1. (8.75 points)

A company has the following paid loss data for its personal auto line of business. The company calculates losses limited to a per occurrence limit to estimate unpaid claim liabilities in two layers: limited to and excess of the limit.

Accident Year	Earned Premium ($000)	Cumulative Unlimited Paid Loss as of year end ($000)			Cumulative Limited Paid Loss as of year end ($000)		
		2015	2016	2017	2015	2016	2017
2015	1,000	350	576	643	321	496	535
2016	1,100		375	614		348	541
2017	1,210			444			393
Total		350	951	1,701	321	844	1,469

- Ultimate loss ratios are expected to be constant across all accident years.
- The limited severity relativity at ultimate is 0.8 across all accident years.
- Loglogistic curves were fit to personal auto industry data for both limited and unlimited triangles:

$$G_{Unlimited}(x) = \frac{x^{1.4}}{x^{1.4} + 6^{1.4}} \qquad G_{Limited}(x) = \frac{x^{1.6}}{x^{1.6} + 5^{1.6}}$$

  where *G* is the cumulative portion of ultimate losses paid and *x* is the average age in months. The curves do not require truncation.
- The company's unpaid claim liabilities as of December 31, 2016 were estimated to be $520,000.
- As of December 31, 2017, the Variance / Mean Parameter for total unpaid claims is 4,000, and parameter standard deviation is $60,000.

a. (1.25 points)

Estimate the total unpaid claims as of December 31, 2017 for limited losses using the LDF method with a loglogistic development pattern.

b. (1 point)

Calculate the limited severity relativities at 12, 24, and 36 months.

c. (0.75 point)

Calculate the cumulative excess age to ultimate loss development factors at 12, 24, and 36 months using the limited severity relativities from part b. above.

d. (1.5 points)

Estimate unpaid claims in the excess layer as of December 31, 2017 using the Cape Cod method and the excess LDFs derived in part c. above.

- The company selects unpaid claim liabilities as of December 31, 2017 equal to the sum of unpaid claim estimates from parts a. and d. above.
- The company incurred expenses equal to 20% of earned premium in 2017.
- Required capital is equal to 50% of unpaid claims.

e. (1.75 points)

Calculate the free cash flow to equity for calendar year 2017.

f. (1 point)

Calculate the independent risk coefficient of variation (CoV) for this line of business

g. (1.5 points)

The company also writes commercial auto which has unpaid claim liabilities of $1,000,000. Independent risk CoV for commercial auto is the same as for part f. above. The correlation between lines is 50% for internal risk and 100% for external risk. The following CoVs are also given:

	Personal Auto	Commercial Auto
Internal Risk	7%	9%
External Risk	5%	6%

Calculate the total CoV for the portfolio.

IQ Sample Q1

a)

ACA (months)	Limited % Paid	Paid Loss	Unpaid Claim
30	0.946	535	30
18	0.886	541	70
6	0.572	393	294
			394

Limited percentage paid$_{30}$ = $30^{1.6}/(30^{1.6} + 5^{1.6})$ = 0.946

Limited percentage paid$_{18}$ = $18^{1.6}/(18^{1.6} + 5^{1.6})$ = 0.886

Limited percentage paid$_{6}$ = $6^{1.6}/(6^{1.6} + 5^{1.6})$ = 0.572

Unpaid Claims$_{30}$ = Paid$_{30}$/lmtd % paid$_{30}$ – paid$_{30}$ = 535/.946-535 = 30

Unpaid Claims$_{18}$ = Paid$_{18}$/lmtd % paid$_{18}$ – paid$_{18}$ = 541/.886-541 = 70

Unpaid Claims$_{6}$ = Paid$_{6}$/lmtd % paid$_{6}$ – paid$_{6}$ = 393/.572-393 = 294

Total Unpaid Claims = 30 + 70 + 294 = **394**

Source: Clark p.59-64

b)

	ACA	Unlimited Pattern	Limited Pattern	Severity Relativity
Ult				0.800
2015	30	0.905	0.946	**0.836**
2016	18	0.823	0.886	**0.861**
2017	6	0.500	0.572	**0.916**

Unlimited Pattern$_{30}$ = $30^{1.4}/(30^{1.4}+6^{1.4})$ = .905

Unlimited Pattern$_{18}$ = $18^{1.4}/(18^{1.4}+6^{1.4})$ = .823

Unlimited Pattern$_{6}$ = $6^{1.4}/(6^{1.4}+6^{1.4})$ = .500

Limited Severity Relativity$_{30}$ = Limited$_{30}$ /Unlimited$_{30}$ * Ultimate = .946/.905*0.8 = **.836**

Limited Severity Relativity$_{18}$ = Limited$_{18}$ /Unlimited$_{18}$ * Ultimate = .886/.823*0.8 = **.861**

Limited Severity Relativity$_{6}$ = Limited$_{6}$ /Unlimited$_{6}$ * Ultimate = .572/.500*0.8 = **.916**

Source: Siewert p.228-229

c)

	Unlimited LDF	Severity Relativity	XS LDF
Ult		0.800	
2015	1.105	0.836	**1.352**
2016	1.215	0.861	**1.747**
2017	2.000	0.916	**4.754**

Unlimited LDF_{30} = 1/Unlimited Pattern: 1/.905 = 1.105

XS LDF_{30} = Unlimited LDF_{30}*(1- Ultimate Severity Relativity)/(1- Limited Severity $Relativity_{30}$)

= 1.105*(1-0.8)/(1-0.836) = **1.352**

XS LDF_{18} = Unlimited LDF_{18}*(1- Ultimate Severity Relativity)/(1- Limited Severity $Relativity_{18}$)

= 1.215*(1-0.8)/(1-0.861) = **1.747**

XS LDF_{6} = Unlimited LDF_{6}*(1- Ultimate Severity Relativity)/(1- Limited Severity $Relativity_{6}$)

= 2.000*(1-0.8)/(1-0.916) = **4.754**

Source: Siewert p.228-229

d)

AY	EP	XS LDF	Used EP	Paid Loss	ELR	Expected Loss	Cape Cod R_x
2015	1,000	1.352	740	108		143	37
2016	1,100	1.747	630	73		157	67
2017	1,210	4.754	255	51		173	137
	3,310		1,624	232	14.3%		**241**

Used EP_{30} = EP_{30} / XS LDF_{30} = 1,000/1.352 = 740

Used EP_{18} = EP_{18} / XS LDF_{18} = 1,100/1.747 = 630

Used EP_{6} = EP_{6} / XS LDF_{6} = 1,210/4.754 = 255

Paid $Loss_{30}$ = Cumulative Unlimited Paid $Loss_{30}$ – Cumulative Limited Paid $Loss_{30}$ = 643-535=108

Paid $Loss_{18}$ = Cumulative Unlimited Paid $Loss_{18}$ – Cumulative Limited Paid $Loss_{18}$ = 614-541=73

Paid $Loss_{6}$ = Cumulative Unlimited Paid $Loss_{6}$ – Cumulative Limited Paid $Loss_{6}$ = 444-393=51

ELR = Total Paid/Total EP = 232/1,624 = 14.3%

Expected $Loss_{30}$ = EP*ELR = 1,000*14.3% = 143

Expected $Loss_{18}$ = EP*ELR = 1,100*14.3% = 157

Expected $Loss_{6}$ = EP*ELR = 1,210*14.3% = 173

Cape Cod Unpaid $Claims_{30}$ = Expected $Loss_{30}$ * (1-1/ XS LDF_{30}) = 143*(1-1/1.352) = 37

Cape Cod Unpaid $Claims_{18}$ = Expected $Loss_{18}$ * (1-1/ XS LDF_{18}) = 157*(1-1/1.747) = 67

Cape Cod Unpaid Claims$_6$ = Expected Loss$_6$ * (1-1/ XS LDF$_6$) = 173*(1-1/4.754) = 137

Total Cape Cod Unpaid Claims = 37 + 67 + 137 = **241**

Source: Patrik p.454-460

e)

		YE 16	YE 17	CY 17
Premium				1,210
Expenses	20%			242=1,210*20%
Paid		951	1,701	750 = 1701-951
Unpaid Claim		520	635	115 = 635-520
Net Income				103
Chg Req Capital		260	317	57
FCFE				**46**

Paid$_{16}$ = 576+375=951

Paid$_{17}$ = 444+614+643=1,701

Unpaid Claim$_{17}$ = 241+394=635

Net Income = Premium – [Expenses+Paid+Unpaid Claim] = 1,210 – [242+750+115] = 103

Change in Required Capital = Unpaid Claim * 50%

FCFE = Net Income – Change in Required Capital = 103-57 = **46**

Source: Goldfarb p.19-21

f)

Unpaid Claim	635,000
Process Variance	2,540,000,000
Parameter Variance	3,600,000,000
StDev	78,358
CoV	**12.3%**

Process Variance = Unpaid Claim*Variance/Mean = 635,000*4,000 = 2.54B

Parameter Variance = Parameter Std Dev^2 = 60,000^2 = 3.6B

StDev = (Process Variance + Parameter Variance)$^{0.5}$ = (2.54B+3.6B)$^{0.5}$ = 78,358

CoV = 78,358/635,000 = **12.3%**

Source: Clark p.65-69

g)

	Unpaid Claims	Share	Independent	Internal	External	
Personal	635,000	39%	12.3%	7.0%	5.0%	
Commercial	1,000,000	61%	12.3%	9.0%	6.0%	
Correlation			0%	50%	100%	
Total			8.9%	7.3%	5.6%	**12.8%**

Independent = $[(.39*.123)^2+(.61*.123)^2+2*0*.39*.61*.123*.123]^{0.5}$= 8.9%

Internal = $[(.39*.07)^2+(.61*.09)^2+2*0.5*.39*.61*.07*.09]^{0.5}$= 7.3%

External = $[(.39*.05)^2+(.61*.06)^2+2*1.0*.39*.61*.05*.06]^{0.5}$= 5.6%

Total = $(\text{independent}^2+\text{internal}^2+\text{external}^2)^{0.5} = (.089^2+.073^2+.056^2)^{0.5}$ = **12.8%**

Source: Marshall p.21