Exam LC

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam LC

Models for Life Contingencies

1.5 HOURS

April 30, 2015

INSTRUCTIONS TO CANDIDATES

1. This 30 point examination consists of 15 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2015 and that the exam name is LC.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam LC" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 15, 2015.

END OF INSTRUCTIONS

1.

You are given the following information:

- $l_{50} = 1{,}000$
- $l_{50.25} = 900$
- $l_{50.50} = 810$
- $l_{50.75} = 750$

Calculate ${}_{0.25|0.50}q_{50}$.

A. Less than 0.10
B. At least 0.10, but less than 0.12
C. At least 0.12, but less than 0.14
D. At least 0.14, but less than 0.16
E. At least 0.16

CONTINUED ON NEXT PAGE

2.

You are given the following information:

- $G = 1 - e^{-\int_0^1 \mu_x(t)dt}$
- $H = 1 - e^{-\int_0^1 (\mu_x(t)+k)dt}$
- k is a constant such that $H = 0.5G$

Determine an expression for k.

A. $\ln\left(\frac{1-q_x}{1-0.5q_x}\right)$

B. $\ln\left(\frac{1-0.5q_x}{1-p_x}\right)$

C. $\ln\left(\frac{1-0.5p_x}{1-p_x}\right)$

D. $\ln\left(\frac{1-p_x}{1-0.5q_x}\right)$

E. $\ln\left(\frac{1-0.5q_x}{1-q_x}\right)$

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3.

You are given the following information:

- Deaths are uniformly distributed over each year of age.
- $q_{60} = 0.0300$
- $\mu_{61.5} = 0.0325$

Calculate $\overset{o}{e}_{60:\overline{1.75}|}$.

A. Less than 1.705
B. At least 1.705, but less than 1.715
C. At least 1.715, but less than 1.725
D. At least 1.725, but less than 1.735
E. At least 1.735

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4.

You are given the following information:

- T_x and T_y are independent time-to-failure random variables.
- T_x has an exponential distribution with mean 20.
- T_y has an exponential distribution with mean 5.

Calculate $Var(T_{xy})$.

A. Less than 14.5
B. At least 14.5, but less than 15.5
C. At least 15.5, but less than 16.5
D. At least 16.5, but less than 17.5
E. At least 17.5

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5.

You are given the following information about two independent lives:

- The male is currently age 70 and the female is currently age 65.
- The survival function for the male is $S_0^M(x) = 1 - \frac{x}{80}, 0 \le x \le 80$.
- The survival function for the female is $S_0^F(y) = 1 - \frac{y}{\omega}, 0 \le y \le \omega$.
- The force of mortality for the female at age 50 is 60% of the force of mortality for the male at age 50.

Calculate the expected time until the second death.

A. Less than 5
B. At least 5, but less than 10
C. At least 10, but less than 15
D. At least 15, but less than 20
E. At least 20

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6.

A company evaluating its employee retention policies categorizes the causes of employee departure as follows:

(1) Departure to a competitor
(2) Retirement
(3) All other causes

You are given the following:

- Each cause j has a constant hazard rate, $\mu^{(j)} = 0.1$.
- After implementing a new employee retention initiative, the company halves $\mu^{(1)}$.
- $\mu^{(2)}$ and $\mu^{(3)}$ are not impacted by the new initiative.
- The probability that an employee leaves due to retirement within five years is M prior to the initiative and N after the initiative.

Calculate $N - M$.

A. Less than -0.030
B. At least -0.030, but less than -0.010
C. At least -0.010, but less than 0.010
D. At least 0.010, but less than 0.030
E. At least 0.030

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7.

You are given the following for a multiple-decrement model:

- There are three causes of leaving the workforce, (1), (2), and (3).

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$	$d_x^{(\tau)}$
55	0.15	0.05	0.10	300
56	0.10		0.12	203
57	0.08	0.08	0.14	
58			0.16	

Calculate the probability that an individual aged (55) leaves the workforce due to cause (2) or cause (3) after one year but less than three years from now.

A. Less than 0.20
B. At least 0.20, but less than 0.25
C. At least 0.25, but less than 0.30
D. At least 0.30, but less than 0.35
E. At least 0.35

CONTINUED ON NEXT PAGE

8.

You are given the following information on a double-decrement model:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
40		200	300
41	2,000	200	
42	1,400		300
43	800		

Calculate the probability that (40) will fail from cause 2 between the ages of 41 and 43.

A. Less than 0.225
B. At least 0.225, but less than 0.250
C. At least 0.250, but less than 0.275
D. At least 0.275, but less than 0.300
E. At least 0.300

CONTINUED ON NEXT PAGE

9.

An insurer's retention of insurance policies follows a non-homogeneous Markov Chain with the following states:

- State 0 = Retained
- State 1 = Canceled by the Insured
- State 2 = Non-renewed by Insurer

Transitions between states occur at the end of each year, according to the following transition probability matrices:

$$Q_0 = \begin{bmatrix} 0.85 & 0.10 & 0.05 \\ 0 & 0 & 1.00 \\ 0 & 0 & 1.00 \end{bmatrix}; Q_n = \begin{bmatrix} 0.90 & 0.07 & 0.03 \\ 0 & 0 & 1.00 \\ 0 & 0 & 1.00 \end{bmatrix}, \text{for } n \geq 1$$

- At time $t = 0$, the insurer has 10,000 policies in State 0.
- The insurer does not write new policies.

Calculate the expected number of policies canceled by the insured during the next two years.

A. Less than 1,200
B. At least 1,200, but less than 1,400
C. At least 1,400, but less than 1,600
D. At least 1,600, but less than 1,800
E. At least 1,800

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10.

For a life (x), the force of mortality μ is determined for each state, as provided below:

State	μ
1	0.1
2	0.5

At the end of each year, (x) transitions between states according to the transition matrix:

$$\begin{pmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{pmatrix}$$

At $t = 0$, (x) is in State 1.

Calculate $\overset{o}{e}_{x:\overline{2}|}$.

A. Less than 1.600
B. At least 1.600, but less than 1.650
C. At least 1.650, but less than 1.700
D. At least 1.700, but less than 1.750
E. At least 1.750

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11.

You are given the following information:

- $A_x = 0.25$
- $A_{x+15} = 0.35$
- $A_{x:\overline{15|}} = 0.50$

Calculate $A^1_{x:\overline{15|}}$.

A. Less than 0.11
B. At least 0.11, but less than 0.12
C. At least 0.12, but less than 0.13
D. At least 0.13, but less than 0.14
E. At least 0.14

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12.

You are given the following information:

- A homeowner's insurance policy provides a payment of \$300,000 at the precise time of loss due to fire.
- The hazard rate due to fire is μ.
- The force of interest is $\delta = 0.5\mu$.
- After installing a sprinkler system, the hazard rate due to fire decreases to 0.6μ and the payment at the precise time of loss due to fire increases to \$350,000.
- M is the actuarial present value of the policy prior to installing the sprinkler system.
- N is the actuarial present value of the policy after installing the sprinkler system, where k is the indicated discount, i.e. $N = (1 - k) * M$.

Calculate k.

A. Less than 0.0%
B. At least 0.0%, but less than 2.5%
C. At least 2.5% but less than 5.0%
D. At least 5.0%, but less than 7.5%
E. At least 7.5%

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13.

You are given the following information for a fully discrete whole life insurance on (10):

- Premium has *not* been calculated using the equivalence principle.
- $A_{10} = 0.21$
- ${}^{2}A_{10} = 0.15$
- $Var(L_{10}) = 0.18$
- L_{10} is the random variable for the present value of loss.
- $i = 4.5\%$

Calculate (L_{10}) .

A. Less than -0.06
B. At least -0.06, but less than -0.02
C. At least -0.02, but less than 0.02
D. At least 0.02, but less than 0.06
E. At least 0.06

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14.

You are given the following information for a continuous whole life annuity of 1 on (x):

- T_x is the time-to-failure random variable for an entity known to be alive at age x.
- The force of interest and force of mortality are constant and equal.
- $\bar{a}_x = 10$.

Calculate the standard deviation of $\bar{a}_{\overline{T_x|}}$.

A. Less than 5
B. At least 5, but less than 15
C. At least 15, but less than 25
D. At least 25, but less than 35
E. At least 35

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15.

You are given the following information:

- Performance of a worker is in one of two states (state 1 or state 2) and migrates annually between the two according to a homogeneous Markov Chain process.
- The transition probability matrix Q is

$$Q = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

- At the end of each year, the company will pay 500 as a performance bonus if the worker's performance is in state 1, and 0 if the performance is in state 2.
- Interest rate $i = 0.05$.

Calculate the absolute difference of actuarial present values at time 0 of the performance bonus over the next two years, between a worker who is currently in state 1 and a worker who is currently in state 2.

A. Less than 100
B. At least 100, but less than 110
C. At least 110, but less than 120
D. At least 120, but less than 130
E. At least 130

END OF EXAMINATION

Spring 2015 Exam LC Solution Key

1. D
2. A
3. A
4. C
5. D
6. D
7. B
8. D
9. C
10. D
11. B
12. C
13. B
14. B
15. C