# Exam ST

**CASUALTY ACTUARIAL SOCIETY**
AND THE
**CANADIAN INSTITUTE OF ACTUARIES**



October 30, 2014

# Exam ST

## Models for Stochastic Processes and Statistics



**2.5 HOURS**

## INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2014 and that the exam name is ST.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam ST" included in your exam packet.

**CONTINUE TO NEXT PAGE OF INSTRUCTIONS**

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. <u>Do not remove this label</u>. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. <u>Candidates must remain in the examination center until the examination has concluded.</u> The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. <u>At the end of the examination, place the short-answer card in the Examination Envelope.</u> Nothing written in the examination booklet will be graded. <u>Only the short-answer card will be graded.</u> Also place any included reference materials in the Examination Envelope. <u>BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.</u>

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. <u>Do not put the self-addressed stamped envelope inside the Examination Envelope.</u>

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. <u>Do not put scrap paper in the Examination Envelope.</u> The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the “Admissions/Exams” section. Please submit your survey by November 17, 2014.

**END OF INSTRUCTIONS**

1.

You are trying to get downtown at rush hour and will take the first vehicle to arrive. You are given:

- Taxis arrivals follow the Poisson process with rate $\lambda = 1$ per 10 minutes.
- Bus arrivals follow the Poisson process with rate $\lambda = 4$ per 30 minutes.
- Streetcar arrivals follow the Poisson process with rate $\lambda = 2$ per hour.

Let W denote your waiting time, in minutes.

Calculate the variance of W.

A. Less than 5
B. At least 5, but less than 10
C. At least 10, but less than 15
D. At least 15, but less than 20
E. At least 20



EXAMINATION CONTINUED ON NEXT PAGE

2.

You are given:

- The number of tickets sold to a show follows a Poisson process.
- Tickets are sold at a rate of $(800 - 45t)$ per day, where $t$ is the number of days.
- Tickets are on sale for 7 days.
- 90% of the tickets sold will actually be used.

Calculate the expected number of tickets used.

A. Fewer than 3,800
B. At least 3,800, but fewer than 4,100
C. At least 4,100, but fewer than 4,400
D. At least 4,400 but fewer than 4,700
E. At least 4,700


EXAMINATION CONTINUED ON NEXT PAGE

3.

The number of tow trucks needed for car accidents in a city follows a compound Poisson process. You are given:

- The expected number of accidents per hour $= \begin{cases} 5 & \text{for} \quad \text{7am to 7pm} \\ 2 & \text{for} \quad \text{7pm to 7am} \end{cases}$

- The number of tow trucks needed per accident follows the Poisson distribution with a mean of 1.5.

- The expected number of accidents per hour follows the Poisson distribution.

Calculate the probability that more than 150 tow trucks will be needed in a 24 hour period using a normal approximation.

A. Less than 1%
B. At least 1%, but less than 5%
C. At least 5%, but less than 10%
D. At least 10%, but less than 20%
E. At least 20%



EXAMINATION CONTINUED ON NEXT PAGE

4.

You are given a random sample of seven observations, 1.2, 1.9, 2.5, 2.7, 2.8, 3.3, and 3.6, from the following distribution:

$$f(x) = \frac{\theta^{-3} x^2 e^{-\frac{x}{\theta}}}{2}, \qquad x > 0$$

Calculate the maximum likelihood estimate of θ.

A. Less than 0.65
B. At least 0.65, but less than 0.70
C. At least 0.70, but less than 0.75
D. At least 0.75, but less than 0.80
E. At least 0.80.

5.

Calculate the Fisher information, I(q), of a binomial distribution with m=1.

A. $\frac{1-2q}{q(1-q)}$

B. $\frac{1}{q(1-q)}$

C. *q(1-q)*

D. $\frac{-1}{q(1-q)}$

E. *0*



**EXAMINATION CONTINUED ON NEXT PAGE**

6.

You are given the following information:

- $X$ is a random variable uniformly distributed on [-a,a].
- Two values are observed, with order statistics $Y_1 < Y_2$.
- $kY_2$ is used as an estimate of a.

Calculate the value of k that results in an unbiased estimate of a.

A. Less than 0.5
B. At least 0.5, but less than 1.5
C. At least 1.5, but less than 2.5
D. At least 2.5, but less than 3.5
E. At least 3.5



EXAMINATION CONTINUED ON NEXT PAGE

7.

You are given the following information:

- $Y_1, Y_2, \ldots, Y_7$ is a random sample of size 7 where the observations are independent and identically distributed from an exponential distribution with mean $\theta$.
- The mean of the sample is 400.

Calculate the minimum variance unbiased estimate of $\theta^2$.

A. Less than 150,000
B. At least 150,000, but less than 175,000
C. At least 175,000, but less than 200,000
D. At least 200,000, but less than 225,000
E. At least 225,000



EXAMINATION CONTINUED ON NEXT PAGE

8.

Let $f(x) = 3e^{3(\alpha - x)}, x \geq \alpha, \alpha > 0$. Let 0.1, 0.6, and 0.5 be a sample of size 3.

Determine the maximum likelihood estimate of α for this function using this data.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8


EXAMINATION CONTINUED ON NEXT PAGE

9.

For a general liability policy the paid claim amounts follow the Pareto distribution with probability density function:

$$f(x) = \frac{\alpha(18)^{\alpha}}{(x+18)^{\alpha+1}} \qquad 0 < x$$

You are given:

- Five paid claim amounts of 1, 6, 7, 8, and 10.
- Three additional paid claim amounts exceed 10.

Calculate the maximum likelihood estimate of $\alpha$.

A. Less than 1

B. At least 1, but less than 2

C. At least 2, but less than 3

D. At least 3, but less than 4

E. At least 4



EXAMINATION CONTINUED ON NEXT PAGE

10.

Let $Y_1, Y_2, \dots, Y_n$ be a random sample of size $n$ from a normal distribution with unknown mean $\mu$ and known variance $\sigma^2 = 25$.

- $H_0$: $\mu = 20$.
- $H_1$: $\mu > 20$.
- Significance level $\alpha = 0.05$.
- Sample mean was 21.49.

Calculate the smallest value of $n$ that would result in rejecting $H_0$ using the Neyman-Pearson lemma.

A. Less than 32
B. At least 32, but less than 33
C. At least 33, but less than 34
D. At least 34, but less than 35
E. At least 35

11.

You are given the following information:

- A random sample consisting of 6 observations is taken from a normal distribution with mean $\mu$ and unknown variance.
- The sample mean is 3.1.
- The unbiased sample variance is 0.15.

The following hypothesis test is performed:

- $H_0$: $\mu = 2.5$
- $H_1$: $\mu > 2.5$

Calculate the *p*-value of this test.

A. Less than 0.005
B. At least 0.005, but less than 0.01
C. At least 0.01, but less than 0.025
D. At least 0.025, but less than 0.05
E. At least 0.05

12.

A sample of size n=12 is drawn from a normal population with unknown mean $\mu$ and known variance $\sigma^2$. The null hypothesis $H_0$: $\mu=8$ is rejected at the level of significance $\alpha=0.06$ if $\sum_{i=1}^{12} X_i \in R$ where R is defined:

$$R = \left\{\sum_{i=1}^{12} X_i \geq 105\right\} \cup \left\{\sum_{i=1}^{12} X_i \leq L\right\}$$

You are given:

- $\Pr\{\sum_{i=1}^{12} X_i \geq 105\} = 2\Pr\{\sum_{i=1}^{12} X_i \leq L\}$

Calculate L.

A. Less than 80
B. At least 80, but less than 90
C. At least 90, but less than 100
D. At least 100, but less than 110
E. At least 110

13.

You are given the following information:

- Losses follow a normal distribution with mean μ and known variance 100.
- The pdf for the normal distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- $H_0$: $\mu = 50$.
- $H_1$: $\mu \neq 50$.
- k is selected so that the significance level of the test is α.

Given a sample of size *n*, find the form of the critical region determined from the likelihood ratio test.

A. $|\bar{X} - 50| \leq \sqrt{\frac{100k}{n}}$

B. $|\bar{X} - 50| \geq \sqrt{\frac{200k}{n}}$

C. $|\bar{X} - 50| > \frac{100k}{n}$

D. $|\bar{X} - 50| \leq \frac{100n}{k}$

E. $|\bar{X} - 50| \leq \sqrt{\frac{200n}{k}}$


EXAMINATION CONTINUED ON NEXT PAGE

14.

You are given the following:

- A discrete distribution has two possible probability density functions: f(x) and g(x).
- Those distribution functions are defined for the space: 1, 2, 3, 4, 5, 6.
- $H_0$ : f(x) is the probability density function.
- $H_1$ : g(x) is the probability density function.
- The two distribution functions are defined in the table shown below.

| X | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| f(x) | 0.05 | 0.05 | 0.4 | 0.3 | 0.15 | 0.05 |
| g(x) | 0.1 | 0.15 | 0.25 | 0.25 | 0.2 | 0.05 |

Calculate the Type II error based on the best critical region as defined by the Neyman-Pearson Theorem of size $\alpha$=0.1.

A. Less than 0.30
B. At least 0.30, but less than 0.40
C. At least 0.40, but less than 0.50
D. At least 0.50, but less than 0.60
E. At least 0.60

15.

You are given:

- 500 random observations are taken from a normal distribution with mean $\mu$and variance 12.
- $H_0 : \mu = 7$.
- $H_1 : \mu > 7$.
- Probability of a Type I error is 15%.
- Reject $H_0$ if $\bar{x} \geq c$, where c is the critical region.

Calculate c.

A. Less than 7.5
B. At least 7.5, but less than 8.5
C. At least 8.5, but less than 9.5
D. At least 9.5, but less than 10.5
E. At least 11.5

16.

You are given the following information:

- Your company has developed a new training program for claims adjusters to increase their productivity.
- Assume that the number of claims closed follows a normal distribution, with equal variance before and after the training program.
- Observations for a given adjuster are not independent.
- $H_0$: the number of claims closed is the same before and after training.
- $H_1$: the number of claims closed is not the same before and after training.

| Adjuster ID | # Claims Closed Before Training | # Claims Closed After Training |
|---|---|---|
| 1 | 7 | 5 |
| 2 | 6 | 3 |
| 3 | 7 | 7 |
| 4 | 8 | 12 |
| 5 | 4 | 10 |
| 6 | 7 | 13 |
| 7 | 5 | 10 |
| 8 | 7 | 14 |
| 9 | 8 | 6 |
| 10 | 2 | 6 |
| TOTAL | 61 | 86 |

Calculate the minimum significance level at which you reject $H_0$.

A. Less than 0.1%
B. At least 0.1%, but less than 1.0%
C. At least 1.0%, but less than 2.0%
D. At least 2.0%, but less than 5.0%
E. At least 5.0%



EXAMINATION CONTINUED ON NEXT PAGE

17.

You are given the following daily temperature readings from two different locations with only three observations from Location A and four from Location B:

| | | | | |
|---|---|---|---|---|
| Location A | 45 | 57 | 62 | |
| Location B | 81 | 60 | 65 | 75 |

- Assume $m$ is defined as the median of the dataset.
- $H_0$: $m_A = m_B$.
- $H_1$: $m_A \neq m_B$.
- You perform a Mann-Whitney-Wilcoxon test using the exact calculation.

Determine the smallest significance level at which you reject $H_0$..

A. Less than 1.0%
B. At least 1.0%, but less than 2.5%
C. At least 2.5%, but less than 5.0%
D. At least 5.0%, but less than 10.0%
E. At least 10.0%



EXAMINATION CONTINUED ON NEXT PAGE

18.

You are given the following information:

- From each of the classrooms X and Y, a random sample of students' test scores were selected for analysis.
- Five scores were selected from each classroom.
- Let $X_i$ be the sampled scores from classroom X and let $Y_i$ be the sampled scores from classroom Y.
- $H_0$: $m_x = m_y$ , where $m_i$ is the median of the difference in the test scores.
- $H_1$: $m_x < m_y$
- $H_0$ is rejected at the $\alpha = 0.05$ significance level.
- W is defined as the Wilcoxon statistic.
- The following sample data is collected:

| | | | | | |
|---|---|---|---|---|---|
| X: | 86.5 | 85.2 | 70.8 | 96.2 | 90.6 |
| Y: | 65.2 | 99.4 | 82.3 | 92.2 | 89.8 |

Calculate the absolute value of the difference between the computed W and the lower-bound of the w implied by the significance level, using the normal approximation.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

19.

You are given the following information:

- A bowler claims to be equally skilled at bowling right-handed and left- handed.
- The bowler bowled 10 games with each hand. The scores are provided in the table below.

| Game | Right Handed | Left Handed |
|---|---|---|
| A | 200 | 175 |
| B | 200 | 175 |
| C | 200 | 175 |
| D | 175 | 175 |
| E | 175 | 175 |
| F | 175 | 175 |
| G | 175 | 175 |
| H | 175 | 175 |
| I | 200 | 175 |
| J | 200 | 175 |

- Assume $m$ is defined as the median of the dataset.
- $H_0$: $m_R = m_L$
- $H_1$: $m_R \neq m_L$

Calculate the smallest value for α for which $H_0$ can be rejected using the Sign Test using the exact calculation.

A. Less than 0.010
B. At least 0.010, but less than 0.025
C. At least 0.025, but less than 0.050
D. At least 0.050, but less than 0.100
E. At least 0.100


EXAMINATION CONTINUED ON NEXT PAGE

20.

For the linear model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$, you are given:

- $n = 6$.
- $\hat{\beta}_1 = 4$.
- $\sum_{i=1}^{n}(x_i - \bar{x})^2 = 50$ .
- $\sum_{i=1}^{n}(y_i - \bar{y})^2 - \hat{\beta}_1 \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 25$.

Calculate the upper bound of the 95% confidence interval for $\beta_1$.

A. Less than 5.1
B. At least 5.1 but less than 5.3
C. At least 5.3 but less than 5.5
D. At least 5.5, but less than 5.7
E. At least 5.7



EXAMINATION CONTINUED ON NEXT PAGE

21.

You are given the following information:

- A claim tracking system is developed that has three different interfaces. A consulting firm tests the speed of each of the three interfaces and assigns a score between 0 and 50 based on the speed.
- Let $X_i$ equal the speed of interface $i$, and assume that the distribution of $X_i$ is normal with mean = $\mu_i$, and variance = $\sigma^2$
- We are testing the null hypothesis, $H_0$: $\mu_1 = \mu_2 = \mu_3$, using 4 independent speed observations of each interface.
- The following sample data is collected:

| | Observation 1 | Observation 2 | Observation 3 | Observation 4 | $\sum X_i$ | $\sum X_i^2$ |
|---|---|---|---|---|---|---|
| $X_1$ | 10 | 25 | 20 | 19 | 74 | 1486 |
| $X_2$ | 23 | 26 | 24 | 27 | 100 | 2510 |
| $X_3$ | 24 | 25 | 28 | 30 | 107 | 2885 |

Calculate the F statistic of this test.

A. Less than 4
B. At least 4, but less than 5
C. At least 5, but less than 6
D. At least 6, but less than 7
E. At least 7

22.

You are given:

i. For a general liability policy, the log of paid claims conditionally follows the normal distribution with mean $\mu$, which varies by policy holder, and variance 1.

ii. The posterior distribution of $\mu$ follows the normal distribution with mean $\frac{n\bar{x}+2}{n+1}$ and variance $\frac{1}{n+1}$ where $\bar{x}$ denotes the sample mean and n denotes the sample size.

iii. The following sample of observed log of paid claims:

| | | | | |
|---|---|---|---|---|
| 3.22 | 4.34 | 5.98 | 7.32 | 2.78 |

Calculate the upper bound of the symmetric 95% Bayesian confidence interval for $\mu$.

A. Less than 4.8
B. At least 4.8, but less than 5.0
C. At least 5.0, but less than 5.2
D. At least 5.2, but less than 5.4
E. At least 5.4



EXAMINATION CONTINUED ON NEXT PAGE

23.

To test whether or not a coin with one side heads and the other side tails is fair, you model the probability of it landing on heads using a Bayesian approach and Beta Distribution with initial parameter estimates of $a = b = 1$.

After 10 trials the coin comes up heads 9 times.

Calculate the Bayesian posterior probability of a heads on the $11^{\text{th}}$ trial.

A. Less than 50%
B. At least 50%, but less than 60%
C. At least 60%, but less than 70%
D. At least 70%, but less than 80%
E. At least 80%

24.

For a large number of home insurance policies, the profit per policy is modeled using a Bayesian approach and a Normal distribution with unknown mean $\mu$ and known variance $\sigma^2 = 10{,}000$.

The prior expected profit per policy follows a Normal distribution with mean $\eta = 100$ and variance $\delta^2 = 100$.

After the first year, a sample of five policies returns a total profit of 5,000 to the company.

Calculate the Bayesian posterior average profit per policy.

A. Less than 75
B. At least 75, but less than 100
C. At least 100, but less than 125
D. At least 125, but less than 150
E. At least 150

25.

You are given the following information for an insurance policy:

- Annual claim frequencies follow a Poisson process with parameter $\lambda$.
- The prior distribution of $\lambda$ follows a gamma distribution with parameters $\alpha$=5 and $\theta$=2.
- In the first two years, a policy had 5 claims.

Calculate the posterior mean of annual claims for this policy.

A. Less than 3
B. At least 3, but less than 5
C. At least 5, but less than 7
D. At least 7, but less than 9
E. At least 9



**END OF EXAMINATION**

Fall 2014 Exam ST Answer Key

| question | Solution |
|---|---|
| 1 | C |
| 2 | B |
| 3 | C |
| 4 | E |
| 5 | B |
| 6 | D |
| 7 | A |
| 8 | A |
| 9 | B |
| 10 | A |
| 11 | B |
| 12 | B |
| 13 | B |
| 14 | E |
| 15 | A |
| 16 | E |
| 17 | E |
| 18 | D |
| 19 | D |
| 20 | A |
| 21 | B |
| 22 | C |
| 23 | E |
| 24 | D |
| 25 | B |