Exam MAS-I

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Expertise. Insight. Solutions.

Canadian Institute of Actuaries / Institut canadien des actuaires

October 24, 2019

MAS-I

Modern Actuarial Statistics I

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 90 point examination consists of 45 multiple choice questions each worth 2 points.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Fall 2019 and that the exam name is MAS-I.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS MAS-I" included in your exam packet.

   - Candidates should not interpolate in the tables unless explicitly instructed to do so in the problem, rather, a candidate should round the result that would be used to enter a given table to the same level of precision as shown in the table and use the result in the table that is nearest to that indicated by rounded result. For example, if a candidate is using the Tables of the Normal Distribution to find a significance level and has a Z value of 1.903, the candidate should round to 1.90 to find cumulative probability in the Normal table.

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 7, 2019.

END OF INSTRUCTIONS

1.

Losses follow a memoryless distribution with mean 1,000. Each loss is insured and subject to a deductible of 500.

Calculate the average insurance payment made on losses that exceed the deductible.

A. Less than 500
B. At least 500, but less than 700
C. At least 700, but less than 900
D. At least 900, but less than 1100
E. At least 1100

CONTINUED ON NEXT PAGE

2.

You are given the following information about waiting times at a subway station:

- Subway trains arrive at a Poisson rate of 20 per hour
- 30% of the trains are Express and 70% are Local
- The arrival times of each train are independent
- An Express train gets you to work in 18 minutes, and a Local train gets you there in 30 minutes
- You always take the first train to arrive and you get to the office in X_1 minutes from the time you arrive at the subway station
- Your coworker always takes the first Express train to arrive and he gets to the office in X_2 minutes from the time he arrives at the subway station

Calculate the expected value of $X_1 - X_2$.

A. Less than -2.0
B. At least -2.0, but less than -1.0
C. At least -1.0, but less than 0.0
D. At least 0.0, but less than 1.0
E. At least 1.0

CONTINUED ON NEXT PAGE

3.

You are given the following information about an online retailer:

- Orders are placed on the website according to a homogeneous Poisson process with mean 50 per hour
- The number of items purchased in each order is independent and has the following distribution:

Number of Items	Probability
1	0.50
2	0.40
3	0.08
4	0.02

Calculate the variance of the total number of items purchased in a four-hour period.

A. Less than 100
B. At least 100, but less than 300
C. At least 300, but less than 500
D. At least 500, but less than 700
E. At least 700

CONTINUED ON NEXT PAGE

4.

A building is powered by three generators with independent lifetimes, each following an exponential distribution with mean of one year. All three generators are started at the same time.

Let T be the time in years between the first and last generator failure.

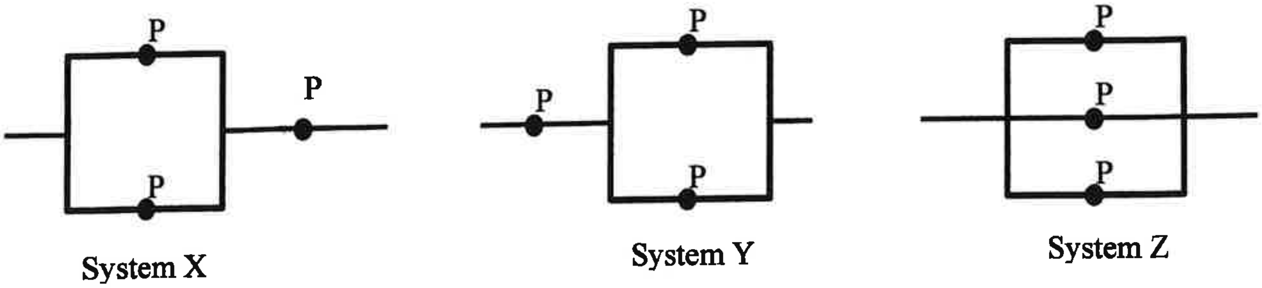
Calculate E[T].

A. Less than 0.8
B. At least 0.8, but less than 1.2
C. At least 1.2, but less than 1.6
D. At least 1.6, but less than 2.0
E. At least 2.0

CONTINUED ON NEXT PAGE

5.

You are given the following bridge systems:



- All three systems X, Y, and Z are built with independent and identical components, P
- The reliability of System i is denoted as r_i, for $i = X, Y, Z$

Determine which of the following best describes the reliabilities of these systems.

A. $r_X = r_Y > r_Z$
B. $r_X < r_Y < r_Z$
C. $r_X = r_Y = r_Z$
D. $r_X = r_Y < r_Z$
E. $r_X > r_Y > r_Z$

CONTINUED ON NEXT PAGE

6.

You are given a system which consists of the following minimal cut sets:

$$\{1\},\{2,3\},\{4\},\{5,6\}$$

The system is comprised of independent and identically distributed components, each with reliability 0.9.

Calculate the lower bound of the reliability of the system by using the first two inclusion-exclusion bounds from the method of inclusion and exclusion.

A. Less than 0.75
B. At least 0.75, but less than 0.77
C. At least 0.77, but less than 0.79
D. At least 0.79, but less than 0.81
E. At least 0.81

CONTINUED ON NEXT PAGE

7.

You are given the following information regarding a series system with two independent machines, X and Y:

- The hazard rate function, in years, for machine i is denoted by $r_i(t)$
- $r_x(t) = \ln(1.06)$, for $x > 0$
- $r_y(t) = \frac{1}{20-t}$, for $0 < y < 20$
- Both machines are currently three years old

Calculate the probability that the system fails when the machines are between five and nine years old.

A. Less than 0.305
B. At least 0.305, but less than 0.315
C. At least 0.315, but less than 0.325
D. At least 0.325, but less than 0.335
E. At least 0.335

CONTINUED ON NEXT PAGE

8.

You are given:

- Mary plays a game repeatedly
- Each game ends with her either winning or losing
- Mary's chances of winning her next game depends on the outcome of the prior game
    - o P[Winning after a win] = min(80%, P[Winning prior game] + 10%)
    - o P[Winning after a loss] = 40%
- Mary just played her 10th game and lost.

Calculate the probability that Mary will lose her 13th game.

A. Less than 40%
B. At least 40%, but less than 45%
C. At least 45%, but less than 50%
D. At least 50%, but less than 55%
E. At least 55%

CONTINUED ON NEXT PAGE

9.

You are given the following Markov chain transition probability matrix:

$$P = \begin{bmatrix} 0.0 & 0.3 & 0.3 & 0.4 & 0.0 \\ 0.4 & 0.0 & 0.0 & 0.6 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.5 & 0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Determine the number of recurrent states in this Markov chain.

A. 1
B. 2
C. 3
D. 4
E. 5

CONTINUED ON NEXT PAGE

10.

Ann and Beatrice are playing a game with multiple rounds:

- At the end of each round, the loser pays 1 coin to the winner
- Each player begins with 5 coins and plays until one of them has no coins remaining
- Beatrice is more experienced at the game, and her probability of winning each round is 0.75
- Each round is independent

Calculate the probability that Beatrice has no coins remaining at the end of the game.

A. Less than 0.01
B. At least 0.01, but less than 0.03
C. At least 0.03, but less than 0.05
D. At least 0.05, but less than 0.07
E. At least 0.07

CONTINUED ON NEXT PAGE

11.

A company with extensive experience is using the Illustrative Life Table (ILT) to price life insurance, but is considering switching to a Modified ILT.

The Modified ILT is identical to the original ILT for Ages beyond 25, but has adjustments for Ages 25 and prior.

The following is an excerpt from the adjusted section of the Modified ILT:

Age x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$
20	9,617,802	1.34	16.5112	65.40
21	9,604,896	1.37	16.4640	68.08
22	9,591,695	1.10	16.4144	70.89
23	9,581,169	1.14	16.3572	74.12
24	9,570,288	0.86	16.2971	77.52
25	9,562,017	0.91	16.2290	81.38

This company only sells whole life policies of 10,000. Someone Age 23 is considering purchasing a whole life policy.

Calculate the change in annual level benefit premium the company charges this customer by using the Modified ILT instead of the original ILT.

A. Less than -0.25
B. At least -0.25, but less than 0.00
C. At least 0.00, but less than 0.25
D. At least 0.25, but less than 0.50
E. At least 0.50

CONTINUED ON NEXT PAGE

12.

An insurance company is using the Illustrative Life Table to price a block of life insurance policies covering 5,000 people aged 50.

Calculate the lower bound of a 90% confidence interval for the number of deaths in this block during the next 15 years, using a normal approximation.

A. Less than 745
B. At least 745, but less than 755
C. At least 755, but less than 765
D. At least 765, but less than 775
E. At least 775

CONTINUED ON NEXT PAGE

13.

An actuary is using the inversion method to simulate the waiting time until the 5th event of a Poisson process with a rate $\lambda = 1$.

Five random draws from $U(0, 1)$ are provided below:

0.2, 0.7, 0.8, 0.3, 0.5

Calculate the simulated waiting time until the 5th event.

A. Less than 2.5
B. At least 2.5, but less than 3.5
C. At least 3.5, but less than 4.5
D. At least 4.5, but less than 5.5
E. At least 5.5

CONTINUED ON NEXT PAGE

14.

You are given the following information:

- X is a random variable from a single-parameter Pareto distribution with $\alpha = 5$ and unknown θ
- $\bar{x}$ is the sample mean of n independent observations from this distribution
- $c\bar{x}$ is an unbiased estimator of θ

Calculate c.

A. Less than 1.5
B. At least 1.5, but less than 2.5
C. At least 2.5, but less than 3.5
D. At least 3.5, but less than 4.5
E. At least 4.5

CONTINUED ON NEXT PAGE

15.

An actuary obtained the following random sample from a normal distribution with mean μ and variance σ^2:

$$1.16, \quad -6.78, \quad 4.04, \quad 7.68, \quad 0.85$$

Calculate the minimum variance unbiased estimate of σ^2.

A. Less than 15
B. At least 15, but less than 20
C. At least 20, but less than 25
D. At least 25, but less than 30
E. At least 30

CONTINUED ON NEXT PAGE

16.

Suppose that $X_1, \dots, X_{10}$ is a random sample from a normal distribution with:

$$\sum_{i=1}^{10} X_i = 100 \quad \text{and} \quad \sum_{i=1}^{10} X_i^2 = 2000$$

The parameters of this distribution are estimated using the method of moments with raw moments only.

Calculate the estimated variance of this distribution.

A. Less than 120
B. At least 120, but less than 140
C. At least 140, but less than 160
D. At least 160, but less than 180
E. At least 180

CONTINUED ON NEXT PAGE

17.

You are given the following information:

- An insurer has observed the following nine losses:
  500, 600, 750, 880, 940, 1000, 1050, 1060, 1400
- A uniform kernel function with bandwidth 75 is used to estimate the distribution of loss sizes
- $\hat{F}(Y)$ is the kernel-smoothed estimate of the cumulative distribution function

Calculate $\hat{F}(1{,}000)$.

A. Less than 0.55
B. At least 0.55, but less than 0.60
C. At least 0.60, but less than 0.65
D. At least 0.65, but less than 0.70
E. At least 0.70

CONTINUED ON NEXT PAGE

18.

You are given the following information about the distribution of losses:

- Losses follow an exponential distribution with mean θ.
- Insurance payments for each loss are subject to a deductible of 500 and a maximum payment of 30,000:

  Insurance payment = min[30,000, max(0, loss – 500)]
- No insurance payments are made for losses less than 500
- A random sample of five insurance payments are drawn:

  1,000 4,900 7,000 19,500 30,000

Calculate the maximum likelihood estimate of θ.

A. Less than 12,500
B. At least 12,500, but less than 13,500
C. At least 13,500, but less than 14,500
D. At least 14,500, but less than 15,500
E. At least 15,500

CONTINUED ON NEXT PAGE

19.

You are testing the following hypotheses about a random variable, X:

- H_0: X is uniformly distributed on [0,16]
- H_1: X is uniformly distributed on [8,16]

You used a single observation and the best critical region with power of 0.95 to evaluate this hypothesis.

Calculate the probability of a Type I error.

A. Less than 0.40
B. At least 0.40, but less than 0.50
C. At least 0.50, but less than 0.60
D. At least 0.60, but less than 0.70
E. At least 0.70

CONTINUED ON NEXT PAGE

20.

An insurance company has classified claims into 5 categories based on their severity.

The null hypothesis H_0 assumes the numbers of claims for Categories 1, 2, 3, 4 and 5 appear in the following ratios:

$$12:8:6:4:1$$

In 2017, the insurance company recorded the numbers of claims as follows:

Category	# of Claims
1	1172
2	829
3	605
4	347
5	102

Calculate the Chi-square goodness-of-fit statistic.

A. Less than 5.5
B. At least 5.5, but less than 6.5
C. At least 6.5, but less than 7.5
D. At least 7.5, but less than 8.5
E. At least 8.5

CONTINUED ON NEXT PAGE

21.

An insurer is estimating the impact of a loss mitigation program. They ran an experiment to evaluate the severities of five losses before and after the program was instituted:

	A	B	C	D	E
Original Severity	400	800	1200	2000	5000
New Severity	280	500	1235	1600	4800

A paired t-test with the following hypotheses was used to evaluate the effectiveness of this program:

- H_0: The program had no impact on losses
- H_1: The program was able to reduce losses

Calculate the smallest significance level at which one would reject the null hypothesis.

A. Less than 1.0%
B. At least 1.0%, but less than 2.5%
C. At least 2.5%, but less than 5.0%
D. At least 5.0%, but less than 10.0%
E. At least 10.0%

CONTINUED ON NEXT PAGE

22.

You are given the following information about a sample, $X_1 \ldots X_n$:

- X_i's are all mutually independent
- $X_i \sim \text{Gamma}(\alpha_i, \theta)$, for $i = 1,2,\ldots n$
- $\alpha_i = \frac{1}{n}$ for all i
- $Y = \sum_{i=1}^{n} X_i$

Calculate the probability that $Y > \theta$.

A. Less than 0.2
B. At least 0.2, but less than 0.4
C. At least 0.4, but less than 0.6
D. At least 0.6, but less than 0.8
E. At least 0.8

CONTINUED ON NEXT PAGE

23.

You are given the following information about losses covered by an insurance company:

- Individual losses follow a lognormal distribution with ($\mu = 8,\ \sigma = 1$)
- For insurance payments, an ordinary deductible of 1,000 per loss applies
- Losses below the deductible are not reported to the insurance company

Calculate the mean payment made by the insurance company.

A. Less than 3,900
B. At least 3,900, but less than 4,200
C. At least 4,200, but less than 4,500
D. At least 4,500, but less than 4,800
E. At least 4,800

CONTINUED ON NEXT PAGE

24.

A student would like to estimate the upper bound of a uniform distribution, $U(0, \theta)$ using the method of maximum likelihood.

The true value of θ is 10.

N is the minimum sample size required such that the absolute value of the bias of the estimator is less than 0.1.

Calculate N.

A. Less than 20
B. At least 20, but less than 40
C. At least 40, but less than 60
D. At least 60, but less than 80
E. At least 80

CONTINUED ON NEXT PAGE

25.

A bank uses a logistic model to estimate the probability of clients defaulting on a loan, and it comes up with the following parameter estimates:

i	Variable	β_i
0	Intercept	-1.6790
1	Income (in 000's)	-0.0294
2	Student [Yes]	-0.3870
3	Number of credit cards	0.7710

The following four clients applied for loans from the bank:

Client	Income	Student	# of credit cards
1	25,000	Y	1
2	10,000	Y	3
3	20,000	N	0
4	75,000	N	3

The bank will reject any loan if the probability of default is greater than 10%.

Calculate the number of clients whose loan requests are rejected.

A. 0
B. 1
C. 2
D. 3
E. 4

CONTINUED ON NEXT PAGE

26.

A number of candidate models were fit using the following variables:

- An intercept term
- Variable A – a Yes/No indicator
- Variable B – a Yes/No indicator
- An interaction of Variables A and B

There are four observations, which were arranged into the following design matrix:

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

This data was fit using three different link functions:

I. Identity
II. Inverse
III. Log

The predicted values, given below, were the same under all three models:

$$\hat{Y} = \begin{bmatrix} 0.50 \\ 0.80 \\ 0.40 \\ 0.70 \end{bmatrix}$$

Determine for which of the above link functions the estimated interaction coefficient is non-zero.

A. Identity, Inverse and Log
B. Identity and Inverse only
C. Identity and Log only
D. Inverse and Log only
E. The answer is not given by (A), (B), (C) or (D)

CONTINUED ON NEXT PAGE

27.

An actuary is asked to model a non-negative response variable and requires that the model form produces an unbiased estimate.

Determine which error structure and link function combination would be the best choice for the modelling request.

A. Poisson and Identity
B. Compound Poisson-Gamma and Log
C. Normal and Identity
D. Gamma and Log
E. Poisson and Log

CONTINUED ON NEXT PAGE

28.

Determine which one of the following statements about Principal Component Regression (PCR) is FALSE.

A. When performing PCR it is recommended that the modeler standardize each predictor prior to generating the principal components.
B. PCR is useful for performing feature selection.
C. PCR assumes that the directions in which features show the most variation are the directions that are associated with the target.
D. PCR can reduce overfitting.
E. The first principal component direction of the data is that along which the observations vary the most.

CONTINUED ON NEXT PAGE

29.

An actuary has a dataset with four observations and wants to use Leave-One-Out Cross Validation (LOOCV) to determine which one of two competing models fits the data better. The model preference will be based on minimizing the mean squared error.

The values of the dependent variable are:

$$y = [y_1, y_2, y_3, y_4] = [1.55, 1.55, 1.60, 1.95]$$

Corresponding fitted values under each model and training data subset are:

Training Obs. Used	Model 1 $\hat{y}_1$	Model 1 $\hat{y}_2$	Model 1 $\hat{y}_3$	Model 1 $\hat{y}_4$	Model 2 $\hat{y}_1$	Model 2 $\hat{y}_2$	Model 2 $\hat{y}_3$	Model 2 $\hat{y}_4$
1,2,3	1.50	1.60	1.20	1.80	1.60	1.70	1.60	Z
1,2,4	2.00	1.50	1.10	1.90	1.80	1.40	1.30	1.70
1,3,4	1.75	1.55	1.70	2.10	1.40	1.30	1.50	1.95
2,3,4	1.70	1.65	1.60	2.00	1.60	1.70	1.20	2.00

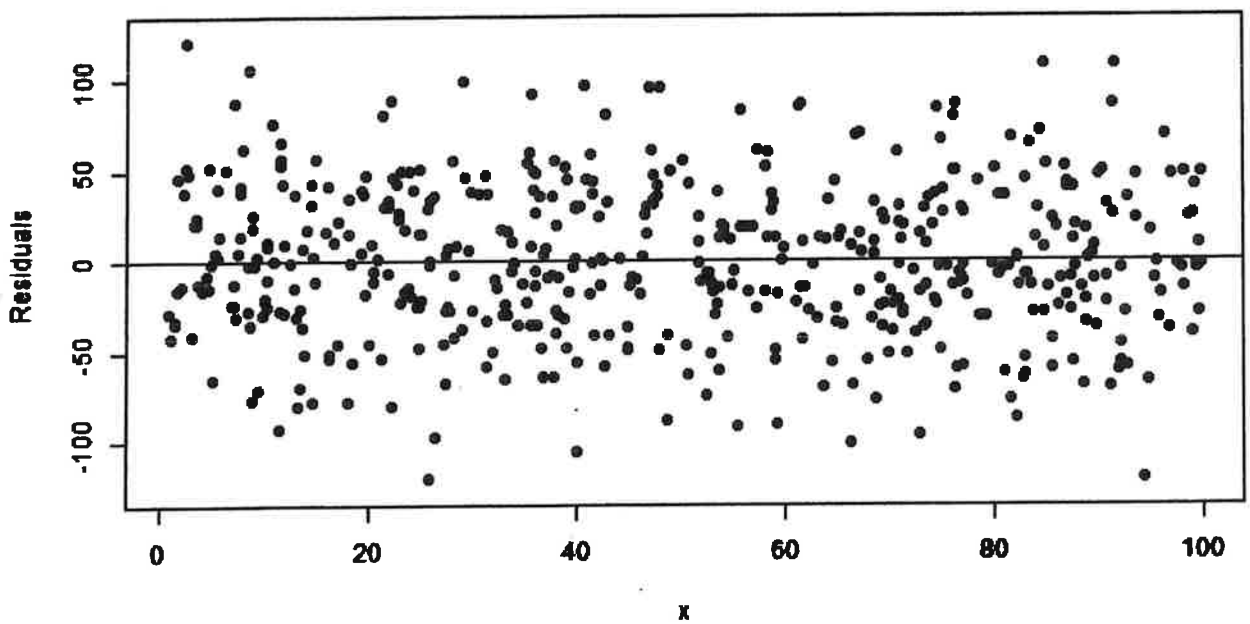
Calculate the maximum value of Z for which the actuary will prefer Model 2.

A. Less than 1.5
B. At least 1.5, but less than 1.8
C. At least 1.8, but less than 2.1
D. At least 2.1, but less than 2.4
E. At least 2.4

CONTINUED ON NEXT PAGE

30.

An actuary has a data set with one predictor variable, X, and a response variable, Y. She divides the data set randomly into training and testing sets. The training subset is used to fit an ordinary least squares regression. In order to evaluate the fit, she plots the residuals from the model against the independent variable, X:



Determine which of the following enhancements to the model would most likely improve the fit to the testing data set.

A. Linear Spline
B. Local Regression
C. Polynomial Regression
D. Step Function
E. There is no evidence that any of (A), (B), (C), (D) will improve the fit

CONTINUED ON NEXT PAGE

31.

In order to predict individual candidates' test scores a regression was performed using one independent variable, Hours of Study, plus an intercept. Below is a partial table of data and model results:

Candidate	Test Score	Hours of Study	Leverage	Standardized Residuals
1	2,041	538	0.6205	-1.3477
2	2,502	548	0.2018	-0.4171
3	2,920	528	0.6486	-1.1121
4	2,284	608	0.2807	1.1472

Calculate the number of observations above that are influential using Cook's Distance with a unity threshold.

A. 0
B. 1
C. 2
D. 3
E. 4

CONTINUED ON NEXT PAGE

32.

You have three competing GLMs that each predict the number of claims under an insurance policy, and are evaluating the models using AIC and BIC. All models are trained on the same dataset of 300 observations. These models are summarized below:

Model	Likelihood	Number of Parameters
1	0.0456	4
2	0.0567	5
3	0.0575	6

The following are three statements about the fit of these models:

I. Model #1 is best based on BIC
II. Model #2 is best based on AIC
III. Model #3 is best based on BIC

Determine which of the above statements are true.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C), or (D)

CONTINUED ON NEXT PAGE

33.

You have a sample of five independent observations, $x_1 \ldots x_5$, each with exponential distribution:

$$f(x_i|\theta_i) = \frac{1}{\theta_i}\exp\left(-\frac{x_i}{\theta_i}\right)$$

- You are fitting this data to a model with $\theta_i = \theta$, for all i, using maximum likelihood estimation:

$$f(x_i|\theta) = \frac{1}{\theta}\exp\left(-\frac{x_i}{\theta}\right)$$

- The five observed values are: 100, 100, 500, 800, 1000
- The deviance of the model, D, is equal to twice the difference between the log-likelihood of the saturated model and the log-likelihood of the fitted model.

Calculate D.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

CONTINUED ON NEXT PAGE

34.

You are given the following statements comparing k-fold cross validation (with $k < n$) and Leave-One-Out Cross Validation (LOOCV), used on a GLM with log link and gamma error.

I. k-fold validation has a computational advantage over LOOCV
II. k-fold validation has an advantage over LOOCV in bias reduction
III. k-fold validation has an advantage over LOOCV in variance reduction

Determine which of the above statements are true.

A. None are true
B. I and II only
C. I and III only
D. II and III only
E. The answer is not given by (A), (B), (C), or (D)

CONTINUED ON NEXT PAGE

35.

Two variables, X and Y, exhibit the following relationship:

$$Y_i = 1.5X_i + 2 + \varepsilon_i$$

where ε_i is a standard normal random variable, and each ε_i is mutually independent.

For some sample of data, an actuary uses ordinary least squares regression of the form:

$$Y_i = \hat{\beta}_1 X_i + \hat{\beta}_0 + e_i$$

to estimate the relationship. The following parameter estimates were formed:

Parameter	Estimate
$\hat{\beta}_0$	2.5
$\hat{\beta}_1$	1.3

Calculate the bias of the estimate.

A. Less than -0.1
B. At least -0.1, but less than 0.1
C. At least 0.1, but less than 0.3
D. At least 0.3
E. There is not enough information provided to calculate the bias.

CONTINUED ON NEXT PAGE

36.

You are given the following three statements regarding shrinkage methods in linear regression:

I. As tuning parameter, λ, increases towards ∞, the penalty term has no effect and a ridge regression will result in the unconstrained estimates.

II. For a given dataset, the number of variables in a lasso regression model will always be greater than or equal to the number of variables in a ridge regression model.

III. The issue of selecting a tuning parameter for a ridge regression can be addressed with cross-validation.

Determine which of the above statements are true.

A. I only
B. II only
C. III only
D. I, II and III
E. The answer is not given by (A), (B), (C) or (D)

CONTINUED ON NEXT PAGE

37.

For a set of data with 40 observations, 2 predictors (X_1 and X_2), and one response (Y), the residual sum of squares has been calculated for several different estimates of a linear model with an intercept. Only integer values from 1 to 3 were considered for estimates of β_0 (the intercept), β_1 and β_2.

The grid below shows the residual sum of squares for every combination of the parameter estimates, after standardization:

		$\hat{\beta}_0 = 1$			$\hat{\beta}_0 = 2$			$\hat{\beta}_0 = 3$		
		$\hat{\beta}_2$			$\hat{\beta}_2$			$\hat{\beta}_2$		
		1	2	3	1	2	3	1	2	3
$\hat{\beta}_1$	1	3,924	1,977	1,250	3,949	1,822	1,174	3,784	1,671	1,107
	2	1,858	1,141	711	1,907	1,187	717	1,827	1,128	668
	3	1,386	822	369	1,363	711	349	1,294	700	344

Let $\hat{\beta}_i^R$ be the estimate of β_i using a ridge regression with budget parameter $s = 5$. Assume the intercept is not subject to the budget parameters.

Calculate the value of $\hat{\beta}_0^R + \hat{\beta}_1^R + \hat{\beta}_2^R$.

A. Less than 6
B. 6
C. 7
D. 8
E. Greater than 8

CONTINUED ON NEXT PAGE

38.

A modeler creates a cubic spline model with Property Claim Frequency as the response variable and Age of Building Construction as a continuous predictor. The modeler puts knots in at Age = {10, 20, 30, 50}.

The modeler believes that she is overfitting on the ends of the distribution and decides to impose an additional constraint that the curve before the first knot and after the last knot will be linear.

Calculate the number of degrees of freedom used by this new model.

A. 3
B. 4
C. 5
D. 6
E. 7

CONTINUED ON NEXT PAGE

39.

An actuary has a dataset with one dependent variable, Y, and five independent variables $(X_1, X_2, X_3, X_4, X_5)$. She is trying to determine which subset of the predictors best fits the data, and is using a Forward Stepwise Selection procedure with no stopping rule. Below is a subset of the potential models:

Model	Dependent variable	RSS	Independent variable	p-value
1	Y	9,823	X_1	0.0430
			X_2	0.0096
2	Y	7,070	X_1	0.0464
			X_2	0.0183
			X_3	0.0456
3	Y	6,678	X_1	0.0412
			X_2	0.0138
			X_4	0.0254
4	Y	4,800	X_1	0.0444
			X_2	0.0548
			X_5	0.0254
5	Y	3,475	X_1	0.0333
			X_2	0.0214
			X_3	0.0098
			X_4	0.0274
			X_5	0.0076

The procedure just selected Model 1 as the new candidate model.

Determine which of the following independent variable(s) will be added to the model in the next iteration of this procedure.

A. No variables will be added
B. X_3 only
C. X_4 only
D. X_5 only
E. X_3, X_4 and X_5

CONTINUED ON NEXT PAGE

40.

You have p = 10 independent variables and would like to select a linear model to fit the data using the following two procedures:

- Best Subset Selection (BSS)
- Forward Stepwise Selection (FSS)

Let N_i be the maximum number of models fit by model selection procedure, *i*.

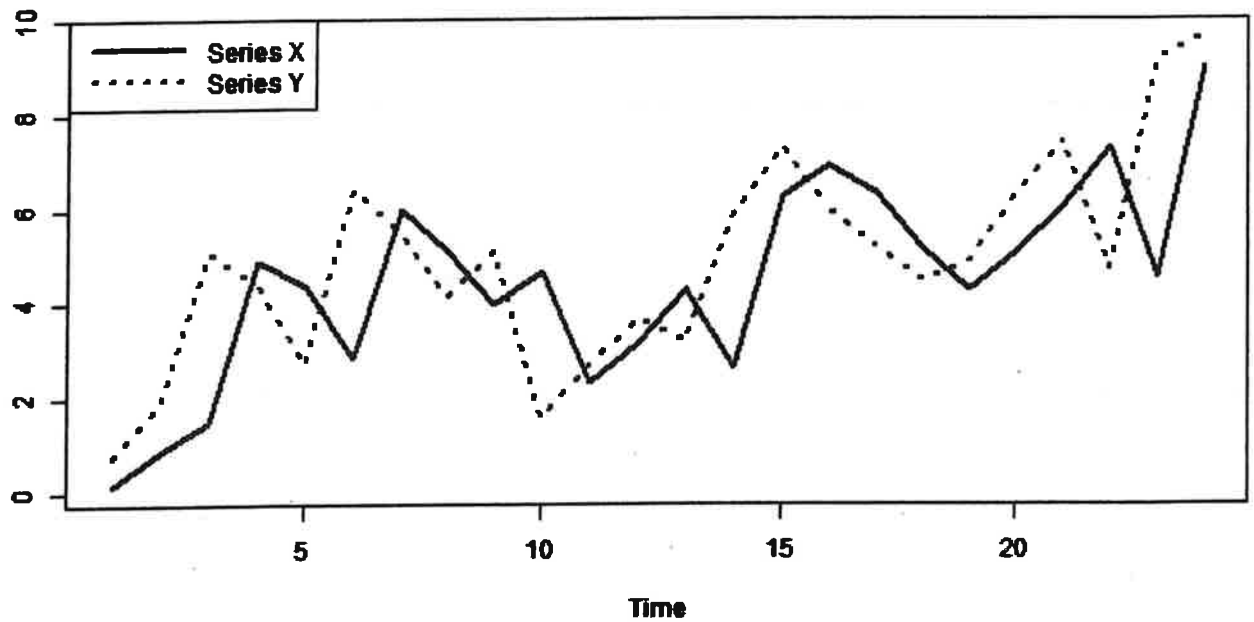
Calculate the ratio $\frac{N_{FSS}}{N_{BSS}}$.

A. Less than 0.005
B. At least 0.005, but less than 0.010
C. At least 0.010, but less than 0.050
D. At least 0.050, but less than 0.100
E. At least 0.100

CONTINUED ON NEXT PAGE

41.

Two time series (X and Y) are shown in the graph below:



The autocorrelation variance function (acvf) and cross covariance function (ccvf) are estimated at lags 0, 1, 2 & 3 in the table below.

Lag	acvf(x)	acvf(y)	ccvf(x,y)
0	4.165	4.655	2.626
1	1.594	1.960	3.422
2	1.123	0.518	0.950
3	0.478	1.064	0.389

Calculate the sample lag 1 cross-correlation.

A. Less than 0.30
B. At least 0.30, but less than 0.55
C. At least 0.55, but less than 0.80
D. At least 0.80, but less than 1.05
E. At least 1.05

CONTINUED ON NEXT PAGE

42.

An actuary uses four separate models to fit a time series. All models have mean $\mu_x = 0$.

- Model 1: A random walk model with no drift
- Model 2: A stationary autoregressive process of order 1 with a root of the characteristic equation of the backwards shift operator equal to 3
- Model 3: A stationary autoregressive process of order 1 with a root of the characteristic equation of the backwards shift operator equal to 2
- Model 4: A non-stationary autoregressive process of order 1 with a root of the characteristic equation of the backwards shift operator greater than 0

The most recent values of x at time t, x_t, are given in the table below:

t	x_t
4	2
5	1
6	4

Determine which model will result in the smallest predicted values of x_7.

A. Model 1
B. Model 2
C. Model 3
D. Model 4
E. There is not enough information given to determine the correct answer.

CONTINUED ON NEXT PAGE

43.

You are given the following ARMA(p, q) model:

$$x_t = \left(\frac{3}{2}\right)x_{t-1} - \left(\frac{1}{2}\right)x_{t-2} + w_t - w_{t-1} + \left(\frac{1}{4}\right)w_{t-2}$$

Determine (p, q) and whether the model is stationary and/or invertible.

A. p = 2, q = 2, Stationary, Invertible
B. p = 3, q = 3, Stationary, Not Invertible
C. p = 3, q = 3, Not Stationary, Not Invertible
D. p = 2, q = 2, Not Stationary, Invertible
E. p = 2, q = 2, Not Stationary, Not Invertible

CONTINUED ON NEXT PAGE

44.

You are given the following annual sales totals for a department store.

Year	Sales
2013	400
2014	375
2015	410
2016	420
2017	410
2018	525

Calculate the sample lag 2 autocorrelation.

A. Less than 0.00
B. At least 0.00, but less than 0.05
C. At least 0.05, but less than 0.10
D. At least 0.10, but less than 0.15
E. At least 0.15

CONTINUED ON NEXT PAGE

45.

A department store's annual sales, x_t, is modeled as an AR(3) process given by:

$$x_t = 450 - 0.9x_{t-1} + x_{t-3} + w_t$$

where w_t is white noise, with:

- $E(w_t) = 0$
- $Var(w_t) = \sigma^2$

You are given the following historical annual sales totals for this store:

Year	Sales
2013	400
2014	375
2015	410
2016	420
2017	410
2018	525

Calculate the two-step-ahead forecast value, x_{2020}.

A. Less than 400
B. At least 400, but less than 450
C. At least 450, but less than 500
D. At least 500, but less than 550
E. At least 550

END OF EXAMINATION

Fall 2019 Final Answer Key

Number	Answer Key
1	D
2	E
3	D
4	C
5	D
6	C
7	D
8	D
9	E
10	A
11	A
12	B
13	C
14	A
15	D
16	A
17	C
18	E
19	B
20	D
21	C
22	B
23	D
24	E
25	D
26	D
27	E
28	B
29	D
30	E
31	C
32	A
33	C
34	C
35	B & E
36	C
37	B
38	B & D
39	D
40	D
41	C
42	B
43	D
44	B
45	D