Exam LC

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

October 30, 2014

Exam LC

Models for Life Contingencies

1.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 30 point examination consists of 15 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2014 and that the exam name is LC.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam LC" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 17, 2014.

END OF INSTRUCTIONS

1.

You are given the following life table for an electronic device:

x	l_x	d_x
0	1,000	20
1		
2	950	50
3		80
4		120
5		

Calculate the probability that a device age 1 will survive to age 5.

A. Less than 0.72
B. At least 0.72, but less than 0.74
C. At least 0.74, but less than 0.76
D. At least 0.76, but less than 0.78
E. At least 0.78

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2.

You are given the following information describing the age-at-failure random variable for a machine component:

- $S(x)=1-\dfrac{x^3}{125} \quad for\ 0 \le x \le 5.$

Calculate the variance of the age-at-failure random variable, T_0.

A. Less than 0.86
B. At least 0.86, but less than 0.88
C. At least 0.88, but less than 0.90
D. At least 0.90, but less than 0.92
E. At least 0.92

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3.

You are given the following information regarding a population of 100 members with independent lifetimes:

- Each member has constant force of mortality, μ.
- P is the probability that the observed average life expectancy of the population is greater than 70.
- Using the normal approximation, it is estimated that $P = 0.10$.

Calculate the force of mortality, μ.

A. Less than 0.015
B. At least 0.015, but less than 0.025
C. At least 0.025, but less than 0.035
D. At least 0.035, but less than 0.045
E. At least 0.045

CONTINUED ON NEXT PAGE

4.
You are given the following information:

- ${}_k p_x = (0.8)^k$, for $k \geq 2$
- ${}_{0.2|0.5}q_x = 0.08$
- Mortality is assumed to follow a uniform distribution between integer ages.

Calculate $\overset{o}{e}_x$

A. Less than 4.00
B. At least 4.00, but less than 4.50
C. At least 4.50, but less than 5.00
D. At least 5.00, but less than 5.50
E. At least 5.50

CONTINUED ON NEXT PAGE

5.

For two new engines A and B, you are given the following information:

- A and B have independent future lifetimes
- Engine A follows a de Moivre survival pattern (uniform distribution) with $\omega = 20$
- Engine B follows a de Moivre survival pattern (uniform distribution) with $\omega = 25$

Calculate the expected time until both engines have failed.

A. Less than 10
B. At least 10, but less than 12
C. At least 12, but less than 14
D. At least 14, but less than 16
E. At least 16

CONTINUED ON NEXT PAGE

6.

You are given two independent lives (x) and (y) with constant forces of mortality μ and $k\mu$ respectively, where $k \geq 1$.

You learn that the expected time to the second death is equal to three times the expected time to the first death.

Calculate k.

A. Less than 1.5
B. At least 1.5, but less than 2.0
C. At least 2.0 but less than 2.5
D. At least 2.5, but less than 3.0
E. At least 3.0

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7.

For a multiple decrement model on (70), you are given the following information:

- $\mu_{70}^{(1)}(t)$ follows the Illustrative Life Table for all $t \geq 0$.
- $\mu_{70}^{(\tau)}(t) = 3\mu_{70}^{(1)}(t), t \geq 0$

Calculate ${}_{15|}q_{70}^{(\tau)}$, the probability that failure occurs during the 16th year due to any cause.

A. Less than 0.0145
B. At least 0.0145, but less than 0.0150
C. At least 0.0150, but less than 0.0155
D. At least 0.0155, but less than 0.0160
E. At least 0.0160

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8.

An auto insurance company writes 1,000 single vehicle policies. Each policyholder may cancel for one of four mutually exclusive reasons:

1) The vehicle's engine dies
2) The vehicle is totaled in an accident
3) The policyholder fails to pay the premium
4) The policyholder leaves for a competing insurer

Assume the cancellation probabilities due to each cause are given by:

- $q_x^{(1)} = .01 + .001x$
- $q_x^{(2)} = .05$
- $q_x^{(3)} = .20 - .02x$
- $q_x^{(4)} = .08 - .007x$

Calculate the expected number of policies that, in their 3rd year, either are cancelled for non-payment of premium or are lost to a competing insurer.

A. Less than 100
B. At least 100, but less than 110
C. At least 110, but less than 120
D. At least 120, but less than 130
E. At least 130

CONTINUED ON NEXT PAGE

9.

Daily changes in a stock index follow a homogeneous Markov chain, with three states. Transitions into each state result in the following changes to the stock index:

State 1:	Loses 100 points
State 2:	Stays the same
State 3:	Gains 100 points

The transition probability matrix is:

$$Q = \begin{bmatrix} 0.4 & 0.6 & 0.0 \\ 0.3 & 0.5 & 0.2 \\ 0.0 & 0.9 & 0.1 \end{bmatrix}$$

The stock index is currently in State 2.

Calculate the probability that the stock index will be exactly 200 points higher after three transitions.

A. Less than 0.03
B. At least 0.03, but less than 0.06
C. At least 0.06, but less than 0.09
D. At least 0.09, but less than 0.12
E. At least 0.12

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10.

You are given a two-life model on (x) and (y) where four states are defined as follows:

- State #1 – Both (x) and (y) are intact.
- State #2 – (x) is intact, but (y) has failed.
- State #3 – (y) is intact, but (x) has failed.
- State #4 – Both (x) and (y) have failed.

Assume that (x) and (y) are independent lives aged 70 and 85, respectively, each subject to de Moivre's law with $\omega = 110$.

Find ${}_2Q_{10}^{(1,3)}$, the transition probability from State #1 in Year 10 to State #3 in Year 12.

A. Less than 0.040
B. At least 0.040, but less than 0.045
C. At least 0.045, but less than 0.050
D. At least 0.050, but less than 0.055
E. At least 0.055

CONTINUED ON NEXT PAGE

11.

You are given the following information:

- $\delta = 0.020$
- $\mu_{x+t} = 0.025$ for all t.

Calculate the probability that $\overline{Y}_x = \bar{a}_{\overline{T_x}|}$ will exceed 30.

A. Less than 0.30
B. At least 0.30, but less than 0.35
C. At least 0.35, but less than 0.40
D. At least 0.40, but less than 0.45
E. At least 0.45

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12.

You are given the following information:

- An individual, age 60, purchases a 2-year *immediate annuity* paying 1,000.
- The individual is subject to two causes of mortality: Decrement 1 and Decrement 2.
- Decrement 1 follows the Illustrative Life Table.
- Decrement 2 is described by the following life table:

x	$q_x^{(2)}$
60	0.04
61	0.05
62	0.06

- Interest rate $i = 0.05$

Calculate the actuarial present value of this immediate annuity.

A. Less than 1,650
B. At least 1,650, but less than 1,670
C. At least 1,670, but less than 1,690
D. At least 1,690, but less than 1,710
E. At least 1,710

CONTINUED ON NEXT PAGE

13.

You are given the following information:

- A product is covered by a 5-year warranty.
- The product has an exponential failure rate with a mean time to failure of 10 years.
- Interest rate, $i = 0.05$
- Payment of 1,000 is made immediately in the event of failure.

Calculate the standard deviation of the present value of the warranty payout.

A. Less than 250
B. At least 250, but less than 300
C. At least 300, but less than 350
D. At least 350, but less than 400
E. At least 400

CONTINUED ON NEXT PAGE

14.

You are given the following information about a non-homogenous Markov Chain:

- Claims close according to the following transition probability matrices (where state 1 is an open claim and state 2 is a closed claim):

$$Q_0 = \begin{bmatrix} 0.9 & 0.1 \\ 0 & 1 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$Q_n = \begin{bmatrix} 0.7 & 0.3 \\ 0 & 1 \end{bmatrix}; n \geq 2$$

- Transitions occur at the end of the period.
- Cash flows occur in the middle of the period.
- The insurer pays 500 in the middle of the period for each claim that is open.
- The insurer has 50 open claims at time $t = 0$.
- Interest rate $i = 0.03$

Calculate the actuarial present value of the cash flows.

A. Less than 70,000
B. At least 70,000, but less than 80,000
C. At least 80,000, but less than 90,000
D. At least 90,000, but less than 100,000
E. At least 100,000

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15.

You are given the following information:

- A machine is in one of 4 states (State 1, State 2, State 3 or State 4) and migrates annually among them according to a homogeneous Markov process.
- The transition probability matrix **Q** is

$$\begin{pmatrix} 0.20 & 0.80 & 0.00 & 0.00 \\ 0.50 & 0.00 & 0.50 & 0.00 \\ 0.75 & 0.00 & 0.00 & 0.25 \\ 1.00 & 0.00 & 0.00 & 0.00 \end{pmatrix}$$

- At time 0, the machine is in State 1.
- At the end of 3 years, a salvage company will pay 1,000 if the machine is in State 1.
- Interest rate $i = 0.06$

Calculate the actuarial present value at time 0 of this payment.

A. Less than 350
B. At least 350, but less than 365
C. At least 365, but less than 380
D. At least 380, but less than 395
E. At least 395

END OF EXAMINATION

Fall 2014 Exam LC Solution Key

1. A
2. E
3. B
4. C
5. D
6. A
7. B
8. B
9. B or C
10. E
11. B
12. D
13. E
14. D
15. D