Exam MAS-I

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

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MAS-I

Modern Actuarial Statistics I

May 7, 2018

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 90 point examination consists of 45 multiple choice questions each worth 2 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2018 and that the exam name is MAS-I.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS MAS-I" included in your exam packet.

   - Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 21, 2018.

END OF INSTRUCTIONS

1.

You are given the following information:

- The amount of time one spends in an IRS office is exponentially distributed with a mean of 30 minutes.
- P_1 is the probability that John will spend more than an hour of total time in the IRS office, given that he has already been in the office for 20 minutes.
- P_2 is the probability that Lucy will spend more than an hour of total time in the IRS office, given that she has just arrived.

Calculate the difference $(P_1 - P_2)$.

A. Less than 0.00
B. At least 0.00, but less than 0.05
C. At least 0.05, but less than 0.10
D. At least 0.10, but less than 0.15
E. At least 0.15

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2.

Insurance claims are made according to a Poisson process with rate λ.

Calculate the probability that exactly 3 claims were made by time $t = 1$, given that exactly 6 claims are made by time $t = 2$.

A. Less than 0.3
B. At least 0.3, but less than 0.4
C. At least 0.4, but less than 0.5
D. At least 0.5, but less than 0.6
E. At least 0.6

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3.

The number of cars passing through the Lexington Tunnel follows a Poisson process with rate:

$$\lambda(t) = \begin{cases} 16 + 2.5t & \text{for} \quad 0 < t \leq 8 \\ 52 - 2t & \text{for} \quad 8 < t \leq 12 \\ -20 + 4t & \text{for} \quad 12 < t \leq 18 \\ 160 - 6t & \text{for} \quad 18 < t \leq 24 \end{cases}$$

Calculate the probability that exactly 50 cars pass through the tunnel between times $t = 11$ and $t = 13$.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.03
D. At least 0.03, but less than 0.04
E. At least 0.04

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4.

You are given the following information about flights on ABC Airlines:

- Flights are delayed at a Poisson rate of four per month.
- The number of passengers on each flight is independently distributed with mean of 100 and standard deviation of 50.
- Each passenger on a delayed flight is compensated with a payment of 200.

Calculate the standard deviation of the total annual compensation for delayed flights.

A. Less than 100,000
B. At least 100,000, but less than 125,000
C. At least 125,000, but less than 150,000
D. At least 150,000, but less than 175,000
E. At least 175,000

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5.

You are given:

- Computer lifetimes are independent and exponentially distributed with a mean of 24 months.
- Computer I has been functioning properly for 36 months.
- Computer II is a brand new and functioning computer.

Calculate the absolute difference between Computer I's failure rate and Computer II's failure rate.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.03
D. At least 0.03, but less than 0.04
E. At least 0.04

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6.

You are given the following information about a system of only two components:

- A and B are two independent components in a parallel system.
- T_A and T_B are the time-to-failure random variables of A and B, respectively.
- T_A has the same distribution as the first waiting time of a Poisson process with rate $\lambda = 1$.
- T_B has a constant hazard rate $\lambda_B = 2$.
- A and B start operation at the same time.

Calculate the expected time until the system fails.

A. Less than 1.00
B. At least 1.00, but less than 1.25
C. At least 1.25, but less than 1.50
D. At least 1.50, but less than 1.75
E. At least 1.75

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7.

You are given the following information about an insurer:

- The amount of each loss is an exponential random variable with mean 2000.
- Currently, there is no deductible and the insurance company pays for the full amount of each loss.
- The insurance company wishes to introduce a deductible amount, *d,* to reduce the probability of having to pay anything out on a claim by 75%. The insurance company only pays the amount per loss exceeding the deductible.
- The insurance company assumes the underlying loss distribution is unchanged after the introduction of the deductible.

Calculate the minimum amount of deductible, *d,* that will meet the requirement of having 75% fewer claims excess of deductible.

A. Less than 2,000
B. At least 2,000, but less than 2,300
C. At least 2,300, but less than 2,600
D. At least 2,600, but less than 2,900
E. At least 2,900

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8.

You are given a system of five independent components, with each component having reliability of 0.90. Three-out-of-five of the components are required to function for the system to function.

Calculate the reliability of this three-out-of-five system.

A. Less than 0.96
B. At least 0.96, but less than 0.97
C. At least 0.97, but less than 0.98
D. At least 0.98, but less than 0.99
E. At least 0.99

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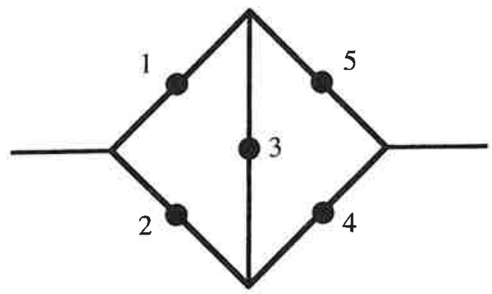
9.

You are given the following system:



and the following statements:

I. {1,5} and {2,4} are minimal path sets.
II. {1,2} and {4,5} are minimal cut sets.
III. {1,3,5} and {2,3,4} are both minimal path sets and minimal cut sets.

Determine which of the above statements are correct.

A. None are correct
B. I and II only
C. I and III only
D. II and III only
E. The answer is not given by (A), (B), (C), or (D)

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10.

You are given the following Markov chain transition probability matrix:

$$P = \begin{bmatrix} 0.7 & 0.3 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.4 & 0.2 & 0.1 & 0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Determine the number of classes of states in this Markov chain.

A. 1
B. 2
C. 3
D. 4
E. 5

CONTINUED ON NEXT PAGE

11.

You are given the following information about a homogenous Markov chain:

- $P = \begin{bmatrix} u & v & 0 \\ 1/3 & 1/3 & w \\ x & y & z \end{bmatrix}$
- The limiting probabilities of all three states are equal.

Calculate z.

A. Less than 0.35
B. At least 0.35, but less than 0.45
C. At least 0.45, but less than 0.55
D. At least 0.55, but less than 0.65
E. At least 0.65

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12.

You are given the following information:

- There are two individuals, age (45) and age (65).
- Using the Illustrative Life Table, the level premium for a whole life of 1 for age (45) is calculated as P_{45}.
- Using the Illustrative Life Table, the level premium for a whole life of 1 for age (65) is calculated as P_{65}.

Calculate the difference ($P_{45} - P_{65}$).

A. Less than -0.04
B. At least -0.04, but less than -0.02
C. At least -0.02, but less than 0.00
D. At least 0.00, but less than 0.02
E. At least 0.02

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13.

A company plans to offer a warranty on their product, which pays 1000 at the end of the year of failure if their product fails within a certain number of years, N. You are given the following information:

- The annual interest rate is 5%
- The rates of failure, q, for each year are given by the following table:

Age x	q_x
0	0.01
1	0.03
2	0.10
3	0.15
4	0.30

- The length of the warranty, N, is set to be the largest number of years such that the actuarial present value of the warranty is less than 200.

Calculate N.

A. 1
B. 2
C. 3
D. 4
E. 5

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14.

You are given the following simulation process to generate random variable X using the rejection method:

- X has density function: $f(x) = 12x(1-x)^2$, for $0 < x < 1$
- The rejection method is based on $g(x) = 1$, for $0 < x < 1$
- The rejection procedure is as follows:
  - Step 1: Generate independent random numbers Y and U, which are both uniform on $(0,1)$.
  - Step 2: If rejection function, $h(Y)$, is true, stop and set $X = Y$. Otherwise return to Step 1.

Determine which of the following is a form of the rejection function $h(Y)$.

A. $U \leq \frac{27}{4} Y(1-Y)^2$

B. $U \geq \frac{27}{4} Y(1-Y)$

C. $U \leq 16\, Y^2(1-Y)^2$

D. $U \geq \frac{27}{4} Y^2(1-Y)^2$

E. The answer is not given by (A), (B), (C), or (D)

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15.

You are given the following information:

- Loss severity follows an exponential distribution with mean of θ.
- Any loss over 10,000 will result in a claim payment of only 10,000 due to policy limits. Otherwise claims are paid in full.
- You observe the following four claim payments:

1,000	3,100	7,500	10,000

Calculate the maximum likelihood estimate of θ.

A. Less than 5,000
B. At least 5,000, but less than 6,000
C. At least 6,000, but less than 7,000
D. At least 7,000, but less than 8,000
E. At least 8,000

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16.

Suppose that a random draw is made from a normal distribution $N(\mu, \sigma^2)$, where σ is known to be 2.

The normal distribution density is given by:

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < \infty$$

Calculate the Fisher information of μ.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

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17.

You are given the following information:

- A random variable, X, is uniformly distributed on the interval $(0, \theta)$.
- θ is unknown.
- For a random sample of size n, an estimate of θ is $Y_n = \max\{X_1, X_2, \ldots, X_n\}$.

Determine which of the following is a consistent estimator of θ.

A. Y_n
B. $Y_n \,/\, (n-1)$
C. $Y_n * (n+1)$
D. $Y_n * (n+1) * (n-1)$
E. $Y_n \,/\, (n+1)$

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18.

A random sample of 10 screw lengths is taken. The sample mean is 2.5 and the unbiased sample variance is 3.0. The underlying distribution is assumed to be normal.

You want to perform a test of the variance of screw lengths, σ^2.

- Null hypothesis is $H_0: \sigma^2 = 6$
- Alternative hypothesis is $H_1: \sigma^2 < 6$

Determine the result of this hypothesis test.

A. Reject H_0 at the 0.005 level
B. Reject H_0 at the 0.010 level, but not at the 0.005 level
C. Reject H_0 at the 0.025 level, but not at the 0.010 level
D. Reject H_0 at the 0.050 level, but not at the 0.025 level
E. Do not reject H_0 at the 0.050 level

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19.

Let $X_1, \ldots, X_n$ be a random sample from a distribution with density function $f(x) = \frac{2\theta^2}{(x+\theta)^3}$, where n is large. You wish to test the hypothesis $H_0: \theta = 1$ against the hypothesis $H_1: \theta \neq 1$.

You perform this test, and the null hypothesis is rejected at a 1% significance using the Chi-squared approximation for the likelihood ratio test. Let Λ be the likelihood ratio for this hypothesis test.

Calculate the maximum possible value of Λ.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.03
D. At least 0.03, but less than 0.04
E. At least 0.04

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20.

You are given the following information:

- A random variable X follows a gamma distribution with parameters $\alpha = 0.6$ and unknown θ.
- Null hypothesis is $\mathrm{H}_0: \theta = 2000$
- Alternative hypothesis is $\mathrm{H}_1: \theta > 2000$
- n values of X will be observed and the null hypothesis is rejected if:

$$\frac{1}{n}\sum_{i=1}^{n} X_i > 1500$$

Calculate the minimum value of n which will result in a probability of Type I error of less than 2.5%, using the normal approximation.

A. Less than 100
B. At least 100, but less than 105
C. At least 105, but less than 110
D. At least 110, but less than 115
E. At least 115

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21.

You would like to determine if coffee consumption varies by start time of job shifts. A random sample of 400 adults were surveyed and the results are as follows:

	Low	Medium	High	Total
Day	100	25	75	200
Night	25	125	50	200
Total	125	150	125	400

Calculate the value of Chi-square statistic used in this test.

A. Less than 60
B. At least 60, but less than 80
C. At least 80, but less than 100
D. At least 100, but less than 120
E. At least 120

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22.

You are given the following information about samples from two populations:

- A random sample of size 8, $X_1 \ldots X_8$, was drawn from a normal population with unknown mean μ_1 and unknown variance σ_1^2.
$$s_1^2 = \frac{\Sigma(x_i - \overline{x})^2}{7}$$
- A random sample of size 10, $Y_1 \ldots Y_{10}$, was drawn from a normal population with unknown mean μ_2 and unknown variance σ_2^2.
$$s_2^2 = \frac{\Sigma(y_i - \overline{y})^2}{9}$$

You want to test the null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$ against the alternative hypothesis H_1: $\sigma_1^2 < \sigma_2^2$ at a 5% significance level by using the statistic $\frac{s_1^2}{s_2^2}$

Calculate the boundary of the best critical region for the test.

A. Less than 1.5
B. At least 1.5, but less than 3.0
C. At least 3.0, but less than 4.5
D. At least 4.5, but less than 6.0
E. At least 6.0

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23.

You are given the following information:

- X, Y, and Z are independent random variables.
- X, Y, and Z follow exponential distributions with means 2, 4, and 5, respectively.
- A single observation is taken from each of the three random variables.

Calculate the probability that the maximum of the three observed values is less than 6.

A. Less than 0.20
B. At least 0.20, but less than 0.30
C. At least 0.30, but less than 0.40
D. At least 0.40, but less than 0.50
E. At least 0.50

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24.

You are given the following output from a model constructed to predict the probability that a Homeowner's policy will retain into the next policy term:

Response variable		retention
Response distribution		binomial
Link		square root
Pseudo R^2		0.6521
Parameter	df	$\hat{\beta}$
Intercept	1	0.6102
Tenure		
< 5 years	0	0.0000
≥ 5 years	1	0.1320
Prior Rate Change		
$< 0\%$	1	0.0160
[0%, 10%]	0	0.0000
$> 10\%$	1	-0.0920
Amount of Insurance (000's)	1	0.0015

Let $\hat{\pi}$ be probability that a policy with 4 years of tenure that experienced a 12% prior rate increase and has 225,000 in amount of insurance will retain into the next policy term.

Calculate the value of $\hat{\pi}$.

A. Less than 0.60
B. At least 0.60, but less than 0.70
C. At least 0.70, but less than 0.80
D. At least 0.80, but less than 0.90
E. At least 0.90

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25.

Three separate GLMs are fit using the following model form:

$$g(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

The following error distributions were used for the three GLMs. Each model also used their canonical link functions:

Model I: gamma
Model II: Poisson
Model III: binomial

When fit to the data, all three models resulted in the same parameter estimates:

$\hat{\beta}_0$	2.0
$\hat{\beta}_1$	1.0
$\hat{\beta}_2$	-1.0

Determine the correct ordering of the models' predicted values at observed point $(X_1, X_2) = (2,1)$.

A. I < II < III
B. I < III < II
C. II < I < III
D. II < III < I
E. The answer is not given by (A), (B), (C), or (D)

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26.

You are given the following daily temperature readings, which are modeled as a function of two independent variables:

		Independent variables	
Observation	Temperature	X_1	X_2
1	5	4	6
2	10	8	2

- The data set consists of only these two observations.
- The first principal component loading for X_1, ϕ_{11}, is 0.750.
- The first principal component loading for X_2, ϕ_{21}, is positive.

Calculate the principal component score for Observation 1.

A. Less than 0.00
B. At least 0.00, but less than 0.20
C. At least 0.20, but less than 0.40
D. At least 0.40, but less than 0.60
E. At least 0.60

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27.

You are given the following information about an insurance policy:

- The probability of a policy renewal, $p(X)$, follows a logistic model with an intercept and one explanatory variable.
- $\beta_0 = 5$
- $\beta_1 = -0.65$

Calculate the odds of renewal at $x = 5$.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

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28.

You are given the following information:

- A statistician uses two models to predict the probability of success, π, of a binomial random variable.
- One of the models uses a logistic link function and the other uses a probit link function.
- There is one predictor variable, X, and an intercept in each model.
- Both models happen to produce same coefficient estimates $\hat{\beta}_0 = 0.02$ and $\hat{\beta}_1 = 0.3$.
- You are interested in the predicted probabilities of success at $x = 4$.

Calculate the absolute difference in the predicted values from the two models at $x = 4$.

A. Less than 0.1
B. At least 0.1, but less than 0.2
C. At least 0.2, but less than 0.3
D. At least 0.3, but less than 0.4
E. At least 0.4

CONTINUED ON NEXT PAGE

29.

You are given the following statements relating to deviance of GLMs:

I. Deviance can be used to assess the quality of fit for nested models.
II. A small deviance indicates a poor fit for a model.
III. A saturated model has a deviance of zero.

Determine which of the above statements are true.

A. None are true
B. I and II only
C. I and III only
D. II and III only
E. The answer is not given by (A), (B), (C), or (D)

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30.

You are considering using k-fold cross-validation (CV) in order to estimate the test error of a regression model, and have two options for choice of k:

- 5-fold CV
- Leave-one-out CV (LOOCV)

Determine which one of the following statements makes the best argument for choosing LOOCV over 5-fold CV.

A. 1-fold CV is usually sufficient for estimating the test error in regression problems.
B. LOOCV and 5-fold CV usually produce similar estimates of test error, so the simpler model is preferable.
C. Running each cross-validation model is computationally expensive.
D. Models fit on smaller subsets of the training data result in greater overestimates of the test error.
E. Using nearly-identical training data sets results in highly-correlated test error estimates.

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31.

You are given a random sample of four observations from a population:

$\{1,3,10,40\}$

and would like to calculate the standard error of an estimate of the mean of the population using a bootstrap procedure. Your calculation is based on only two bootstrapped data sets.

Calculate the maximum possible value of your standard error estimate.

A. Less than 10
B. At least 10, but less than 20
C. At least 20, but less than 30
D. At least 30, but less than 40
E. At least 40

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32.

Two different data sets were used to construct the four regression models below. The following output was produced from the models:

Data Set	Model	Dependent variable	Independent variables	Total Sum of Squares	Residual Sum of Squares
A	1	Y_A	X_{A1}, X_{A2}	35,930	2,823
	2	X_{A1}	X_{A2}	92,990	7,070
B	3	Y_B	X_{B1}, X_{B2}	27,570	13,240
	4	X_{B1}	X_{B2}	87,020	34,650

Determine which one of the following statements best describes the data.

A. Collinearity is present in both data sets A and B.
B. Collinearity is present in neither data set A nor B.
C. Collinearity is present in data set A only.
D. Collinearity is present in data set B only.
E. The degree of collinearity cannot be determined from the information given.

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33.

Consider a multiple regression model with an intercept, 3 independent variables and 13 observations. The value of $R^2 = 0.838547$.

Calculate the value of the F-statistic used to test the hypothesis $H_0: \beta_1 = \beta_2 = \beta_3 = 0$.

A. Less than 5
B. At least 5, but less than 10
C. At least 10, but less than 15
D. At least 15, but less than 20
E. At least 20

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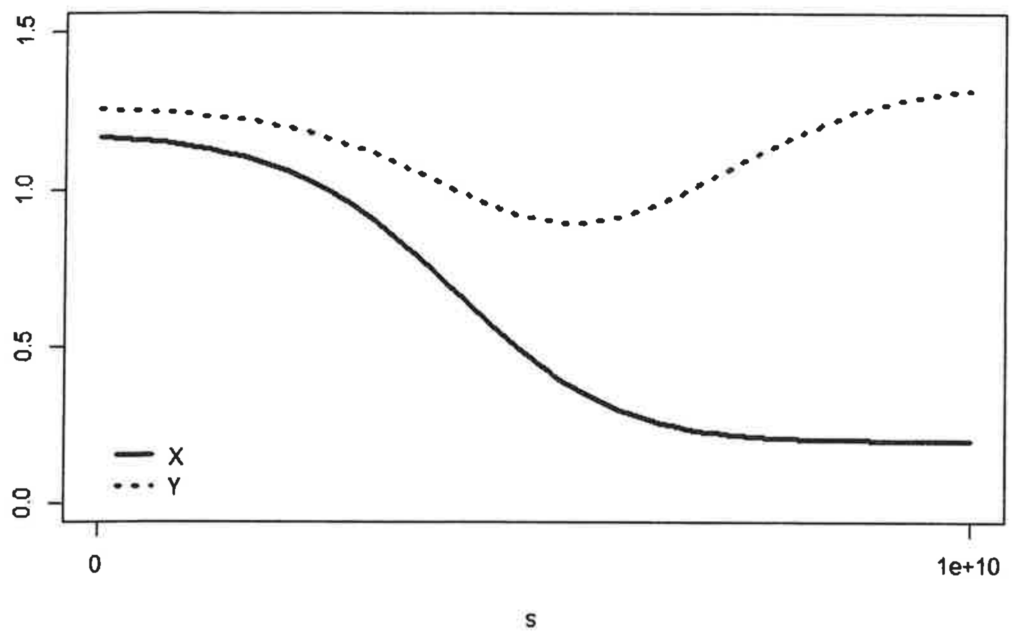
34.

You are estimating the coefficients of a linear regression model by minimizing the sum:

$$\sum_{i=1}^{n}\left(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij}\right)^2 \text{ subject to } \sum_{j=1}^{p}\beta^2 \le s$$

From this model you have produced the following plot of various statistics as a function of the budget parameter, s:



Determine which of the following statistics X and Y represent.

A. X = Squared Bias, Y = Test MSE
B. X = Test MSE, Y = Variance
C. X = Training MSE, Y = Squared Bias
D. X = Training MSE, Y = Variance
E. X = Variance, Y = Training MSE

CONTINUED ON NEXT PAGE

35.

You are given the following three statistical learning tools:

I. Cluster Analysis
II. Logistic Regression
III. Ridge Regression

Determine which of the above are examples of supervised learning.

A. None are examples of supervised learning
B. I and II only
C. I and III only
D. II and III only
E. The answer is not given by (A), (B), (C), or (D)

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36.

For a set of data with 40 observations, 2 predictors (X_1 and X_2), and one response (Y), the residual sum of squares has been calculated for several different estimates of a linear model with no intercept. Only integer values from 1 to 5 were considered for estimates of β_1 and β_2.

The grid below shows the residual sum of squares for every combination of the parameter estimates, after standardization:

		$\hat{\beta}_2$				
		1	2	3	4	5
$\hat{\beta}_1$	1	2,855.0	870.3	464.4	357.2	548.6
	2	1,059.1	488.4	216.3	242.8	567.9
	3	657.0	220.0	81.6	241.9	700.8
	4	368.4	65.1	60.5	354.5	947.1
	5	193.2	23.7	152.8	580.6	1,307.0

Let:

$\hat{\beta}_1^L$ = Estimate of β_1 using a lasso with budget parameter s = 5
$\hat{\beta}_2^L$ = Estimate of β_2 using a lasso with budget parameter s = 5

Calculate the ratio $\hat{\beta}_1^L / \hat{\beta}_2^L$.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. At least 2.0

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37.

You fit a linear model using the following two-level categorical variables:

$$X_1 = \begin{cases} 1 & \text{if Account} \\ 0 & \text{if Monoline} \end{cases}$$
$$X_2 = \begin{cases} 1 & \text{if Multi} - \text{Car} \\ 0 & \text{if Single Car} \end{cases}$$

with the equation:

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2$$

This model produced the following parameter estimates:

$$\hat{\beta}_0 = -0.10$$
$$\hat{\beta}_1 = -0.25$$
$$\hat{\beta}_2 = 0.58$$
$$\hat{\beta}_3 = -0.20$$

Another actuary modeled the same underlying data, but coded the variables differently as such:

$$Z_1 = \begin{cases} 0 & \text{if Account} \\ 1 & \text{if Monoline} \end{cases}$$
$$Z_2 = \begin{cases} 0 & \text{if Multi} - \text{Car} \\ 1 & \text{if Single Car} \end{cases}$$

with the equation:

$$E(Y) = \alpha_0 + \alpha_1 Z_1 + \alpha_2 Z_2 + \alpha_3 Z_1 Z_2$$

Afterwards you made a comparison of the individual parameter estimates in the two models.

Calculate how many pairs of coefficient estimates $(\hat{\alpha}_i, \hat{\beta}_i)$ switched signs, and how many pairs of estimates stayed identically the same, when results of the two models are compared.

A. 1 sign change, 0 identical estimates
B. 1 sign change, 1 identical estimate
C. 2 sign changes, 0 identical estimates
D. 2 sign changes, 1 identical estimate
E. The answer is not given by (A), (B), (C), or (D)

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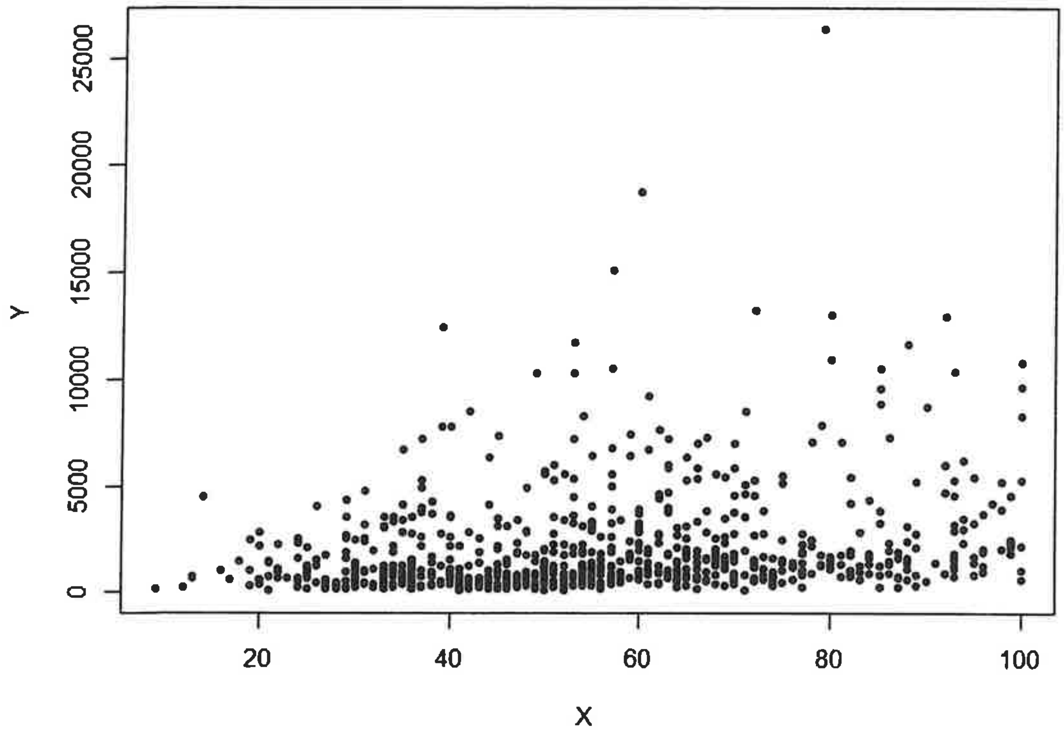
38.

You are given the following data and want to perform a piecewise polynomial regression with knots at $X = \{20,40,60,80\}$.



Determine which of the following models will use the most degrees of freedom.

A. Cubic Spline
B. Linear Spline
C. Natural Cubic Spline
D. Piecewise Cubic Regression
E. Piecewise Linear Regression

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39.

A GLM was used to estimate the expected losses per customer across gender and territory. The following information is provided:

- The link function selected is log
- Q is the base level for *Territory*
- Male is the base level for *Gender*
- Interaction terms are included in the model

The GLM produced the following predicted values for expected loss per customer:

Gender		Territory Q	Territory R
	Male	148	545
	Female	446	4,024

Calculate the estimated beta for the interaction of Territory R and Female.

A. Less than 0.85
B. At least 0.85, but less than 0.95
C. At least 0.95, but less than 1.05
D. At least 1.05, but less than 1.15
E. At least 1.15

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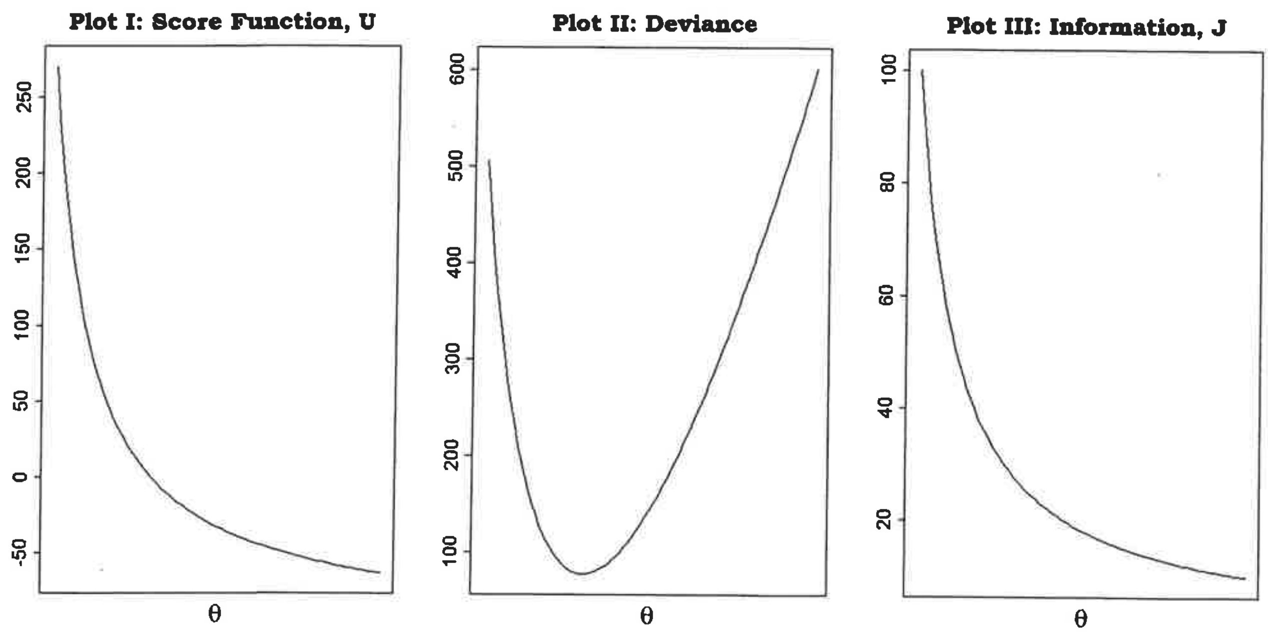
40.

An actuary fits a Poisson distribution to a sample of data, X.

$$f(x) = \frac{\theta^x e^{-\theta}}{x!}$$

To assure convergence of the maximum likelihood fitting procedure, the actuary plots three quantities of interest across different values of θ.



Determine which of the three plots the actuary can use to visually approximate the maximum likelihood estimate for θ.

A. None can be used
B. I and II only
C. I and III only
D. II and III only
E. The answer is not given by (A), (B), (C), or (D)

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41.

You are given the following information from a time series:

t	x_t
1	3.0
2	2.5
3	2.0
4	3.5
5	4.0
6	3.0
$\bar{x}$	3.0

Calculate the sample lag 3 autocorrelation.

A. Less than -0.30
B. At least -0.30, but less than -0.10
C. At least -0.10, but less than 0.10
D. At least 0.10, but less than 0.30
E. At least 0.30

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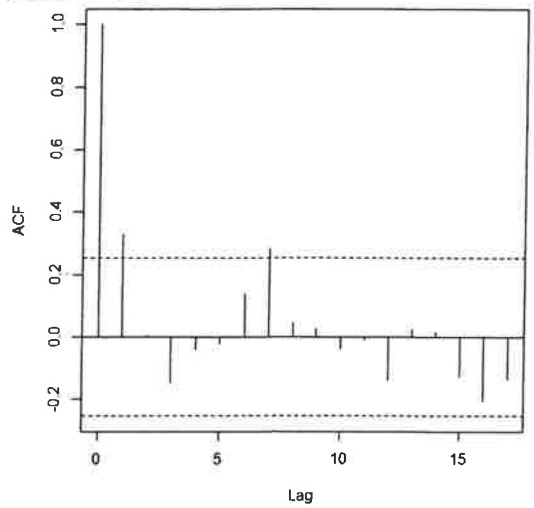
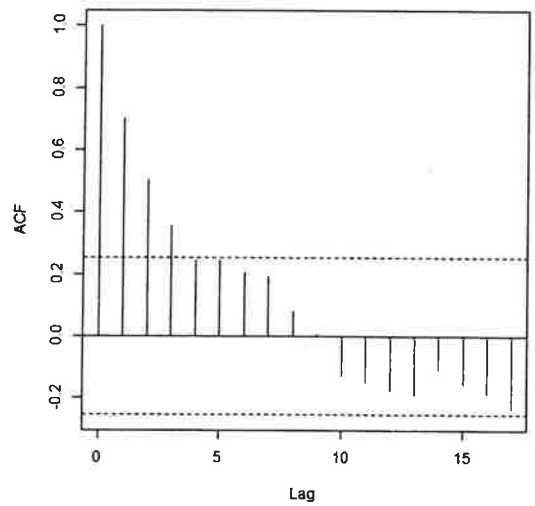
42.

You are given the following correlograms for three separate data sets, each with n observations.

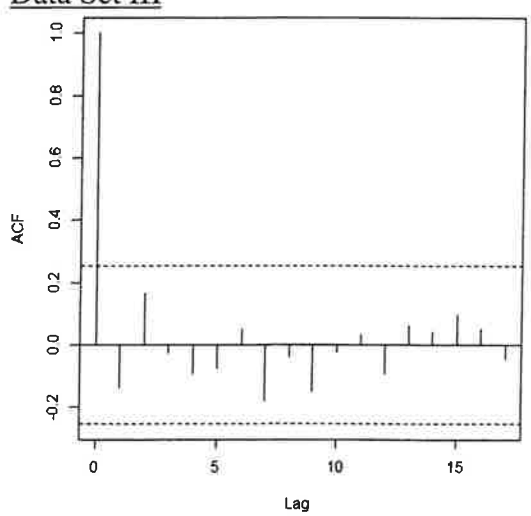
Data Set I



Data Set II



Data Set III



The dotted lines in each correlogram correspond to the values: $\left(-\frac{1}{n} \pm \frac{2}{\sqrt{n}}\right)$.

Determine which of the above data sets exhibit statistically significant autocorrelation.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C), or (D)

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43.

You are given the following stock prices of company CAS:

Day	Stock Price
1	538
2	548
3	528
4	608
5	598
6	589
7	548
8	514
9	501
10	498

- The average stock price in those days is 547

Calculate the sample autocovariance at lag 3.

A. Less than -500
B. At least -500, but less than -250
C. At least -250, but less than 0
D. At least 0, but less than 250
E. At least 250

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44.

You are given the follow fitted AR(1) model:

$$\widehat{Z_t} = 5 + 0.85 z_{t-1}$$

The estimated mean squared error is 13.645.

Calculate the two-step ahead forecast standard error.

A. Less than 4.0
B. At least 4.0, but less than 7.0
C. At least 7.0, but less than 10.0
D. At least 10.0, but less than 13.0
E. At least 13.0.

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45.

You are given the following statements about time series:

I. Stochastic trends are characterized by explainable changes in direction.
II. Deterministic trends are better suited to extrapolation than stochastic trends.
III. Deterministic trends are typically attributed to high serial correlation with random error.

Determine which of the above statements are true.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C), or (D)

END OF EXAMINATION

MAS-I Final Answer Key

Question	Answer
1	D
2	B
3	C
4	D
5	A
6	B
7	D
8	E
9	B
10	D
11	E
12	B
13	C
14	A
15	D
16	B
17	A
18	E
19	D
20	B
21	D
22	A
23	E
24	C
25	B
26	A
27	C
28	B
29	C
30	D
31	C
32	C
33	D
34	A
35	D
36	B
37	E
38	D
39	B
40	B
41	B
42	E
43	B
44	B
45	B