CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam ST

Models for Stochastic Processes and Statistics

April 30, 2014

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth two points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2014 and that the exam name is ST.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam ST" included in your exam packet.

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5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 14, 2014.

END OF INSTRUCTIONS

1.

You are given the following information for a policy covering two types of claims:

- Total number of claims is given by a Poisson process with claims intensity $\lambda(t) = 10t$, $t > 0$.
- At any time the probability of a claim being from Claim Type A is 0.6 and from Claim Type B is 0.4.
- Frequency and severity of claims are independent.
- Claim severities follow the distributions given in the tables below

Claim Type A

Claim Amount	Probability
< 500	0.3
At least 500, but less than 1000	0.5
At least 1000	0.2

Claim Type B

Claim Amount	Probability
< 1000	0.1
At least 1000, but less than 2000	0.6
At least 2000	0.3

Calculate the probability that by time 0.5 there will be fewer than two claims with severity at least equal to 1000.

A. Less than 0.55
B. At least 0.55, but less than 0.65
C. At least 0.65, but less than 0.75
D. At least 0.75, but less than 0.85
E. At least 0.85

Exam Continued on Next Page

2.

You are given the following information:

- Buses arrive according to a Poisson process at a rate of 5 per hour.
- Taxis arrive according to a Poisson process at a rate of 10 per hour.
- The arrival of buses and taxis are independent.
- You get a ride to work from either a bus or a taxi, whichever arrives first.

Calculate the probability you will have to wait more than 10 minutes for a ride to work.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15 but less than 0.20
E. At least 0.20

Exam Continued on Next Page

3.

Let $S(t) = X_1 + X_2 + \ldots + X_{N(t)}$ be a compound Poisson process where:

- X_i is uniform on [0, 3.14]
- N(t) is a Poisson process with rate $\lambda(t) = e^{-t}, t > 0$

Calculate $E[S(4)^2]$.

A. Less than 2.5
B. At least 2.5, but less than 3.5
C. At least 3.5, but less than 4.5
D. At least 4.5, but less than 5.5
E. At least 5.5

Exam Continued on Next Page

4.

You have a random sample of five observations:

2.5 7.5 12.5 16.0 17.5

- The probability density function below is fit to the random sample.

$$f(x) = \frac{2}{\beta^2}\, x_i\, e^{-\left(\frac{x_i}{\beta}\right)^2}$$

Calculate the maximum likelihood estimate of β.

A. Less than 10
B. At least 10, but less than 11
C. At least 11, but less than 12
D. At least 12, but less than 13
E. At least 13

Exam Continued on Next Page

5.

You are given the following information:

- A random variable, X, follows a two-parameter Pareto distribution with parameters $\alpha = 5$ and θ unknown.
- A random sample of 15 independent observations is taken.
- θ is estimated as $c\overline{X}$, where c is a constant.

Calculate the value of c that minimizes the mean squared error of the estimator of θ.

A. Less than 1.0
B. At least 1.0, but less than 2.0
C. At least 2.0, but less than 3.0
D. At least 3.0, but less than 4.0
E. At least 4.0

Exam Continued on Next Page

6.

You are given the following probability distribution:

X	$P(X = x)$
0	θ
1	2θ
2	$1 - 3\theta$

A random sample was taken with the following results:

- The value 0 was observed 2 times.
- The value 1 was observed 5 times.
- The value 2 was observed 3 times.

Calculate the maximum likelihood estimate of θ.

A. Less than 0.15
B. At least 0.15, but less than 0.18
C. At least 0.18, but less than 0.21
D. At least 0.21, but less than 0.24
E. At least 0.24

Exam Continued on Next Page

7.

You are given a distribution with the following probability density function, where θ is unknown:

$$f(x) = \theta x^{\theta-1}, 0<x<1, \theta>0$$

and a random sample of five observations

0.13 0.42 0.60 0.71 0.88

Calculate the maximum likelihood estimate of θ.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

Exam Continued on Next Page

8.

You are given the following:

- X is a uniformly distributed random variable with the pdf

$$f_X(x) = \frac{1}{\theta} \qquad 0 \le x \le \theta$$

- $\hat{\theta} = cX_{max}$ where X_{max} is the largest X_i from a sample of size n.

Determine the value of c that makes $\hat{\theta}$ an unbiased estimator of θ when $n = 100$.

A. Less than 1.005
B. At least 1.005, but less than 1.025
C. At least 1.025, but less than 1.045
D. At least 1.045, but less than 1.065
E. At least 1.065

Exam Continued on Next Page

9.

You are given the following information about the number of auto claims:

	Age Group		
Gender	***Young***	***Old***	***Total***
Male	20	15	35
Female	12	22	34
Total	32	37	69

A Chi-squared hypothesis test is performed to determine if gender and age group are independent.

Calculate the p-value of this test.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.025
D. At least 0.025, but less than 0.050
E. At least 0.050

Exam Continued on Next Page

10.

You are given the following information about a hypothesis test:

- The data is from a Normal Distribution with variance 25
- H_0: $\mu = 4$
- H_1: $\mu = 7$
- H_0 is rejected when $\overline{X} \geq 7.25$
- The sample size is 80.

Calculate the probability of making a Type II error.

A. Less than 0.35
B. At least 0.35, but less than 0.45
C. At least 0.45, but less than 0.55
D. At least 0.55, but less than 0.65
E. At least 0.65

Exam Continued on Next Page

11.

You are given the following information:

- X and Y are random variables with unknown means μ_x and μ_y
- X and Y have unknown but equal variances.
- Samples from X and Y are taken with sizes 40 and 50, respectively
- $\frac{\sum X_i}{40} = \overline{X} = 80.1$ and $\frac{\sum Y_i}{50} = \overline{Y} = 78.8$
- $\frac{\sum(X_i - \overline{X})^2}{40} = 34.8$ and $\frac{\sum(Y_i - \overline{Y})^2}{50} = 25.0$
- H_0: $\mu_x = \mu_y$
- H_1: $\mu_x > \mu_y$

Determine the p-value of the hypothesis test.

A. Less than 0.05
B. At least 0.05, but less than 0.10
C. At least 0.10, but less than 0.15
D. At least 0.15, but less than 0.20
E. At least 0.20

Exam Continued on Next Page

12.

You are given the following claim observations for 2 territories:

- Territory X: A random sample of 100 claims is drawn from a normal distribution with variance 10,000, and has a sample mean of 300
- Territory Y: A random sample of 70 claims is drawn from a normal distribution with variance 5000, and has a sample mean of 250

H_0: mean of Territory X = mean of Territory Y + 30
H_1: mean of Territory X > mean of Territory Y + 30

Determine the p-value of this hypothesis test.

A. Less than 5.0%
B. At least 5.0%, but less than 5.5%
C. At least 5.5%, but less than 6.0%
D. At least 6.0%, but less than 6.5%
E. At least 6.5%

Exam Continued on Next Page

13.

Two six-sided dice (X and Y) are rolled 1000 times each. The outcomes are in the table below.

Outcome	X	Y
1	94	157
2	244	168
3	135	197
4	158	136
5	128	220
6	241	122

An outcome of 4 or more is considered a success. Let P be the probability of a success.

$H_0 : P_X = P_Y$

$H_1 : P_X \neq P_Y$

Calculate the minimum significance level at which the null hypothesis will be rejected.

A. Less than 0.025
B. At least 0.025, but less than 0.035
C. At least 0.035, but less than 0.045
D. At least 0.045, but less than 0.055
E. At least 0.055

Exam Continued on Next Page

14.

You are given the following information about two loss severity distributions fit to a sample of 275 closed claims:

- For the Exponential distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is -828.37.
- For the Weibull distribution, the natural logarithm of the likelihood function evaluated at the maximum likelihood estimate is -826.23.
- The Exponential distribution is a subset of the Weibull distribution.

Calculate the significance level at which the Weibull distribution provides a better fit than the Exponential distribution.

A. Less than 0.5%
B. At least 0.5%, but less than 1.0%
C. At least 1.0%, but less than 2.5%
D. At least 2.5%, but less than 5.0%
E. At least 5.0%

Exam Continued on Next Page

15.

You are given the following information:

Student	Number of Practice Exams	Score on Final Exam
1	8	9
2	6	8
3	5	7
4	4	6
5	3	5
6	2	10
7	1	4
8	0	3

Students are ranked in descending order for both the number of practice exams and the score on the final exam.

Calculate Spearman's rank correlation coefficient between the number of practice exams and the score on final exam.

A. Less than 0.0
B. At least 0.0, but less than 0.3
C. At least 0.3, but less than 0.6
D. At least 0.6, but less than 0.9
E. At least 0.9

Exam Continued on Next Page

16.

A random sample of size eight is drawn from a population in order to test the null hypothesis that the median of the population is 2.3.

You are given the following sample values:

1.8	0.2	1.5	2.5	2.7	0.4	2.4	3.0

Calculate the Wilcoxon statistic.

A. Less than -30
B. At least -30, but less than -25
C. At least -25, but less than -20
D. At least -20, but less than -15
E. At least -15

Exam Continued on Next Page

17.

The probability density of Y_k, the k[th] order statistic for a sample of size n is:

$$g_k(y_k) = \frac{n!}{(k-1)!\,(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y)$$

Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of five independent observations from a uniform distribution on (0,10).

Calculate the variance of Y_4.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

Exam Continued on Next Page

18.

Let $Y_1 < Y_2 < \ldots < Y_{12}$ be the order statistics of a random sample from a uniform distribution [0,1].

Calculate the probability that $0.6 < Y_{12} < 0.75$.

A. Less than 0.010
B. At least 0.010, but less than 0.015
C. At least 0.015, but less than 0.020
D. At least 0.020, but less than 0.025
E. At least 0.025

Exam Continued on Next Page

19.

You are given the following sales information from three car dealers:

Dealer	Number of Cars Sold	Average Sales Price (000s)	SS(E)
A	6	23.8	6.83
B	4	25.3	2.75
C	5	27.4	37.20

You wish to test whether the average sales prices are the same among the dealers:

H_0: $\mu_A = \mu_B = \mu_C$ (The mean sales price between the three dealers is the same)
H_1: The mean sales price between the three dealers is not the same.

Create an ANOVA table and assume the total sum squares, SS(TO) = 81.6

Which of the following statements is TRUE?

A. Reject H_0 when $\alpha = 0.05$ and reject H_0 when $\alpha = 0.01$
B. Reject H_0 when $\alpha = 0.05$ but do not reject H_0 when $\alpha = 0.01$
C. Do not reject H_0 when $\alpha = 0.05$ but reject H_0 when $\alpha = 0.01$
D. Do not reject H_0 when $\alpha = 0.05$ and do not reject H_0 when $\alpha = 0.01$
E. None of above

Exam Continued on Next Page

20.

The model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ was fit using 6 observations. The estimated parameters are as follows:

- $\hat{\beta}_0 = 2.31$
- $\hat{\beta}_1 = 1.15$
- $\hat{\sigma}_{\beta_0} = 0.057$
- $\hat{\sigma}_{\beta_1} = 0.043$

The following hypothesis test is performed:

- $\mathrm{H}_0: \beta_1 = 1$
- $\mathrm{H}_1: \beta_1 \neq 1$

Determine the minimum significance level at which the null hypothesis would be rejected.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.05
D. At least 0.05, but less than 0.10
E. At least 0.10

Exam Continued on Next Page

21.

For $i = 1, 2, 3, 4$ let X_i denote the speed of cars passing location i.

- Assume that the distribution of each X_i is normal with mean μ_i and common variance σ^2
- H_0 : $\mu_1=\mu_2=\mu_3=\mu_4$ (The means are all equal)
- H_1 : The means are not all equal.
- x_{ij} is observation j at location i
- You have the following observations:

Location	Speed by Car Observed					$\sum_{j=1}^{5} x_{ij}$	$\sum_{j=1}^{5} x_{ij}^2$
1	52.4	55.7	60.2	69.9	59.4	297.6	17,886.66
2	62.5	65.1	56.5	52.8	69.0	305.9	18,885.35
3	61.0	63.5	84.0	68.1	77.6	354.2	25,468.62
4	63.0	52.1	65.2	59.5	52.8	292.6	17,262.54
					Totals	1250.3	79,503.17

Determine the F-statistic used to test H_0.

A. Less than 2.5
B. At least 2.5, but less than 2.7
C. At least 2.7, but less than 2.9
D. At least 2.9, but less than 3.1
E. At least 3.1

Exam Continued on Next Page

22.

You are given the following information for an insurance policy:

- Monthly claim frequencies follow a Poisson process with parameter λ.
- The prior distribution of λ follows a gamma distribution with parameters $\alpha=3$ and $\theta=2$.
- In the first month, a policy had 27 claims.

Calculate the posterior mean monthly claims for this policy.

A. Less than 9
B. At least 9, but less than 13
C. At least 13, but less than 17
D. At least 17, but less than 21
E. At least 21

Exam Continued on Next Page

23.

You are given the following information:

- $X_1, \ldots, X_5$, are a random sample from a Poisson distribution with parameter λ, where λ follows a gamma distribution with parameters $\alpha = 2$ and θ.
- The mean of this Poisson-gamma conjugate pair can be represented as a weighted average of the maximum likelihood estimator for the mean and the mean of the prior distribution.
- Let W_{MLE} be the weight assigned to the maximum likelihood estimator.
- The maximum likelihood estimate for the mean is 1.2.
- The variance of the prior gamma is 8.

Calculate W_{MLE}.

A. Less than 0.60
B. At least 0.60, but less than 0.70
C. At least 0.70, but less than 0.80
D. At least 0.80, but less than 0.90
E. At least 0.90

Exam Continued on Next Page

24.

You are given the following information about a random variable X:

- X follows the Normal distribution with parameters $N(\Theta,\sigma^2)$ where σ^2, the variance, is known to be 2.
- Θ is assumed to follow a Normal distribution with parameters N(1, 4) prior to gathering a random sample
- A random sample of size 10 is collected to develop a posterior estimate of Θ and the posterior estimate of Θ is equal to 1.2

Calculate the sample mean of the random sample.

A. Less than 1.25
B. At least 1.25, but less than 1.30
C. At least 1.30, but less than 1.35
D. At least 1.35, but less than 1.40
E. At least 1.40

Exam Continued on Next Page

25.

You are given the following information:

- Y follows a binomial distribution with parameters n and $p = \theta$.
- The prior distribution of θ follows a beta distribution with parameters $\alpha = 4$ and $\beta = 6$.
- A sample of size 15 is drawn from Y with $\sum y_i = 10$

Calculate the estimate of θ for the posterior distribution.

A. Less than 0.50
B. At least 0.50, but less than 0.60
C. At least 0.60, but less than 0.70
D. At least 0.70, but less than 0.80
E. At least 0.80

End of Examination

Spring 2014 Exam ST Revised Answer Key

Question	Answer
1	E
2	B
3	E
4	D
5	D
6	D
7	A
8	B
9	E
10	E
11	C
12	D
13	B
14	D
15	D
16	E
17	B
18	E
19	B
20	C
21	D
22	D
23	E
24	A
25	B