhere else.
- Premium growth – more efficient plan will grow premium healthily rather than by getting risks no one else wants.
- Loss ratio deterioration – more efficient plan => less adverse selection => less LRs deterioration

  I recommend adopting the new plan as its cost is only about 2% of premium and it addresses more concerns.

EXAMINER'S REPORT

Candidates were expected to assess the effectiveness of the experience rating plan and support a recommendation on whether or not to implement.

Part a

Candidates were expected to perform an efficiency test. To receive full credit, candidates needed to correctly calculate the variances (of manual and standard loss ratios) and efficiency test statistics for the current and proposed plans. A correct conclusion was needed based on the efficiency ratios. There were several reasonable ways to calculate variance, and full credit was given for both population or sample statistics, weighting with manual premium, and using the problem's supplied mean. Most candidates did very well on this part.

Common mistakes included:

- Calculating standard deviation or SSE instead of variance
- Switching the numerator and denominator in the efficiency ratios
- Stating a statistic is better without stating or showing the method of comparison

Part b

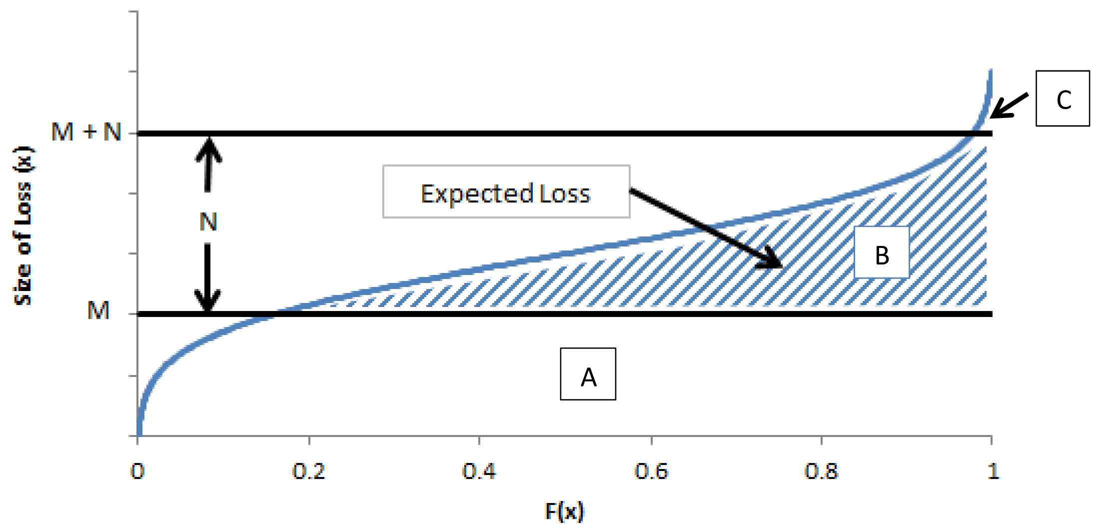
In order to receive full credit, candidates were expected to:

- Provide a CLEAR recommendation. While most candidates simply recommended keeping the current or adopting the new plan, credit was given for adopting the new plan with an off-balance or with reduced credibility.
- Provide adequate support for the recommendation.

Candidates did well on this part, with the most common mistake being just listing pros and cons without making a specific recommendation.

QUESTION 12	
TOTAL POINT VALUE: 3.25	LEARNING OBJECTIVE(S): B1b, B2b
SAMPLE ANSWERS	

Part a: 1.0 point



Expected Insured Loss = B

Part b: 0.5 point

Sample 1

i. $\int_{M}^{M+N} 1 - F(x)dx$

ii. $\int_{M}^{M+N} xdF(x) + (M + N) * \big(1 - F(M + N)\big) - M * \big(1 - F(M)\big)$

Sample 2

i. $\int_{M}^{M+N} S(x)dx$

ii. $\int_{M}^{M+N} (x - m)dF(x) + (N) * S(M + N)$

Part c: 0.5 point

Sample 1

The layer method may be preferred when the survival function is easy to integrate.

The size method may be preferred when empirical data is not available and integrals need to be evaluated algebraically.

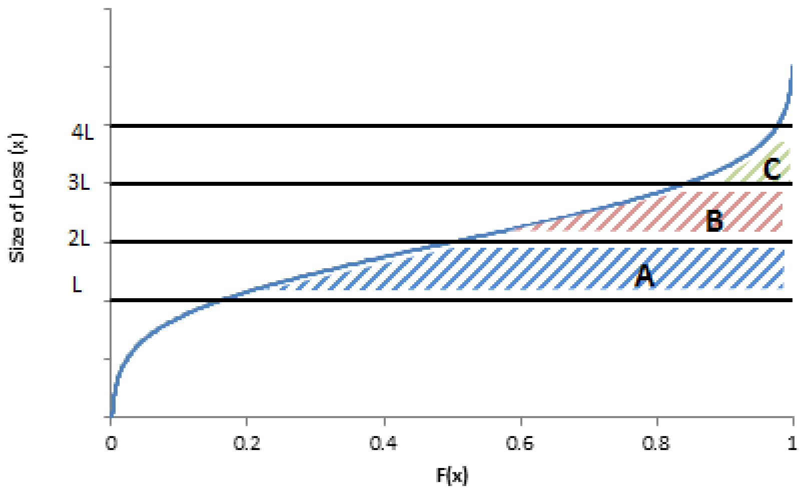
Sample 2

The layer method may be preferred when calculating expected losses at many limits.

The size method may be preferred when calculating expected losses at one limit as it is more intuitive to explain.

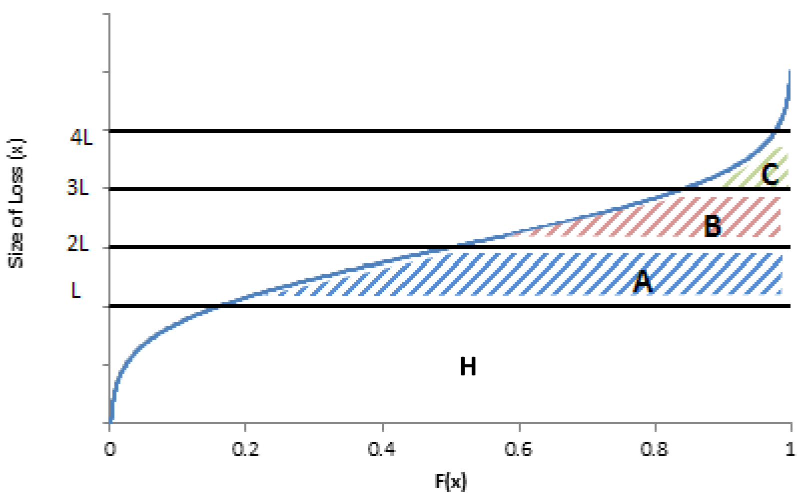
Part d: 1.25 points

Sample 1



The consistency test of ILFs states that the premium calculated from the layer formula applied to successive excess layers of constant width is a decreasing function of the attachment point limit. Area A > Area B > Area C therefore Premium for Layer L to 2L > Premium for Layer 2L to 3L > Premium for Layer 3L to 4L and the consistency test is satisfied.

Sample 2



The consistency test says that ILFs should increase at a decreasing rate.

$$ILF\ (2L) = \frac{A + H}{H}$$

$$ILF\ (3L) = \frac{A + B + H}{H}$$

$$ILF\ (4L) = \frac{A + B + C + H}{H}$$

$$\frac{ILF(3L) - ILF(2L)}{3L - 2L} > \frac{ILF(4L) - ILF(3L)}{4L - 3L} = \frac{B}{L} > \frac{C}{L}$$

Since are B is greater than area C the ILFs pass the consistency test.

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of Lee diagrams for various policy provisions and have an understanding of the size and layer method.

In part (d), candidates were expected to describe the consistency test for ILFs and tie it to a Lee diagram with various limits.

Part a

Most candidates answered part (a) by drawing a Lee diagram for a single loss and labeling the size of loss on the y-axis. Candidates were generally successful in correctly labeling the deductible ***M*** but often did not correctly label ***M+N*** as the policy limit.

The term *insured* was ambiguous and could've been interpreted as meaning losses covered by either the policyholder or the insurance company. Candidates who labeled the expected insured loss as either sections ***A+C*** or ***B*** in the diagram above received full credit.

Some candidates interpreted part (a) as a request for an aggregate Lee diagram for the total losses to the policy. Candidates who interpreted this question as requesting an aggregate Lee diagram were able to receive full credit. The exam syllabus does not give clear examples of aggregate Lee diagrams with the existence of per occurrence deductibles and limits and thus graders accepted a variety of solutions.

Part b

Candidates were expected to provide two equations that would evaluate the amount of covered loss for the policy described in this question. Loss amounts covered under this policy are those above the deductible ***M***, and below the policy limit of ***M+N***. The provided equations were expected to account for these two values. Under the layer method the provided integral should have been evaluated with respect to the size of loss, while under the size method the integral should have been expressed in terms of the distribution function.

Candidates generally provided a correct expression for determining the amount of covered loss under the layer method, but few candidates received full credit in part (b). Common reasons for not receiving full credit included using incorrect bounds or providing an expression that did not evaluate to the amount of covered loss. Note that when candidates used incorrect values of the deductible and policy limit in part (a), they were not penalized for using these values as bounds in the equations provided in part (b).

Part c

Candidates were expected to provide one reason supporting the use of the layer method and an additional reason supporting the use of the size method.

A variety of responses were considered reasonable and candidates often received full credit for part (c).
Part d
Candidates were asked to draw a Lee Diagram to demonstrate the consistency of ILFs. To receive full credit, candidates were expected to draw an appropriately labeled Lee Diagram, provide a brief description of how to validate the consistency of ILFs, and connect these requirements back to the diagram. In general, candidates performed well on this part, receiving a majority of the available points. Candidates who did not receive full credit often failed to provide an adequate explanation of how the Lee Diagram illustrated the consistency of ILFs.

QUESTION 13	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): B5a**
SAMPLE ANSWERS	
Part a: 2.0 points	

Sample 1

E[A]= (25 + 50 + 100 + 100 + 150 + 175 + 200 + 300 + 350 + 550) / 10 = 200K

A	r	# Risks	# Risks over r	% Risks over r	ϕ(r)
0	0.0	0	10	100%	1.0
25	0.125	1	9	90%	0.875
50	0.25	1	8	80%	0.7625
100	0.5	2	6	60%	0.5625
150	0.75	1	5	50%	0.4125
175	0.875	1	4	40%	0.35
200	1.0	1	3	30%	0.3
300	1.5	1	2	20%	0.15
350	1.75	1	1	10%	0.1
550	2.75	1	0	0%	0.0

$\phi(0.25) = 0.7625$

$\psi(0.25) = 0.7625 + 0.25 – 1 = 0.0125$

$\phi(1.5) = 0.15$

b = e – (c-1)E[A] + cI

I = E[A](ϕ(1.5) - ψ(0.25))

b = 10K – (1.05 – 1)*200K + 1.05*(200K)*(.15 – 0.0125) = 28,875

G = (b + cL_G)

G = 28,875 + 1.05*300,000 = 343,875

Sample 2

$E(A) = 200{,}000$

$L_H = r_H*E(A) \rightarrow r_H = 0.25$

$L_G = r_G*E(A) \rightarrow r_G = 1.5$

Construct Table M:

r	ϕ(r)
0.125	0.875
0.25	0.7625
0.5	0.5625
0.75	0.4125
0.875	0.35
1.0	0.3
1.5	0.15
1.75	0.1
2.75	0.0

$\phi(1.5) = 0.15$

$\phi(0.25) = 0.7625$

Assume Tax multiplier = 1

$\phi(r_H) - \phi(r_G) = [(e + E[A]) - H] / (C*E[A]*T)$

$0.7625 - 0.15 = (210{,}000 - H) / 210{,}000$

$H = 81{,}375$

$r_G - r_H = [G - H] / (C*E[A]*T)$

$1.5 - 0.25 = (G - 81{,}375) / (1.05*200{,}000)$

$G = 343{,}875$

<u>*Sample 3*</u>

E[A]= (25 + 50 + 100 + 100 + 150 + 175 + 200 + 300 + 350 + 550) / 10 = 200,000

Charge = ((350,000 – 300,000) + (550,000 – 300,000))/10 = 30,000

Savings = (50,000 – 25,000) /10 = 2,500

I = 30,000 – 2,500 = 27,500

b = e – (c – 1)*E + cI

b = 10,000 – (1.05 – 1)*200,000 + 1.05*27,500 = 28,875

Assume no prem tax:

G = b + cL_G

G = 28,875 + 1.05*300,000 = 343,875

Part b: 0.5 point

<u>*Sample 1*</u>

E[A] from sample in part (a) = 200,000.
E[A] from 5 new risks in part (b) = 515,000

The actuary is incorrect. You should combine if the expected value of losses and the loss distribution are the same. In this case, the new risks E[A] is significantly larger than that of part (a).

They could possibly have a similar distribution since coming from same industry; however since the expected loss is so much larger, they should be using different table Ms.

<u>*Sample 2*</u>

The new book of business has an aggregate loss distribution with much higher losses than the current book. A separate loss curve and separate Table M should be used. The new book is not reflective of the old book of business and combining them will significantly affect risk charges leading to inaccurate premiums that are not reflective of the distribution of potential losses.

SAMPLE ANSWERS AND EXAMINER'S REPORT

EXAMINER'S REPORT
The candidate was expected to use the concepts of Table M insurance charges and savings to construct a loss sensitive rating plan, and to understand the actuarial principles and concepts underlying the construction of a retrospective rating plan. Overall, candidates performed well on this question.
Part a
Candidates were expected to correctly calculate the parameters of a retrospective rating plan to find the maximum premium under the plan. If a candidate stated an assumption regarding a tax rate that would have resulted in a different answer from the sample solutions, credit was awarded provided the assumption was applied properly. Common mistakes included: • Using incorrect formulas for the basic premium and/or maximum premium • Using expected limited loss, instead of total expected loss • Failing to calculate the net insurance charge on a per risk basis (not averaging the charge and/or savings across all risks)
Part b
Candidates were expected to demonstrate the understanding that Table M should be calculated using experience from risks of a similar size and subject to similar hazards, and to assess the validity of the actuary's statement. Common mistakes included: • Not providing an assessment of the actuary's statement • Failing to state, in some form, that creating a combined Table M using risks with different aggregate loss distributions in the same Table M is not appropriate

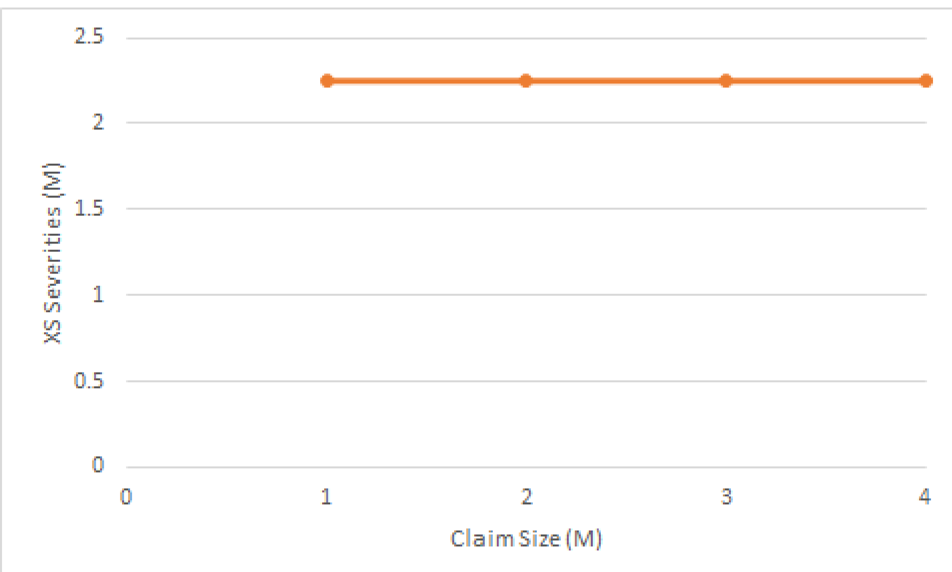
QUESTION 14	
TOTAL POINT VALUE: 3.0	LEARNING OBJECTIVE(S): B1d
SAMPLE ANSWERS	
Part a: 1.5 points	

Sample 1

Limit, L	1-F(L)	E[X;L]	XS Severity at L
1,000,000	.05	387,525	2,249,500
2,000,000	.03205	427,890	2,249,922
3,000,000	.02055	453,765	2,249,878
4,000,000	.0132	470,300	2,250,000

E[X] = 10,000,000,000/20,000 = 500,000
E[X; 1M] = (6,750,500,000 + 1000(1,000,000))/20,000 = 387,525
E[X; 2M] = (6,750,500,000 + 525,300,000 + 2,000,000(230 + 147 + 264))/20,000 = 427,890
E[X; 3M] = (6,750,500,000 + 525,300,000 + 566,500,000 + 3,000,000(147 + 264))/20,000 = 453,765
E[X; 4M] = (6750,500,000 + 525,300,000 + 566,500,000 + 507,700,000 + 264(4,000,000))/20,000 = 470,300

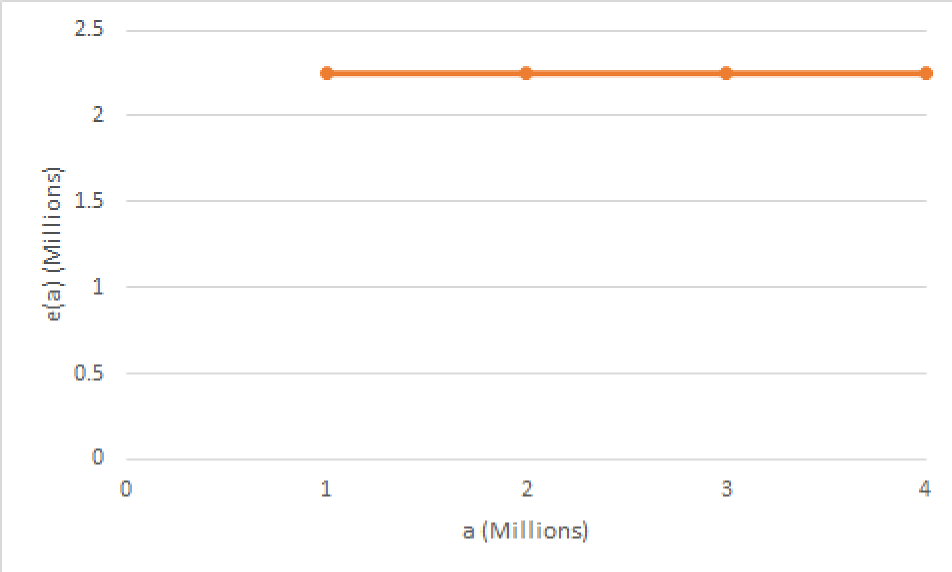
XS Severity at L = (E[X] – E[X; L])/(1-F(L))



Sample 2
Excess sev = e(a) = Sum(loss above a – a * claims above a)/# above

e(1M) = (10M – 6,750,500 – 1000 * (359 + 230 + 147 + 264))/(359 + 230 + 147 + 264) = 2,249.5k
e(2M) = (566,600 + 507,700 + 1,650,000 – 2000 * (230 + 147 + 264))/(230 + 147 + 264) = 2,249.9k

e(3M) = (507,700 + 1,650,000 – 3000 * (147 + 264))/(147 + 264) = 2,249.9k
e(4M) = (1,650,000 – 4000 * 264)/264 = 2,250k



Part b: 1.5 points

Sample 1

From a. excess severity is flat

- ➔ Losses follow exponential distribution above 1M with beta = 2.25M
- ➔ e(5M) = e(4M) = … = e(1M) = 2.25M
- ➔ P(X>5M|X>1M) = $e^{-(5-1)M/2.25M}$ = 0.169
- ➔ E[N|X>5M] = (20k – 19k) * 0.169 = 169.01
- ➔ Reinsurer E[L] = 169.01 * 2.25M = 380.28M

Sample 2

(1)	(2)	(3)	(4) = (3)/(2)
Limit (L)	Claims over L	Claims over L + 1M	% of claims in L that are over L + 1M
1	20k – 19k = 1000	1000 – 359 = 641	641/1000 = 0.64
2	641	641 – 230 = 411	411/614 = 0.641
3	411	411 – 147 = 264	264/411 = 0.642
4	264	264 * .641 = 169.22	.641 <- selected based on constant pattern

Based on memoryless feature of exponential distribution, we estimate that there are $E[N_{5M}]$ = 169.22 claims above 5M. From before, we estimated that $E[X_{5M}]$ = 2.25M

So we estimate reinsurer expected loss as $E[S] = E[X_{5M}]*E[N_{5M}]$ = 2.25M * 169.22 = 380,745,000

EXAMINER'S REPORT

Candidates were expected to calculate excess severities given claims data at various loss layers, determine the underlying distribution from the excess severity pattern, and calculate total expected losses in a given loss layer.

Candidates generally performed well on part (a). Common reasons for not receiving full credit on part (a) included calculating and graphing the incorrect metric, misinterpreting claims information provided as layer losses instead of ground up, and using an incorrect formula for excess severity.

Part (b) was generally difficult for candidates. Common reasons for not receiving full credit on part (b) included:

- Assuming the exponential distribution applied to ground up loss instead of loss excess 1M
- Assuming all losses above 4M also exceeded 5M

This question was based on Chapter 5 of the Bahnemann text.

Part a

Candidates were expected to calculate the excess severities above each of the indicated limits given the loss data provided and to graph them with properly labeled axes.

Candidates who calculated and plotted the incorrect metric, such as the CDF or excess ratio, received no credit as this did not demonstrate knowledge of what an excess severity was.

Common mistakes included misinterpreting the claims information provided as layer losses instead of ground up, or using an incorrect formula for excess severity (such as calculating ground up severity instead of excess severity for claims that crossed a given limit). The former is an incorrect assumption because even if a candidate misinterpreted the table, the severities produced by this method would result in layer severities greater than the 1M layer (ex: 525,300,000/359 = 1.46M average which is > the 1M layer span).

Candidates who incorrectly calculated excess severities but correctly plotted and labelled them received partial credit.

Part b

Candidates were expected to correctly identify the underlying loss distribution given the graphed excess severities and apply it to calculating the total expected excess losses above 5M.

Candidates who assumed all losses and claims above 4M were also above 5M did not receive full credit as this was not an assumption that could be drawn from the data provided.

Common mistakes included:

- Not identifying a distribution from the graphed excess severities
- Assuming the exponential distribution applied to ground up loss instead of loss excess 1M
- Using the overall ground up mean as the beta parameter for the exponential distribution instead of 2.25M

Candidates who identified a correct distribution and inferred a reasonable excess severity in part (b) based off of incorrectly calculated excess severities from part (a) received partial credit.

Some candidates identified a Pareto distribution and then calculated a constant rate of decrease in the survival function to get number of claims excess of 5M. As a constant rate of decrease is a property of an exponential function and not a Pareto, only partial credit was provided.

QUESTION 15	
TOTAL POINT VALUE: 3.25	**LEARNING OBJECTIVE(S): B6**
SAMPLE ANSWERS	
Part a: 1.75 points	

<u>*Sample 1*</u>
T=1/(1-.03)=1.031
C=1+.08=1.08
Assume basic prem already include charge for per occ limit. Since we use E(A_d)=400K to determine initial premium, assume no need to subtract expected loss exceed max ratable loss

E(R) = (B + c*E(L_d))*T = (375K+1.08*400K)*1.031 = 832,017
Also, insured does not elect to stabilize prem
Tim0: insured pays insurer $832,017 initial retro prem

Time 18:
L_d=375,000<600K
R=(375K+1.08*375K)*1.031 = 804,180
832,017-804,180=27,837
Insurer refund insured $27,837

Time 30:
L_d=475K<600K
R=(375K+1.08*475K)1.031 = 915,528
915,528-804,180=111,348
Insured pays insurer $111,348

Time:42
L_d=700K>600K
R=(375K+1.08*600K)*1.031=1,054,713
1,054,713-915,528=139,185
Insured pays insurer $139,185

<u>*Sample 2*</u>
P0=(375,000+400,000*1.08)*(1/(1-3%))=831,959
P18=(375,000+375,000*1.08)*(1/(1-3%))=804,124
Cash flow = -27,835
P30=(375,000+475,000*1.08)*(1/(1-3%))=915,464
Cash flow = 111340
P42=(375,000+600,000*1.08)*(1/(1-3%))=1,054,639
Cash flow = 139,175

Part b: 1 point

<u>*Sample 1*</u>
I propose a large dollar deductible policy with per-occ limit of 150K and agg limit on ded of 600,000

Justify: Insured would be responsible for same losses, but insurer would originally pay all claims and insured would later reimburse for amounts below deductible, thus letting insured hold on to cash longer

Sample 2

LDD with 150K ded and max ded losses of 600K. Insured doesn't have to pay for expected ded losses up front, only expected excess. This lowers the premium at onset giving cash flow advantage

Sample 3

A self-insured retention policy (w/ per occ ret 150,000 and agg limit 600,000) would decrease up-front expenses since there is no credit risk to the insurer and the insurer only pays ALAE on losses above the retention. The insured's excess loss risk remains the same.

Sample 4

A retrospective plan with the same limit parameters but based on paid loss instead. Since paid loss always lags incurred loss, the insured's payment to the insurer will be delayed by that lag.

Part c: 0.5 point

Sample 1

Imposes credit risk for insurer (higher credit risk) – pay for all losses ground up, risk of unable to collect payment from insured for loss < ded. Retro also has credit risk but smaller, less amount for retro prem adjustments

Sample 2

Insurer may be subject to credit risk, the insured may not be able to pay money back (deductible) The holdback provision in part b increase the credit risk for the insurer. By collecting retro adjustments over time, the insurer makes sure to get some payment and knows sooner when payment issues are likely. Waiting until 42 months to collect adjustment premium, the insurer doesn't know whether the additional premium will be collectible

Sample 3

Investment income lost to insurer – lost cash flow reduces investment opportunity from premium up front float

Sample 4

In SIR, the insurer won't be able to adjust the claims that are below the retention, thus it doesn't have the ability to control the loss

EXAMINER'S REPORT

Candidates were expected to know how to calculate retrospective premium adjustments, how the cash flows go back and forth between insured and insurer under various loss sensitive rating plans, criteria for selecting various loss sensitive rating plans, and the advantages and disadvantages of each to both insured and insurer.

Part a

Candidates were expected to be able to calculate the loss conversions factor, calculate the tax multiplier using the given tax rate, calculate the incremental cash flows between insured and insurer due from the initial premium payment and retrospective rating adjustments at t= 18, 30 & 42 months by applying the limited losses, L_t, at each adjustment time in the formula: $E(R) = (B+ c^* L_t)^*T$ The question did not provide information to calculate the excess charge. Candidates were expected to recognize this and therefore assume the excess charge is included in Basic Premium. However some candidates made an assumption of expected excess loss using actual losses provided. Since these excess charges will cancel when calculating <u>incremental</u> cash flows, candidates could still arrive at the correct incremental cash flows. These candidates did not receive full credit, since this is not a valid assumption: this would imply the insurer knows about actual excess losses at policy inception. Some candidates seemed to rely on the assumptions of past papers no longer on the exam and ignore the "Fisher et al." paper new to the syllabus which holds that basic premium may include the per-occurrence excess charge if losses influencing the premium are subject to a per-occurrence loss limit. Given the question asked for the cash flow payments made by the insured, a payment at t=18 of $0 was also accepted since the insured is not making a payment, but instead receiving money from the insurer. Common mistakes included: • Using actual excess losses at each time in the retro adjustment calculation • Assuming loss conversion factor of 1.0 • Calculating the tax multiplier as 1+ tax rate instead of 1/(1 - tax rate) • Using expected limited loss at t=0 of 150K • Using B*T for t=0 premium • Confusing with LDD loss reimbursements and applying losses paid to claimants to calculate net incremental cash flow between insured and insurer • Not capping losses at maximum ratable loss of 600K at 42 months
Part b
Candidates were expected to identify an alternative policy, justify that it results in an increased cash flow benefit to the insured by explaining the cash flow advantage and justify that it will not increase excess losses retained by the insured by specifying limits under the alternate policy structure that match the current incurred loss retro policy. Alternate policy options accepted include large dollar deductible, self-insured retention, paid loss retro, increasing minimum ratable loss, dividend plan, and holdback provision. The following were not accepted: • Guaranteed cost policy option as it results in higher premium at policy inception • Increasing the maximum ratable loss or the per-occurrence limit since it results in additional excess loss retained by insured

- Using a loss development factor to stabilize premium since it does not result in a cash flow advantage due to higher premiums paid initially by developing losses

Common mistakes included:

- Not providing justification that the insured does not retain any additional excess risk by stating the occurrence and aggregate limit for alternate policy options or noting that these limits should be the same as in current incurred loss retro policy. Some candidates simply responded by saying the insured's excess risk is the same which lacks the necessary justification.
- Stabilizing premium by using LDFs, reducing swings in payments or other reasons
- Not demonstrating the cash flow advantage under the alternate option relative to the current policy

Part c

Candidates were expected to identify one disadvantage to the insurer under the alternate policy in part (b), and explain why it is a disadvantage.

Partial credit was given for either only identifying the disadvantage or not providing the right explanation of the disadvantage. Simply stating that it leads to cash flow disadvantage for the insurer without a clear explanation of why there is a cash flow disadvantage to the insurer was not accepted.

Common mistakes included:

- Responding with insured's disadvantage
- Not explaining the insurer's disadvantage clearly

QUESTION 16	
TOTAL POINT VALUE: 1.75	**LEARNING OBJECTIVE(S): B2**
SAMPLE ANSWERS	

Part a: 0.5 point

Sample 1

1) Lack of data. The distribution estimated can be difficult or far away from reality due to sparse data.
2) Need more calculation. Maybe time consuming.

Sample 2

1) Computational complexity – it requires to calculate aggregate losses more than necessary as the additional informal becomes available. This computation is cumbersome and burdensome for the portfolio with a large number of claims coming in.
2) Sensitive to input parameters – the distribution can be changed significantly due to the initial set of parameters and it can be significantly different than the initial expectation.

Sample 3

1) Data is thin especially for the highest claim amounts, the charge for highest entry ratio will have high standard errors.
2) The fitted curve will depend a lot on highest few points, which are the most volatile so shape of curve may have significant bias.

Sample 4

1) Parameter risk – need to estimate parameters.
2) Computational intensive. Need a computer to calculate

Part b: 1.25 points

Sample 1

1) For all n risks (assume similar sizes), calculate the average aggregate loss amount. 2) Order the risk in order of their entry ratio actual/expected. 3) For each desired entry ratio (r_i), determine the % of risks whose entry ratio is above that $r_{i.}$ 4) Starting from the highest ratio, assume its charge is 0. The charge for r_{n-1} can be determined using the layer method referred in the formula: charge (r_n)=charge (r_{n+1}) + (r_{n+1} -r_n)(% of risks above r_n). Repeat until all charges are complete. If desired, the table M savings can be included by computing saving (r_n) = charge (r_n)+ r_n -1. The aggregate excess loss cost of an layer at G can be determined by finding the entry ratio r_G and multiplying the charge (r_G)* E(A)

Sample 2

Get average expected aggregate loss at all policies in that size of risk group selected. For each policy, divide agg loss by avg agg loss, this is r entry ratio. Sort entry ratio in increasing order. Determine what % of risks are above at each r. Charge at bottom r is 0, one above is (% of risk above at that r)*(that r – r – r one below) + plus charge at r below. Continue working up to fill out charges. Estimating loss above certain limit can be estimated by r=excess limit / expected loss. Expected loss * (charge that corresponds to r) = Agg loss cost of excess layer. For layer with bottom and top, do above for both limit and agg loss cost at lower limit- agg loss cost at high limit.

Sample 3

You need to get an expected aggregate loss from empirical data. Then compute entry ratios, actual/ expected losses. Then put entry ratio in order smallest to largest. For each entry ratio, calculate the % of claims bigger and the difference between the subsequent entry ratio. Now start at the highest entry ratio, multiply % bigger by size and (difference in entry ratio) and sum with the value from the subsequent row. This is the charge. To estimate agg loss cost, take aggregate/expected loss to get entry ratio. Then look up table to get the charge. Now multiply expected loss by charge to get loss cost for that agg.

Sample 4

1) From a sample of aggregate losses, calculate the average loss and use this to calculate the entry ratio for each loss, actual/expected.
2) Order the entry ratios from lowest to highest starting at 0 and going up to the max entry ratio.
3) Create columns for the # of risks at each r, # of risks above r, and the % of risks above r and calculate these values for each r.
4) starting at the bottom with the max entry ratio, set charge=0. Then working up the column use charge (r_i)=charge (r_{i+1}) + (r_{i+1} –r_i)(% of risks above r_i) to calculate the charge at each entry ratio.
5) Then the aggregate loss cost of an excess layer is given by k*E(A)+E(A)*(1-k)*charge(r_{agg}) where k=excess loss ratio and r_{agg} =Agg limit/E(A)

EXAMINER'S REPORT

Candidates were expected to know disadvantages of using a continuous approximation model and techniques to estimate aggregate loss distributions directly from aggregate data (e.g., Table M, Table L).

Part a

Candidates were expected to briefly describe the disadvantages of using a continuous approximation model versus Table M.

Some candidates responded with disadvantages in both models. Some candidates gave disadvantages for GLM but the question doesn't mention that the continuous model is fitted using a GLM.

Common mistakes included:

- Stating that the model is not easy to communicate and understand (black box) or the need to select a distribution to use, but the question states that the company is already using the continuous approximation model
- Stating that the model doesn't allow for loss-free scenarios (but this is excess layer insurance, so loss-free scenarios are not appropriate) or does not split the impacts of frequency and severity (those individual distributions may not be known, so this is actually an advantage of the model)
- Stating that it would be difficult for the model to reflect changing or varying per-occurrence limits, or cannot be easily adjusted for changes to expected loss due to

inflation - but this is a problem for both Table M and the continuous approximation model

- Mentioning that the distribution may not fit the empirical data well or may be hard to fit, but not explaining why

Part b

Candidates were expected to describe the process of constructing an empirical Table M and estimating the aggregate loss cost.

Many candidates gave the formula for savings as well, but this was not needed for full credit.

Since the meaning of "excess layer" is not clear in the question, different formulas were accepted for calculating the loss cost; (1- charge + saving)*expected loss was also accepted.

Common mistakes included:

- Forgetting to calculate aggregate loss cost
- Misstating that the charge is the entire loss cost
- Calculating layer loss cost using the charge at the top of the layer minus the charge (instead of the savings) at bottom
- No mention of the data needing to be split into groups of insureds with similar loss potential, or any data requirement

SAMPLE ANSWERS AND EXAMINER'S REPORT

QUESTION 17	
TOTAL POINT VALUE: 2.0	**LEARNING OBJECTIVE(S): B5a**
SAMPLE ANSWERS	
Part a: 1.5 points	

Adj Exp Losses = 700kx.95x($\frac{1+0.8*k}{1-k}$) = 1,143,800 -> ELG=28

Where k=1-500/700=.2857

e(A_D) = 500k

loss cost=xs + e(A_D)Φ(Agg Limit/Ltd Loss)

=200k +500k Φ(800/500)

=266,100

Part b: .5 point

Sample 1

Presence of per-occurrence limit reduces variance of severity distribution, thus reducing variance of aggregate loss distribution. ICRLL procedure approximates a limited table M by increasing the expected losses used to determine the ELG, since policies with larger expected losses have lower variance in aggregate loss distribution (just like a policy with per-occurrence limit)

Sample 2

The ICRLL procedure is used to adjust expected loss for a given risk via its state hazard group differential (accounts for riskiness and location) and its expected XS ratio. It uses the adjusted expected loss rather than unadjusted expected losses to find table M charges. It basically shifts the column of charges that we are looking at to better align variance of loss distribution.

EXAMINER'S REPORT

Candidates were expected to know how to use the ICRLL procedure to calculate the total expected loss cost, and how to construct a table of insurance charges.

Candidates were expected to understand how an occurrence limit necessitates adjustments before using a Table M. Some candidates confused the ICRLL formulas with other insurance charge calculations from the text.

Part a

Candidates were expected to use the ICRLL adjustment to calculate the total expected loss cost. Candidates that used a procedure other than the ICRLL adjustment did not receive full credit.

Common mistakes included:

- Calculating an incorrect entry ratio
- Not having the correct formula for the ICRLL adjustment (incorrect excess ratio)
- Looking up the wrong ELG

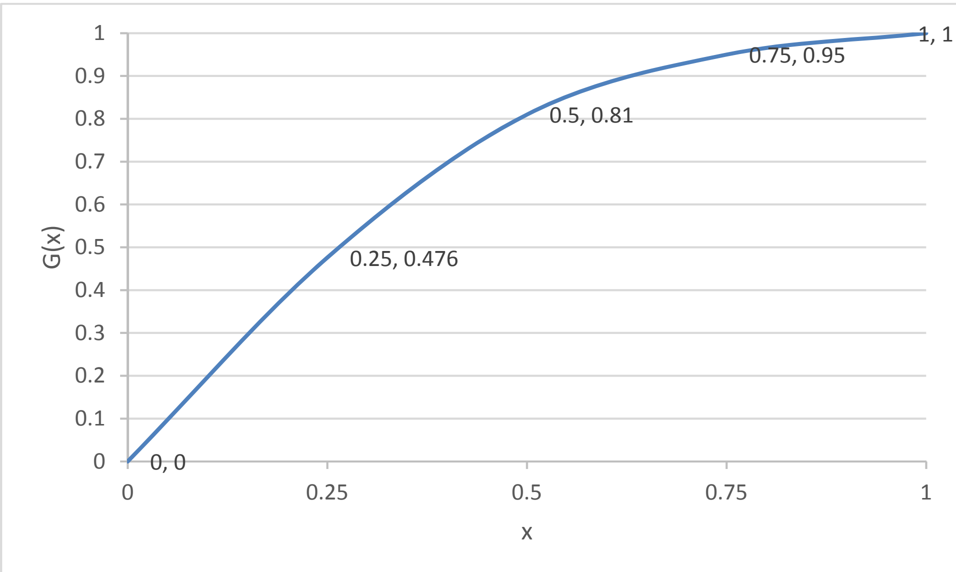
Part b

Candidates were expected to describe how the ICRLL procedure is used to adjust expected losses for a workers compensation policy.

Not fully describing the procedure resulted in only partial credit. For example, just mentioning the state hazard group adjustment without fully describing what the adjustment does did not receive full credit.

QUESTION 18	
TOTAL POINT VALUE: 3.25	**LEARNING OBJECTIVE(S): C5a, C5d**
SAMPLE ANSWERS	
Part a: 2 points	

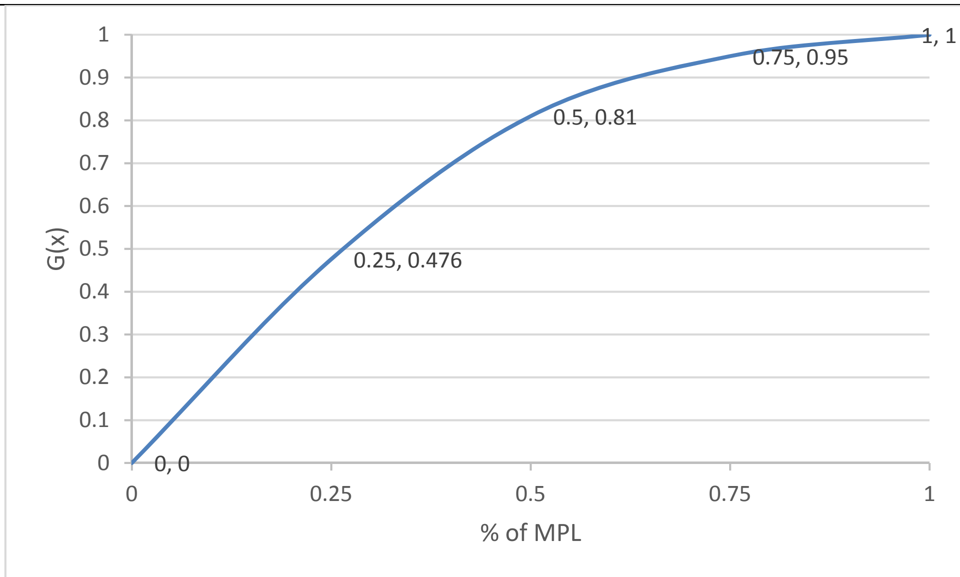
Sample 1



Loss	Probability
25%	6%/(1-80%) = 30%
50%	8%/(1-80%) = 40%
75%	4%/(1-80%) = 20%
100%	2%/(1-80%) = 10%

E(loss) = 52.5%
G(0%) = 0
G(25%) = 25%/52.5% = 0.476
G(50%) = (25%*30% + 50%*70%)/52.5% = 0.81
G(75%) = (25%*30% + 50%*40% + 75%*30%)/52.5% = 0.952
G(100%) = 1

Sample 2



G(x) = E[x; 1] / E[x]
E[x] = .8*0 +.06*.25 + … = .105
E[x;0] = 0
E[x;0.25] = .8*0+.25(.06+.08+.04+.02) = .05
E[x;0.50] = .8*0+.25*.06+.50(.08+.04+.02) = .085
E[x;0.75] = .8*0+.25*.06+.50*.08+.75(.04+.02) = .1
E[x] = 1
G(0) = 0
G(.25) = .05/.105 = .476
G(.5) = .81
G(.75) = .952
G(1) = 1

Part b: 0.75 point

Sample 1
g = 1/p, p = 0.1= 0.02/(1-.8), g = 10
μ = 1/G'(0)
G'(0) = (1-F(0))/E(x) = (1-0.8)/0.105 = 1.905
μ = 1/1.905 = 0.525
Table lookup b = 0.0147

Sample 2
p =G'(1)/G'(0)
g = 1/p
μ = 1/G'(0)
G'(1) = (1-.952)/0.25 = 0.192
G'(0) = 0.476/0.25
μ = 0.525
g = 10

Table lookup b = 0.0147

Sample 3

If part a was not re-scaled, full credit was accepted for:

g= 1/p, p =.02, g=50

μ = (0.6(0.25)+.08(0.5)+0.04(0.75)+0.02(0.1))/0.2 = 0.525

Table lookup b = 0.0003

Part c: 0.5 point

Sample 1

G(0.5) – G(0.25) = 0.81-0.4762=0.334

Sample 2

$G\left(\frac{Limit+Retention}{MPL}\right) - G\left(\frac{Retention}{MPL}\right) = G\left(\frac{50}{100}\right) - G\left(\frac{25}{100}\right)$

=G(0.5) – G(.25) = .81-.476 = .334

Credit was also given if points from the graph in part (a) was used.

EXAMINER'S REPORT

Candidates were expected to demonstrate an understanding of exposure curves. In order to answer part (a) correctly they must have also realized that the provided exposure distribution needed to be restated to a loss distribution. This was a very difficult transformation to realize in an exam setting. Candidates were also expected to recall that the g parameter in the MBBEFD exposure curves equals 1/p.

Part a

Candidates were expected to restate the provided exposure distribution to a loss distribution, calculate the points on the exposure curve, graph the points, and correctly label the x- and y-axes on the graph.

Common mistakes included:

- Not restating with respect to the loss distribution. If they did not restate the loss distribution, then they did not get credit for the points on the graph. Credit was still given to the points (0,0) and (1,1)
- Candidates often missed the axis labels. Credit was not given for % of losses, % exposure, cumulative distribution, cumulative exposure distribution, F(x)
- Credit was not given for plotting a stepwise function, or not connecting the points

Part b

Candidates were expected to recall and calculate the formula for g, calculate μ and look up these values in a table.

Common mistakes include:

- Most candidates did not correctly calculate the conditional mean; if they did not get part (a), credit was still given for μ =0.105 (instead of 0.525). This μ is not on the table provided, but if a candidate explicitly assumed μ equals some number on the table, they received partial credit for looking up the b value.
- Many candidates used p = 0.02 (instead of 0.1). If they did not rescale in part (a), they still received credit for g=(1/p)=50 in part (b).

Part c

Candidates were expected to write the formula for the total losses in a layer and identify the correct points on their exposure curve.

Common mistakes included:

- Calculation errors were common in part (a), but if part (a) was used in part (c) full credit was given. A common answer that was given full credit was G(0.5)-G(0.25) = 0.94 – 0.86 = 0.08
- Many candidates only wrote the formula without trying to identify the correct points on the exposure curve

QUESTION 19	
TOTAL POINT VALUE: 3	**LEARNING OBJECTIVE(S): C3**
SAMPLE ANSWERS	

Part a: 1.5 points

Sample 1

The only claims above $500k retention are 2012 #3, 2014 #1, 2015 #4, and 2016 #2.

	Layer Loss	Premium	Loss Ratio
AY	$=\sum_{claims} \max(0, \min(500k, loss - 500k))$	$= \max(100k, \min(325k, \text{Layer Loss x } 1.3))$	
2012	512,017-500k=12,017	100k -> min	12,017/100k=0.120
2013	0	100k -> min	0
2014	204,895	266,364	0.769
2015	500,000	325k -> max	1.548
2016	154,158	200,405 = 154,158*1.3	0.769

Expected Reinsurer L/R = (0.120 + 0 +0.769 + 1.538 + 0.769)/5 = <u>0.639</u>

Sample 2

500,000 xs 500,000 *Assuming claims are ground-up

AY	500xs500 Loss	Premium
2012	12,017	100,000 (min)
2013	0	100,000 (min)
2014	204,895	266,364
2015	500,000	325,000 (max)
<u>2016</u>	<u>154,158</u>	<u>200,405</u>
Avg	174,218	198,354

(because equal weights)

E[LR] = 174,218/198,354 = 87.83%

*Using SUM(500xs500 Loss)/SUM(Premium) also works

Part b: 0.5 point

Sample 1

No, I don't think it is appropriate. The On-Level Premium has been growing rapidly, which may mean that the underlying book mix has been shifting as well. In this case, the older years won't be as relevant and we won't have enough data to construct an empirical distribution.

<u>Sample 2</u>

The empirical distribution model is not appropriate because there are few claims (and even fewer claims in the layer) so the data isn't credible. It does not capture all possible outcomes.

Part c: 1 point

Sample 1

1. Collective risk model assumes that frequency and severity are independent, and that all observations are independent – these assumptions may not be true
2. Complexity of calculation can lead to "black box" mentality. Be sure to check results for reasonability.

Sample 2

1. Some use numerical methods which may have high errors for low frequency scenarios. Need to check the output of the model.
2. The collective risk model only reflects the process variance of the distribution and does not reflect the parameter variance. Have to give additional care in estimating the parameter.

Sample 3

Using Panjer's Recursive Approach as an example of a collective risk:

1. Difficult to calculate for higher frequency policies as need to recursively calculate prior frequencies.
2. Can only use a single loss severity model/assumption: The limitation could be significant for long tailed risk.

Sample 4

1. Needs more data and it takes more time to build frequency and severity models separately. For higher loss layer, the scarcity of data could be even more of an issue.
2. Black box mentality due to the complexity of a collective risk model. Need to check reasonableness of outcome.

EXAMINER'S REPORT

Candidates were expected to be able to evaluate retrospective reinsurance policies based on an empirical distribution model, evaluate the appropriateness of using the empirical distribution, and be able to explain two concerns regarding about applying aggregate distribution models.

Part a

Candidates were expected to calculate layer losses per year, apply the loading of 1.3 to obtain the retrospective premium, and properly subject the retrospective premium to the min and max provided. From there, using either the straight average or the weighted average to derive the final LR was accepted.

Common mistakes included:

- Using the on-level premium instead of calculating the adjusted retrospective premium
- Forgetting that there is a cap of 500k on the excess of 500k portion of the 2015 loss of 1,246,852 - should use 500,000 instead of 746,852
- Forgetting to adjust the 2012 premium of 12,017 x 1.3 = 15,622 to the minimum premium of 100,000
- Applying 1.3 straight to the total of layered losses and arrive 1/1.3 as the final answer, should apply 1.3 to each AY's layered losses and subject to min/max afterwards

Part b

Candidates were expected to know that the empirical distribution is not appropriate for this account and provide a reason.

Some candidates compared the balance factor of 1 / 1.3 = 76.9% to the derived loss ratio. This confirms that the swing plan is appropriately constructed and is not related to the use of an empirical distribution.

Loss ratio (or just loss) being too volatile does not show specific understanding of the concept. Having volatile losses does not equate to taking into account all possible outcomes. In fact, if the LR or losses were volatile, they are more likely to show all possible outcomes.

Stating that the use of the empirical distribution is appropriate since there is no free cover was not accepted. Even if that is true, it does not outweigh the growing premium and the changing exposure associated to it.

Common mistakes included:

- Comparing 1/1.3 to the LR derived
- Stating LR too volatile without linking it to possible future outcomes or thin data/credibility
- Stating that there is no free cover for the plan

Part c

Candidates were expected to know and explain two concerns of using the collective risk model.

Collective risk models are distributions for which frequency and severity are explicitly recognized. Thus, the answers had to be specific to the collective risk model and could not be general answers related to distributions, models and losses/experience.

The recursive calculation method is also a collective risk model, thus concerns related to it were accepted.

Stating that the model takes more time to run/is more complex was not accepted by itself. Further explanations are needed on the reason why collective risk model takes longer in order for credit to be warranted.

Common mistakes included:

- Providing answers that were too general and not specific to collective risk models (e.g. risks are different from others, distributions for the loss is uncertain to estimate, need to incorporate LDFs, trends, etc.)
- Forgetting to further explain or elaborate on the concerns beyond just naming them

QUESTION 20	
TOTAL POINT VALUE: 2.5	**LEARNING OBJECTIVE(S): C1a, C2c**
SAMPLE ANSWERS	
Part a: 0.75 point	

Sample 1

- A: the damage ratio at each intensity
- B: the uncertainty in the damage ratio at the given intensities
- C: the uncertainty in the felt intensity given a particular earthquake

Sample 2

- A: damage ratio curve
- B: distribution of damage ratios for the three given intensities
- C: probability distribution for the intensity of the earthquake given an initial estimate

Part b: 1.75 points

Sample 1

Intensity	Probability	Expected Loss
3.5	10%	200,000
6.0	5%	450,000
8.5	2%	800,000
Total Expected Loss		58,500

$$\text{SD} = \sqrt{0.10 \times 200{,}000^2 + 0.05 \times 450{,}000^2 + 0.02 \times 800{,}000^2 - 58{,}500^2} = 153{,}306$$

$$\text{Risk Load} = 0.08 \text{ x } 153{,}306 = 12{,}264$$

$$\text{Premium} = \frac{58{,}500 + 12{,}264}{1.0 - 0.2} = 88{,}456$$

Sample 2

This solution is very similar to Sample 1, the exception being that a damage ratio of 40% was selected for the 6.0 intensity earthquake.

Intensity	Probability	Expected Loss
3.5	10%	200,000
6.0	5%	400,000
8.5	2%	800,000
Total Expected Loss		56,000

$$\text{SD} = \sqrt{0.10 \times 200{,}000^2 + 0.05 \times 400{,}000^2 + 0.02 \times 800{,}000^2 - 56{,}000^2} = 147{,}187$$

$\text{Risk Load} = 0.08 \text{ x } 147{,}187 = 11{,}775$

$\text{Premium} = \frac{56{,}000+11{,}775}{1.0-0.2} = 84{,}719$

EXAMINER'S REPORT

Candidates were expected to demonstrate knowledge of the damage ratio and the way in which it is used in catastrophe modeling. Candidates were then asked to use information about the damage ratio to calculate policy premium.

Part a

Candidates were expected to name and/or describe various curves on a graph. The graph showed a curve of the damage function (A), distributions describing the uncertainty of the actual damage to the structure (B), and the uncertainty of the forces experienced by the structure during the event (C).

Common mistakes included:

- Referring to curve C as the distribution of all earthquakes out of the model
- Referring to curve C as a combined distribution of total loss after deductible and/or reinsurance recoverable

Part b

Candidates were expected to calculate the earthquake premium associated with a policy, calculate the expected loss and risk load, and to correctly apply a provision for expenses.

A very small number of candidates assumed that multiple events could occur in the same year, i.e. a 3.5 and a 6.0 intensity earthquake could both occur. Credit was given when candidates completed the response correctly under this assumption.

Common mistakes included:

- Incorrectly calculating the standard deviation
- Failing to apply the risk load factor
- Applying the expense load by multiplying by (1 + 0.2) instead of dividing by (1 - 0.2)
- Attempting to calculate the expected loss by using the intensities rather than the associated loss amounts