CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Virginia R. Prevosto
Vice President-Admissions

Steven D. Armstrong
Chairperson
Examination Committee

William Wilder
Assistant Chairperson
Examination Committee

April 30, 2014

Exam LC

Models for Life Contingencies

1.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 30 point examination consists of 15 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

    - Fill in that it is Spring 2014 and that the exam name is LC.
    - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
    - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
    - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

    - Verify that you have a copy of "Tables for CAS Exam LC" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 14, 2014.

END OF INSTRUCTIONS

1.

You are given the following information describing the age-at-failure random variable T_0 for a machine component:

- $F_0(t) = 1 - \frac{1}{2}(8-t)^{1/3}, \quad 0 \le t \le 8.$

Calculate $\lambda(6)$, the conditional density of failure at age 6 given survival to age 6.

A. Less than 0.200
B. At least 0.200, but less than 0.250
C. At least 0.250, but less than 0.300
D. At least 0.300, but less than 0.350
E. At least 0.350

CONTINUED ON NEXT PAGE

2.

You are given the following information:

- $\mu_x = \begin{cases} 0.030; & 0 < x \leq 35 \\ 0.035; & x > 35 \end{cases}$

Calculate $\overset{o}{e}_{30:\overline{20|}}$.

A. Less than 14.50
B. At least 14.50, but less than 15.00
C. At least 15.00, but less than 15.50
D. At least 15.50, but less than 16.00
E. At least 16.00

CONTINUED ON NEXT PAGE

3.

You are given the following density function:

$$f(x)=\begin{cases}\dfrac{1}{100} & \textit{for } 0 \le x < 45 \\ 0.055e^{-0.1(x-45)} & \textit{for } x \ge 45\end{cases}$$

Calculate ${}_{45|11}q_0$.

A. Less than 0.250
B. At least 0.250, but less than 0.300
C. At least 0.300, but less than 0.350
D. At least 0.350, but less than 0.400
E. At least 0.400

CONTINUED ON NEXT PAGE

4.

You are given the following information:

- $\mu_{60.25} = 0.80$
- Deaths are uniformly distributed between integer ages.

Calculate $\left(q_{60} - \overset{o}{e}_{60:\overline{1}|}\right)$.

A. Less than -0.30
B. At least -0.30, but less than -0.10
C. At least -0.10, but less than 0.10
D. At least 0.10, but less than 0.30
E. At least 0.30

CONTINUED ON NEXT PAGE

5.

You are given the following information on a multi-life model:

- Mortality for a single life follows the Illustrative Life Table.
- Deaths are uniformly distributed between integer ages.
- Individual future lifetimes are independent.

Calculate ${}_{2.25}p_{60:70}$.

A. Less than 0.88
B. At least 0.88, but less than 0.91
C. At least 0.91, but less than 0.94
D. At least 0.94, but less than 0.97
E. At least 0.97

CONTINUED ON NEXT PAGE

6.

You are given the following information on the longevity of two mechanical devices, Device I and Device II:

- $\lambda_0^I(x) = (2 + x)^{-1}$; $x \geq 0$ is the hazard rate function for Device I.
- Device I is currently two years old.
- $\lambda_0^{II}(y) = k(2 + y)^{-1}$; $y \geq 0$ is the hazard rate function for Device II.
- Device II is currently one year old.
- The future lifetimes of Device I and Device II are independent.
- The probability that neither device will fail within the next 12 years is 1%.

Calculate *k*.

A. Less than 0.50
B. At least 0.50, but less than 1.50
C. At least 1.50, but less than 2.50
D. At least 2.50, but less than 3.50
E. At least 3.50

CONTINUED ON NEXT PAGE

7.

You are given the following information:

- The future lifetimes of (x) and (y) are independent.
- ${}_{10}p_x = 0.60$
- ${}_{10}p_y = 0.90$
- $q_{x+10} = 0.70$
- $q_{y+10} = 0.40$

Calculate ${}_{10|}q_{\overline{xy}}$.

A. Less than 0.25
B. At least 0.25, but less than 0.30
C. At least 0.30, but less than 0.35
D. At least 0.35, but less than 0.40
E. At least 0.40

CONTINUED ON NEXT PAGE

8.

You are given the following double-decrement table:

x	$l_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$d_x^{(1)}$	$p_x^{(\tau)}$
70		y		1,000	0.75
71	7,500	0.15	$2.5y$		
72			0.26		0.58

Calculate $d_{72}^{(1)}$.

A. Less than 714
B. At least 714, but less than 718
C. At least 718, but less than 722
D. At least 722, but less than 726
E. At least 726

CONTINUED ON NEXT PAGE

9.

You are given the following information on a multiple-decrement model:

- Total mortality from all causes combined follows the Illustrative Life Table.
- There are three causes of death:
  - Cause 1: Accident
  - Cause 2: Illness
  - Cause 3: All Other
- The relationship between the three causes of death is as follows:
  - $q_x^{(1)} = 2q_x^{(2)}$, for $20 \leq x \leq 30$
  - $q_x^{(2)} = 4q_x^{(3)}$, for $20 \leq x \leq 30$
- A population consists of 100,000 people that are currently age 20.

Calculate how many members of this population can be expected to die from Accidents before reaching age 30.

A. Less than 700
B. At least 700, but less than 750
C. At least 750, but less than 800
D. At least 800, but less than 850
E. At least 850

CONTINUED ON NEXT PAGE

10.

An insurance company classifies their customers into three categories:

- Best tier (State 1)
- Average tier (State 2)
- Worst tier (State 3)

Customers transition between tiers at the end of each year according to a homogeneous Markov process with the transition probability matrix below:

$$Q = \begin{pmatrix} 0.500 & 0.350 & 0.150 \\ 0.100 & X & Y \\ 0.020 & 0.380 & 0.600 \end{pmatrix}$$

The probability that a customer classified as Best tier at time 0 will NOT be classified as Average tier at time 2 (i.e. after two transitions) is 0.5405.

Calculate Y.

A. Less than 0.210
B. At least 0.210, but less than 0.360
C. At least 0.360, but less than 0.510
D. At least 0.510, but less than 0.660
E. At least 0.660

CONTINUED ON NEXT PAGE

11.

Consider a loan default model in which borrowers move among three statuses (Current, Delinquent, and Default) at the end of each month according to a homogeneous Markov process.

The transition probabilities are as follows:

- 70% of Current borrowers remain Current; 30% move to Delinquent; 0% move to Default.
- 50% of Delinquent borrowers remain Delinquent; 30% move to Current; 20% move to Default.
- Once in Default, the borrower remains in Default forever.

Calculate the probability that a borrower who is Delinquent at the beginning of a calendar quarter will NOT be in Default at the end of the same calendar quarter (i.e. after three transitions have occurred).

A. Less than 0.650
B. At least 0.650, but less than 0.680
C. At least 0.680, but less than 0.710
D. At least 0.710, but less than 0.740
E. At least 0.740

CONTINUED ON NEXT PAGE

12.

For a special 3-year term life insurance policy on (x), you are given the following information:

- $Z^{1}_{x:\overline{3}|}$ is the present value random variable.
- $q_{x+m} = 0.03(m+1)$; $m = 0,1,2$
- The death benefit b_k is payable at the end of the year of death k and is given by the following table:

k	b_k
1	200,000
2	250,000
3	300,000

- Interest rate $i = 0.06$.

Calculate $E[Z^{1}_{x:\overline{3}|}]$.

A. Less than 35,000
B. At least 35,000, but less than 37,000
C. At least 37,000, but less than 39,000
D. At least 39,000, but less than 41,000
E. At least 41,000

CONTINUED ON NEXT PAGE

13.

You are given the following information:

- An individual, age 55, purchases a three-year temporary contingent annuity-due that pays a benefit of 500 annually.
- The individual is subject to two causes of mortality: Decrement 1 and Decrement 2.
- Decrement 1 follows the Illustrative Life Table.
- Decrement 2 is described by the following table:

x	$q_x^{(2)}$
55	0.03
56	0.04
57	0.05

- Interest rate $i = 0.05$

Calculate the actuarial present value of this annuity-due.

A. Less than 1,300
B. At least 1,300, but less than 1,325
C. At least 1,325, but less than 1,350
D. At least 1,350, but less than 1,375
E. At least 1,375

CONTINUED ON NEXT PAGE

14.

For a special fully discrete 35-payment whole life insurance on (30), you are given the following information:

- The death benefit is 100,000 for the first 20 years and increases to 1,000,000 thereafter
- The benefit premium is P payable at the beginning of each of the first 20 years and $10P$ payable at the beginning of each of the 15 subsequent years
- Benefit premiums are calculated using the equivalence principle
- Mortality follows the Illustrative Life Table
- Interest rate $i = 0.06$
- $\ddot{a}_{30:\overline{35}|} = 14.835$
- $A^{1}_{30:\overline{20}|} = 0.02933$

Calculate P.

A. Less than 1,800
B. At least 1,800, but less than 1,850
C. At least 1,850, but less than 1,900
D. At least 1,900, but less than 1,950
E. At least 1,950

CONTINUED ON NEXT PAGE

15.

Immigration depends upon the growth rate of the economy, which follows the homogeneous Markov chain described below:

- State 1: Rapid Growth
- State 2: Normal Growth
- State 3: Slow Growth
- Following a year of Rapid Growth, immigration is +10,000.
- Following a year of Normal Growth, immigration is +4,000.
- Following a year of Slow Growth, immigration is zero.
- The economy transitions between states according to the following transition probability matrix:

$$Q = \begin{bmatrix} 0.15 & 0.65 & 0.20 \\ 0.35 & 0.40 & 0.25 \\ 0.05 & 0.85 & 0.10 \end{bmatrix}$$

- The economy just finished a year of Rapid Growth.

Calculate the expected total immigration as a result of the next two transitions.

A. Less than 8,725
B. At least 8,725, but less than 8,750
C. At least 8,750, but less than 8,775
D. At least 8,775 but less than 8,800
E. At least 8,800

END OF EXAMINATION

Exam LC

Answer Key

1. A
2. B
3. D
4. C
5. B
6. C
7. C
8. C
9. B
10. B
11. A
12. D
13. D
14. C
15. E